Wed 6 May 52 42 95

Plan

Ising model ground states vs. lattice structures

Fully connected § solution highlights

$H=0 \rightarrow E = -J \sum_{(i,k)} s_i s_k$

Ground states for $J>0$ (include $J=1$)

All spins aligned, degeneracy 2

→ lattice structure doesn't matter

Ordered (ferromagnetic) phase, OP $|m|=1$

$J<0$: Aligned spins increase energy

Ground state maximizes pairs of opposing n.n. spins

→ All spins alternating if possible

Anti-ferromagnetic phase, degeneracy 2

$m = \frac{n_+ - n_-}{n_+ + n_-} = 0$ same as disordered phase but clear order

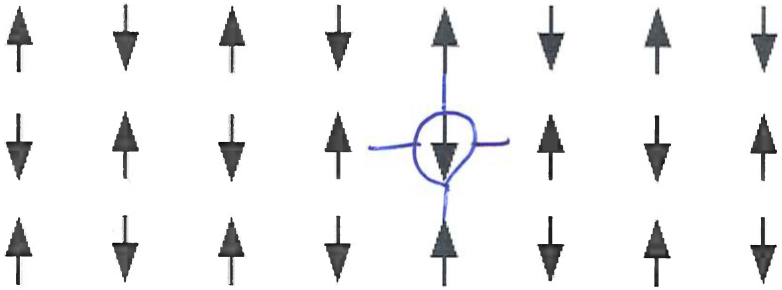
OP: $m_s = \frac{1}{N} \sum_n (-1)^n s_n$ "staggered magnetization"

§ even/odd lattice labels

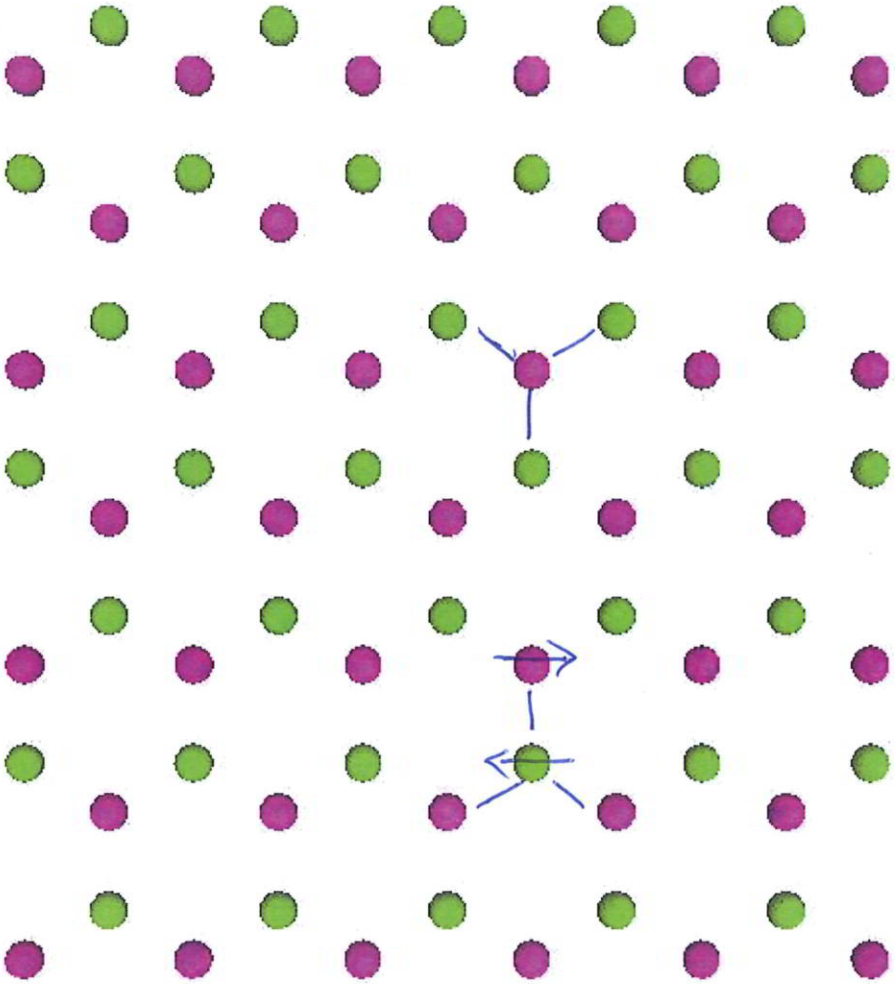
Ground state $|m_s|=1$ vs. $m_s=0$ in disordered phase

Relate to $F = -T \log Z$ by adding "staggered mag. field"

$$E = -J \sum_{(i,k)} s_i s_k - H_s \sum_n (-1)^n s_n$$



commons.wikimedia.org/wiki/File:Antiferromagnetic_ordering.svg



$\langle m_s \rangle \propto \frac{\partial}{\partial H_s} \log Z$ then set $H_s = 0$

$J < 0$ triangular lattice can't have fully alternating spins
"Geometrical frustration" because non-bipartite

Single unit cell → 6 degenerate configurations
with $\epsilon = -J(-2+1) = J < 0$

With $N \gg 1$, no order or order param.
Many degenerate lowest-energy w_i
→ $S > 0$ even at absolute zero T

More general frustration in "spin glasses"

$$E_{s_i} = -\sum_{j \neq k} J_{jk} s_j s_k$$

randomly $J_{jk} - J_{kj}$ with positive and negative

Finding w_i with $E_i \leq C$ is NP-complete

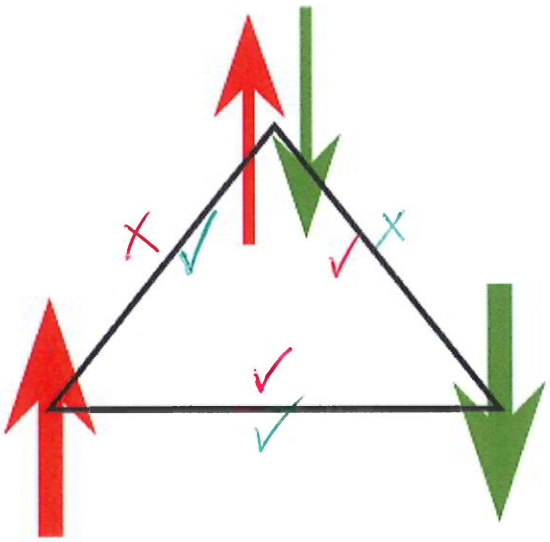
both NP and NP-hard
(solves all NP)

Motivates quantum computing

Spin glasses a.k.a. neural networks

Gaussian J_{jk} → Sherrington-Kirkpatrick model
(solved by Parisi, 2021 Nobel)
special case of Hopfield network (2024 Nobel)
(recurrent neural network)

?



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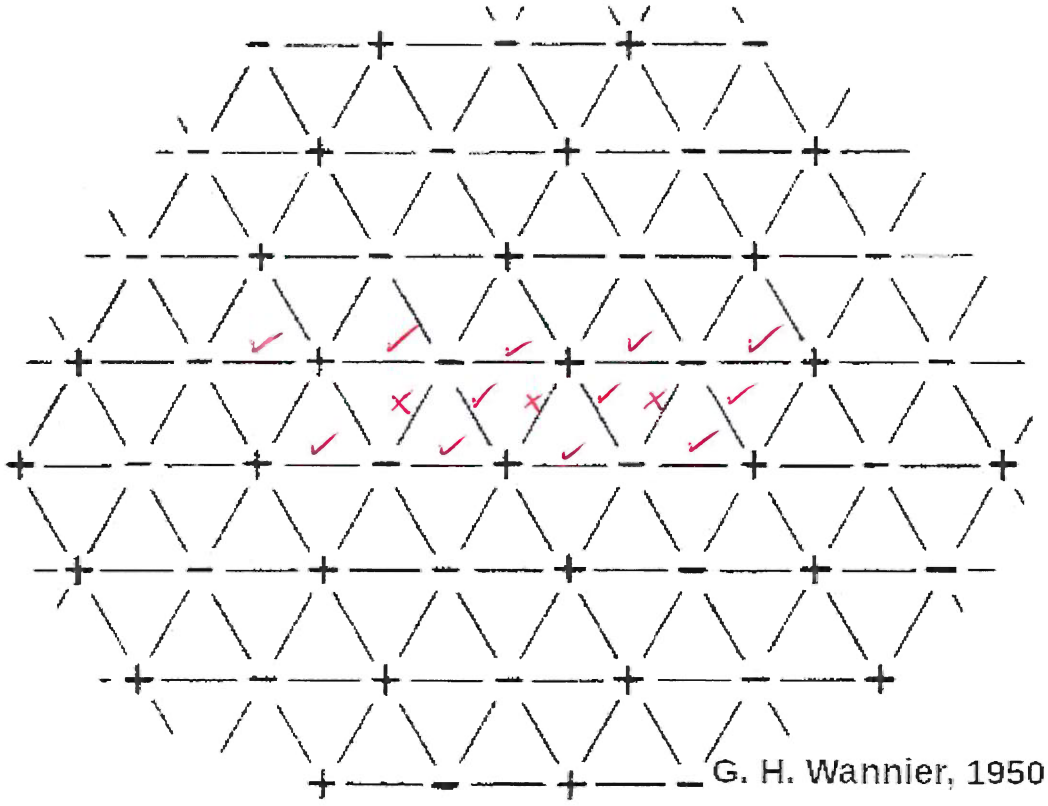

G. H. Wannier, 1950

Add site-dependent background field

$$E_{BM} = -\sum_{j<k} J_{jk} s_j s_k - \sum_n H_n s_n$$

"Boltzmann machine" (Hinton 2024 Nobel)

"restricted" into layers for efficiency

→ "deep learning"