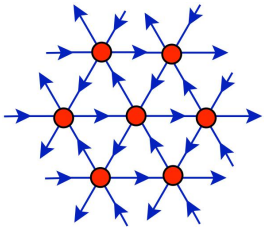
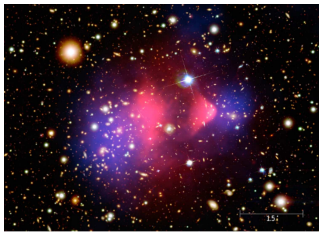
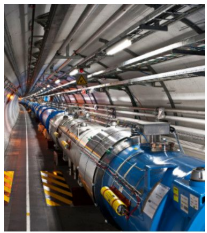
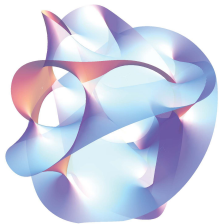
Broader applications of lattice field theory

David Schaich (University of Liverpool)





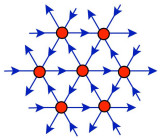




DiRAC Science Day, Liverpool, 12 December 2023

david.schaich@liverpool.ac.uk www.davidschaich.net

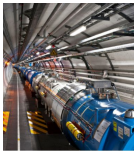
Overview and plan



Lattice field theory is a broadly applicable tool
to study strongly coupled quantum field theories

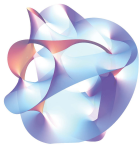
Composite dark matter and gravitational waves

[RAC16 2024–2025]



Near-conformal composite Higgs

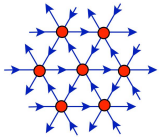
[RAC13 2021–2022; RAC15 2023–2024]



Supersymmetry and holographic duality

[RAC14 2022–2023]

Overview and plan



Lattice field theory is a broadly applicable tool
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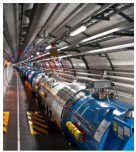
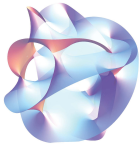
Composite dark matter and gravitational waves
[arXiv:2006.16429, arXiv:2212.09199, arXiv:2311.10243]



Near-conformal composite Higgs
[arXiv:2007.01810, arXiv:2106.13534, arXiv:2305.03665]

Supersymmetry and holographic duality
[arXiv:2208.03580, arXiv:2303.13880, arXiv:2312.04980]



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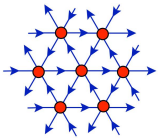

Lattice field theory is a broadly applicable tool
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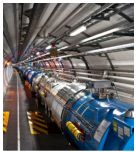
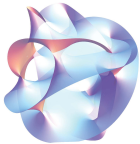


Near-conformal composite Higgs

Supersymmetry and holographic duality

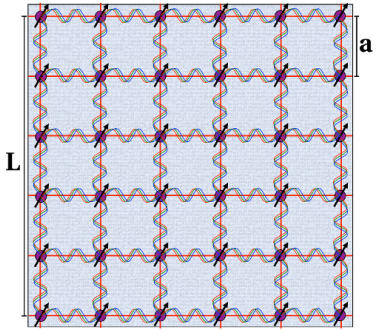


Interaction encouraged — complete coverage unnecessary

Lattice quantum field theory in a nutshell

Formally $\langle \mathcal{O} \rangle = \frac{1}{\mathcal{Z}} \int \mathcal{D}\Phi \;\; \mathcal{O}(\Phi) \;\; e^{-S[\Phi]}$

Regularize by formulating theory in finite, discrete, euclidean space-time
↖ Gauge invariant, non-perturbative, d-dimensional



P. Vranas LLNL

Spacing between lattice sites ("a")
$\longrightarrow$ UV cutoff scale $1/a$

Remove cutoff: $a \to 0 \quad (L/a \to \infty)$

Discrete $\longrightarrow$ continuous symmetries ✓

Numerical lattice field theory calculations

High-performance computing $\longrightarrow$ evaluate up to $\sim$billion-dimensional integrals
(Dirac operator as $\sim 10^9 \times 10^9$ matrix)

Results to be shown, and work in progress, require state-of-the-art resources

Many thanks to USQCD–DOE, DiRAC–STFC–UKRI, and computing centres!

Lassen @Livermore

USQCD @Fermilab

DiRAC @Cambridge

Barkla @UoL

Numerical lattice field theory calculations

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Importance sampling Monte Carlo

Standard algorithms sample field configurations with probability $\frac{1}{\mathcal{Z}} e^{-S[\Phi]}$

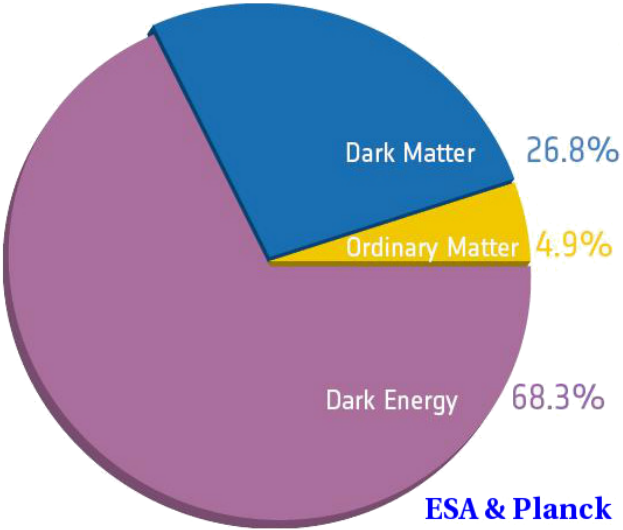
$$\langle \mathcal{O} \rangle = \frac{1}{\mathcal{Z}} \int \mathcal{D}\Phi \;\; \mathcal{O}(\Phi) \;\; e^{-S[\Phi]} \longrightarrow \frac{1}{N} \sum_{i=1}^{N} \mathcal{O}(\Phi_i) \;\; \text{with stat. uncertainty} \;\; \propto \frac{1}{\sqrt{N}}$$

Application: Dark matter

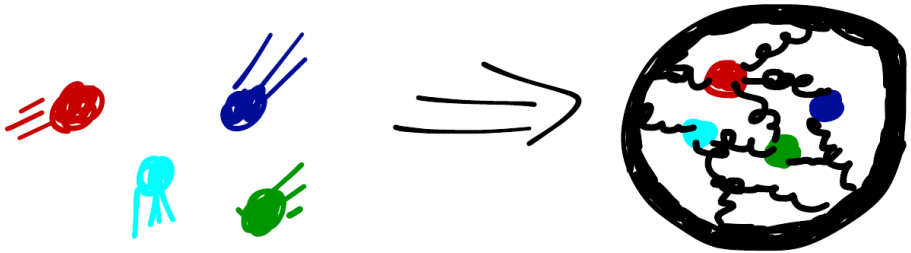
Consistent gravitational evidence from kiloparsec to Gpc scales

$$\frac{\Omega_{\text{dark}}}{\Omega_{\text{ordinary}}} \approx 5 \quad \ldots \text{not } 10^5 \text{ or } 10^{-5}$$

Explained by non-gravitational interactions in the early universe



Composite dark-sector phase transition



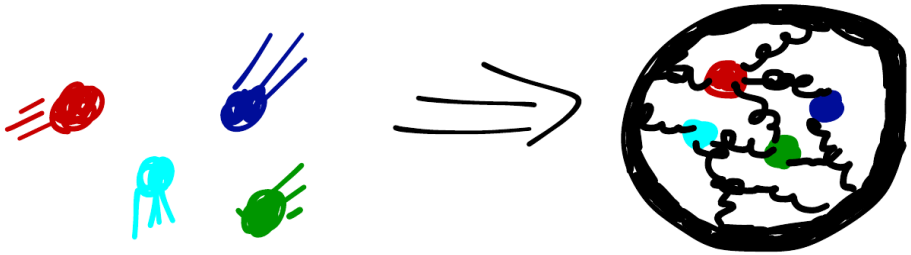
Early universe

Deconfined charged fermions $\longrightarrow$ non-gravitational interactions

Present day

Confined neutral ‘dark baryons’ $\longrightarrow$ no experimental detections

Composite dark-sector phase transition



Non-perturbative lattice targets — focus on SU(4) gauge theory

Dark baryon form factors for direct detection and collider searches

Baryon–baryon scattering (or binding) for small-scale structure

Dark-sector **phase transition** for gravitational-wave background

Gravitational waves

[arXiv:2006.16429]

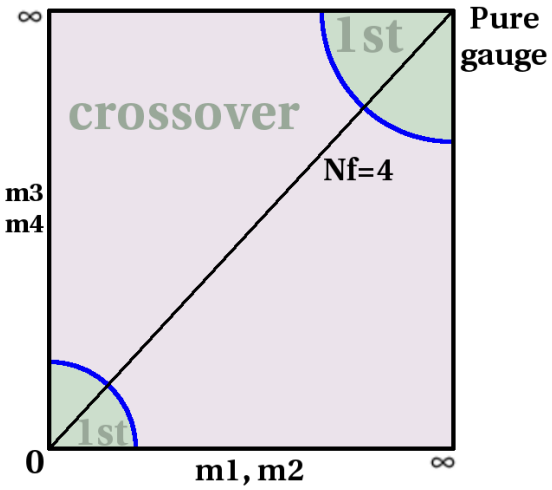
First-order confinement transition $\longrightarrow$ stochastic background of grav. waves

SU(N) pure-gauge transition for $N \geq 3$

Becomes stronger as N increases

First-order transition persists for sufficiently heavy fermions

Four-flavour SU(4) lattice studies $\longrightarrow$ need $M_P/M_V \gtrsim 0.9$

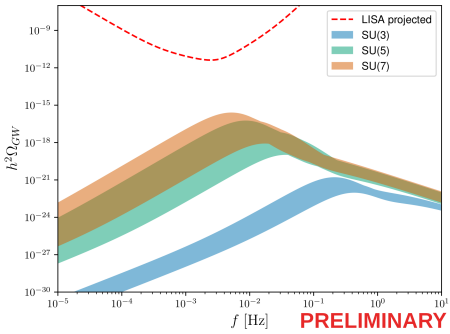


DiRAC composite dark matter project

First-order transition located $\longrightarrow$ now need lattice analyses of its properties

Proposal to compute **latent heat**

Gravitational wave spectrum
also sensitive to supercooling,
bubble nucleation rate & wall speed



Complementary project by Felix Springer [arXiv:2311.10243]

Density-of-states algorithm to avoid super-critical slowing down

Application: Composite Higgs sectors

Large Hadron Collider priority

Study fundamental nature of Higgs boson

Composite Higgs sector
can stabilize electroweak scale

New strong dynamics must differ from QCD

—Flavour-changing neutral currents
—Electroweak precision observables
—SM-like Higgs boson with $M \approx 0.5 v_{\mathrm{EW}}$

Near-conformal dynamics for composite Higgs

New strong dynamics must differ from QCD

—Flavour-changing neutral currents
—Electroweak precision observables
—SM-like Higgs boson with $M \approx 0.5 v_{\mathrm{EW}}$

Near-conformal dynamics
can help with all three issues

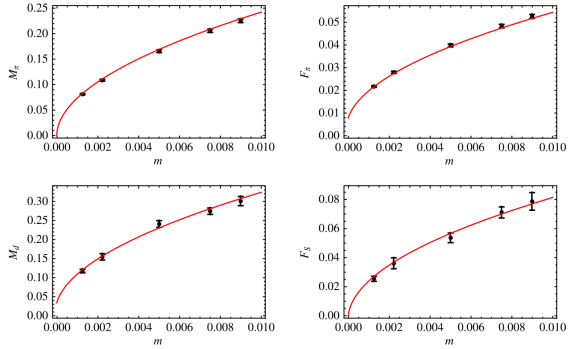
Near-conformality $\longrightarrow$ natural scale separation, novel IR dynamics

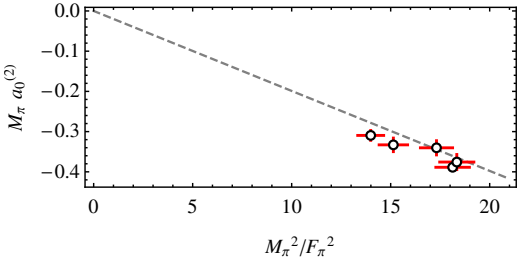
Requires reformulating low-energy effective field theory (EFT)

SU(3) lattice results to test candidate EFTs

Dilaton-EFT includes light pseudo-Nambu–Goldstone boson (PNGB) of broken scale invariance, in addition to usual PNGBs

Good description of spectrum and PNGB scattering for $N_F = 8$ [arXiv:2305.03665]

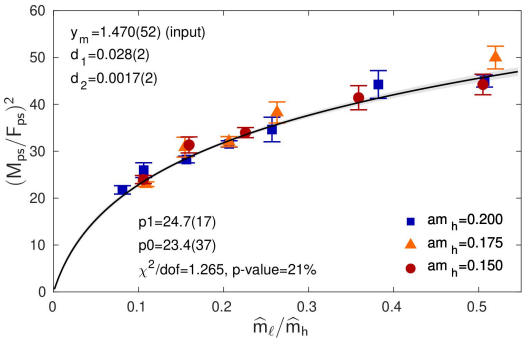




SU(3) lattice results to test candidate EFTs — DiRAC projects

Dilaton-EFT includes light pseudo-Nambu–Goldstone boson (PNGB) of broken scale invariance, in addition to usual PNGBs

Good description of spectrum for $N_F = 4 + 6$ [arXiv:2007.01810]



Using DiRAC to analyze

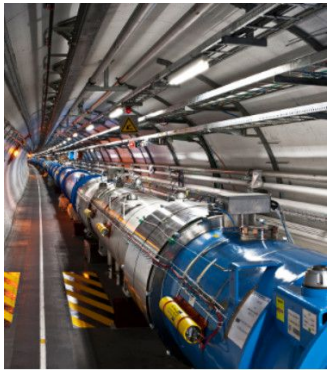
PNGB scattering

Electroweak S parameter (EFT coeff. related to W mass)

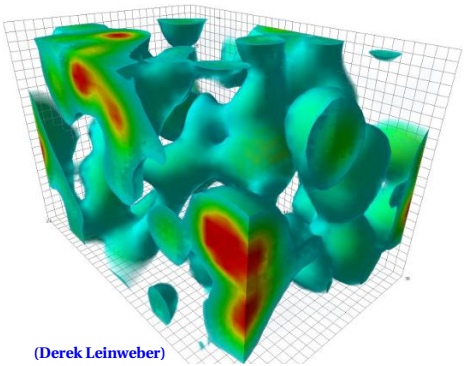
Application: Lattice supersymmetry

Lattice field theory promises first-principles predictions
for strongly coupled supersymmetric QFTs

BSM

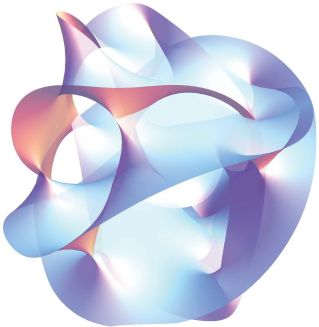


QFT



(Derek Leinweber)

Holography



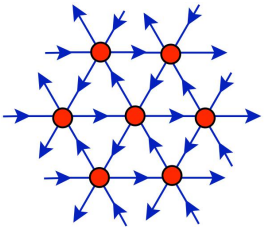
A brief history of lattice supersymmetry

Supersymmetries 'square' to infinitesimal translations, $\left\{Q^{\mathrm{I}}_{\alpha}, \overline{Q}^{\mathrm{J}}_{\dot{\alpha}}\right\} = 2\delta^{\mathrm{IJ}}\sigma^{\mu}_{\alpha\dot{\alpha}}P_{\mu}$

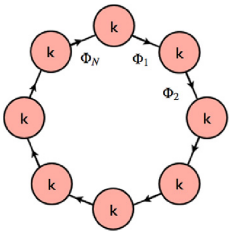
$\longrightarrow$ **do not exist** in discrete space-time

Solution: Reformulate theory to preserve subset of supersymmetries

$\Longrightarrow$ recover others in continuum limit



Review:
Catterall–Kaplan–Ünsal,
arXiv:0903.4881



Testing holographic duality

Holographic duality conjecture

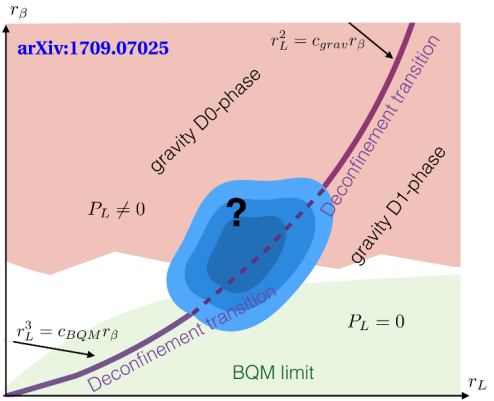
Thermodynamics of supersymmetric QFT $\longleftrightarrow$ black holes in dual supergravity

2d example: For decreasing r_L
at low $t = 1/r_\beta$ and large N

homogeneous black string (D1)
$\longrightarrow$ localized black hole (D0)

$\updownarrow$

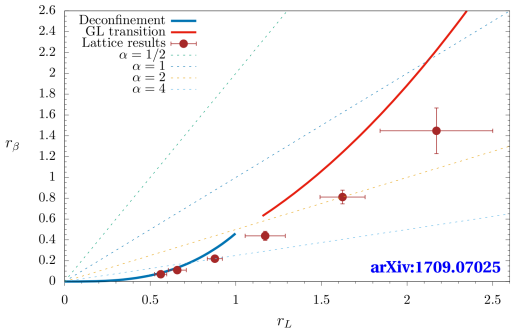
"spatial deconfinement"
signalled by Wilson line

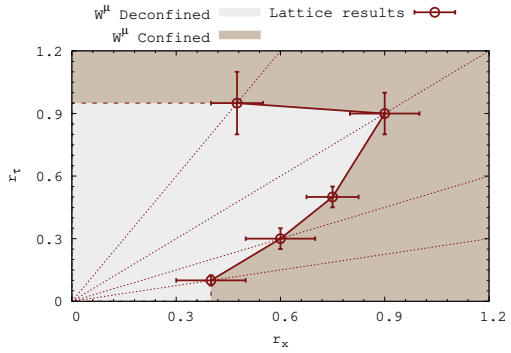


Two-dimensional super-Yang–Mills phase diagram

Left: ‘Maximal’ theory with 16 supersymmetries is consistent with holography

Right: Theory with 4 supersymmetries is not [arXiv:2312.04980]

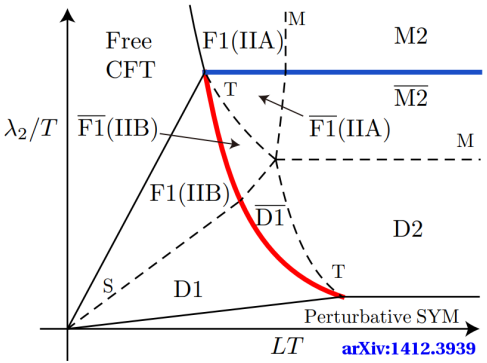


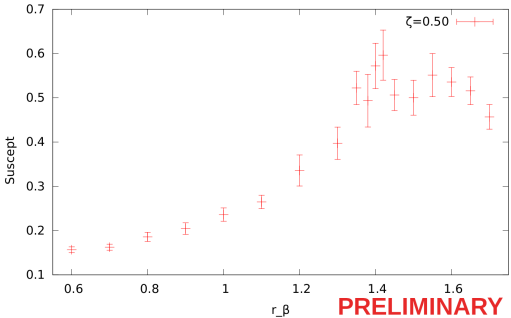


DiRAC project: Three-dimensional super-Yang–Mills

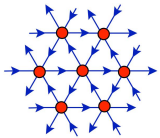
Richer phase diagram conjectured by holography (left, for $\infty \times L\lambda \times \frac{\lambda}{T}$ 3-torus)

Work in progress by Angel Sherletov (right, for $r_L \times r_L \times \frac{r_L}{2}$ 3-torus)



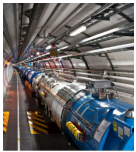


Outlook: Much more still to explore

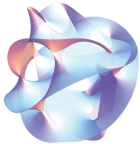


Lattice field theory is a broadly applicable tool
to study strongly coupled quantum field theories

Composite dark matter and gravitational waves
$\longrightarrow$ baryon–baryon scattering and the density of states



Near-conformal composite Higgs
$\longrightarrow$ anomalous dimensions for partial compositeness



Supersymmetry and holographic duality
$\longrightarrow$ supersymmetric QCD and sign problems

Thanks for your attention! Any further questions?

Collaborators on highlighted work

Felix Springer, Angel Sherletov,
Roman Marcarelli, Navdeep Singh Dhindsa, Raghav Jha, Anosh Joseph

Lattice Strong Dynamics Collaboration
(esp. Kimmy Cushman, Ethan Neil, Enrico Rinaldi, James Ingoldby, Oliver Witzel)

Funding and computing resources

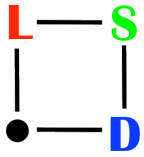
UK Research and Innovation

LIVERPOOL THEORETICAL PHYSICS

DEPARTMENT OF ENERGY · UNITED STATES OF AMERICA

USQCD

Backup: Lattice Strong Dynamics Collaboration



Argonne	Xiao-Yong Jin, James Osborn
Bern	Andy Gasbarro
Boston	Venkitesh Ayyar, Rich Brower, Evan Owen, Claudio Rebbi
Colorado	Anna Hasenfratz, Ethan Neil, Curtis Peterson
UC Davis	Joseph Kiskis
Durham	James Ingoldby
Fermilab	George Fleming
Livermore	Pavlos Vranas
Liverpool	DS, Felix Springer
Nvidia	Evan Weinberg
Oregon	Graham Kribs
Quantinuum	Enrico Rinaldi
Siegen	Oliver Witzel
Yale	Thomas Appelquist, Kimmy Cushman

Exploring the range of possible phenomena in strongly coupled field theories

Backup: Dark matter form factors

[PRL **115** 171803; PRD **92** 075030]

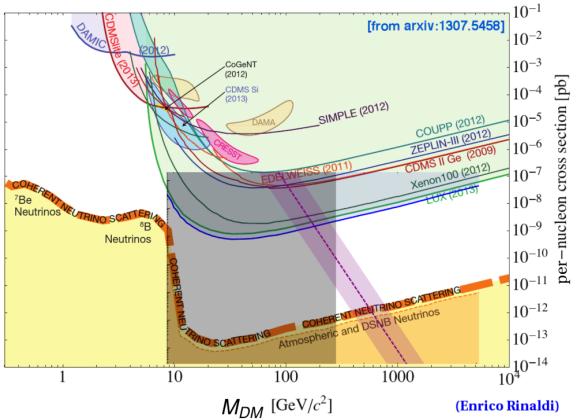
Lattice studies of four-flavour SU(4) dark sector
$\longrightarrow$ lightest **scalar** 'baryon' is stable dark matter candidate

Direct detection

Symmetries
$\longrightarrow$ electric **polarizability**
is leading interaction

Collider searches

Charged 'meson' Drell–Yan
rules out shaded region



Backup: Light scalar in near-conformal composite Higgs models

Nearly conformal dynamics makes scale separation natural

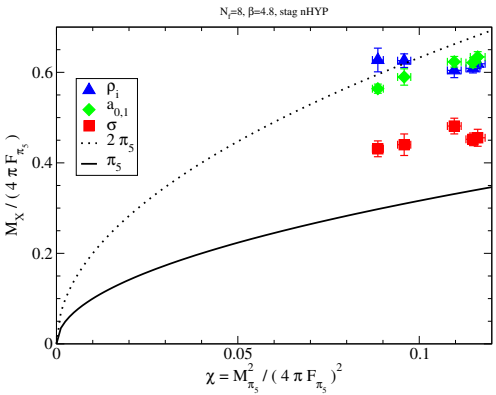
$\longrightarrow$ consistent with non-observation of new particles at LHC

Near-conformal lattice studies generically observe light scalar

Review: Witzel, arXiv:1901.08216

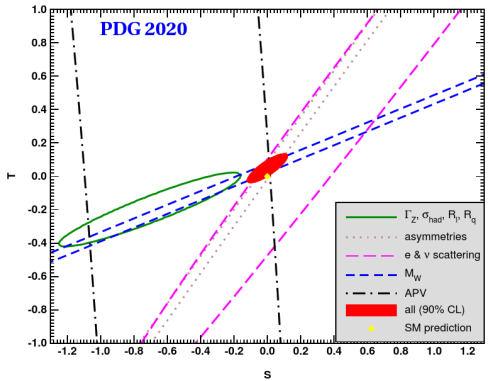
Right: LSD Collab., arXiv:2306.06095

Requires reformulating low-energy effective field theory



Backup: The S parameter

Constrain Higgs sector from vector-minus-axial vacuum polarization $\Pi_{V-A}(Q)$



Experimental $S = -0.01 \pm 0.10$

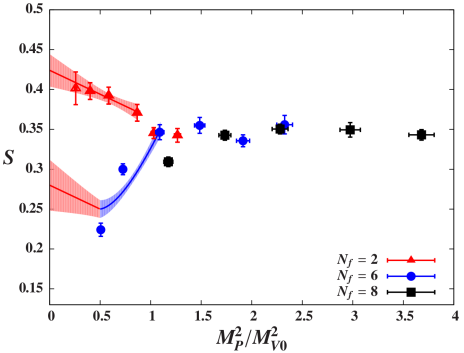
vs. QCD-like $S \approx 0.43\sqrt{\xi}$

Related to W boson mass [arXiv:2204.03796]

Domain-wall fermion symmetries important

Backup: S parameter on the lattice

$$\mathcal{L}_\chi \supset -\frac{S}{32\pi^2} g_1 g_2 B_{\mu\nu} \mathrm{Tr}\left[U\tau_3 U^\dagger W^{\mu\nu}\right] \longrightarrow \gamma,\, Z \sim\sim \text{new} \sim\sim \gamma,\, Z$$



Prior LSD study of $N_F = 2, 6, 8$ [arXiv:1405.4752]
$N_F = 4 + 6$ planned for the near future

$S/\sqrt{\xi} = 0.42(2)$ for $N_F = 2$ matches QCD ✓

Significant reduction from larger N_F,
chiral extrapolation again challenging

$V\!-\!A$ vacuum polarization also contributes to Higgs potential [arXiv:1903.02535]

Backup: Public code for lattice supersymmetry

so that the full improved action becomes

$$S_{\rm imp} = S'_{\rm exact} + S_{\rm closed} + S'_{\rm soft} \tag{18}$$

$$S'_{\rm exact} = \frac{N}{4\lambda_{\rm lat}} \sum_n {\rm Tr}\Bigg[-\overline{\mathcal{F}}_{ab}(n)\mathcal{F}_{ab}(n) - \chi_{ab}(n)\mathcal{D}^{(+)}_{[a}\psi_{b]}(n) - \eta(n)\overline{\mathcal{D}}^{(-)}_a\psi_a(n) + \frac{1}{2}\left(\overline{\mathcal{D}}^{(-)}_a\mathcal{U}_a(n) + G\sum_{a\neq b}(\det\mathcal{P}_{ab}(n) - 1)\,\mathbb{I}_N\right)^2\Bigg] - S_{\rm det}$$

$$S_{\rm det} = \frac{N}{4\lambda_{\rm lat}} G \sum_n {\rm Tr}\left[\eta(n)\right] \sum_{a\neq b} \left[\det\mathcal{P}_{ab}(n)\right] {\rm Tr}\left[\mathcal{U}_b^{-1}(n)\psi_b(n) + \mathcal{U}_a^{-1}(n+\widehat{\mu}_b)\psi_a(n+\widehat{\mu}_b)\right]$$

$$S_{\rm closed} = -\frac{N}{16\lambda_{\rm lat}} \sum_n {\rm Tr}\left[\epsilon_{abcde}\,\chi_{de}(n+\widehat{\mu}_a+\widehat{\mu}_b+\widehat{\mu}_c)\overline{\mathcal{D}}^{(-)}_c\chi_{ab}(n)\right],$$

$$S'_{\rm soft} = \frac{N}{4\lambda_{\rm lat}}\mu^2 \sum_n \sum_a \left(\frac{1}{N}{\rm Tr}\left[\mathcal{U}_a(n)\overline{\mathcal{U}}_a(n)\right] - 1\right)^2$$

$\gtrsim$100 inter-node data transfers in the fermion operator — non-trivial...

Public parallel code to reduce barriers to entry: github.com/daschaich/susy

Evolved from MILC QCD code, user guide in arXiv:1410.6971