

# MATH327: StatMech and Thermo, Spring 2026

## Extra practice — Linear potential

Consider  $N$  indistinguishable particles that can move in the region  $x \geq 0$  in one dimension at fixed temperature  $T$ . Each particle has mass  $m$  and energy

$$E(p, x) = \frac{1}{2m}p^2 + \frac{\alpha}{2}x,$$

where the momentum  $p$  and position  $x \geq 0$  of the particle are both real, and  $\alpha$  is a real positive constant. The single-particle partition function is

$$Z_1(T) = \int \exp[-E(p, x)/T] dp dx.$$

- (a) What is the Helmholtz free energy for the  $N$  indistinguishable particles?
- (b) Calculate the internal energy  $\langle E \rangle$  and the entropy  $S$ .
- (c) Calculate  $\langle x \rangle$  and  $\langle x^2 \rangle$ .
- (d) Calculate  $\langle p \rangle$  and  $\langle p^2 \rangle$ .