MATH327: StatMech and Thermo, Spring 2026

Extra practice — Boson gases

The grand-canonical potential for a gas of bosons with mass m in a box of volume V, with temperature $T = 1/\beta$ and chemical potential μ, is

$$\Phi(\beta, \mu) = \frac{V}{\beta} \frac{\sqrt{2m^3}}{2\pi^2\hbar^3} \int_0^\infty \log\left[1 - e^{-\beta(E-\mu)}\right] \sqrt{E}\, dE\,.$$

Consider the case of vanishing chemical potential, $\mu = 0$.

(a) Calculate the particle density $\frac{\langle N \rangle}{V}$.

(b) Calculate the internal energy density $\frac{\langle E \rangle}{V}$ to show $\langle E \rangle = \gamma \langle N \rangle T$ and determine the constant γ.

(c) Calculate the entropy of the gas. What is the condition of constant entropy?

(d) Calculate the pressure P to show $PV = \kappa \langle N \rangle T$ and determine the constant κ.

Now consider confining these particles to a two-dimensional surface of area A, with everything else the same. The grand-canonical potential becomes

$$\Phi(\beta, \mu) = \frac{A}{\beta} \frac{m}{2\pi\hbar^2} \int_0^\infty \log\left[1 - e^{-\beta(E-\mu)}\right] dE\,.$$

What is the average particle number $\langle N \rangle$ in the case of vanishing chemical potential, $\mu = 0$?

Hint: Expect something unexpected. (You can find more information in doi:10.1016/0370-1573(77)90052-7.)