MATH327: StatMech and Thermo, Spring 2025

Extra practice — Randomly walking inventory

Suppose a company has developed a revolutionary zero-emission airplane, which is in high demand. Let $I(t)$ be the inventory of planes available to sell at time t, with negative values representing a backlog of planes already ordered that still need to be produced. Ten pre-orders produce a backlog of $I_0 \equiv I(0) = -10$ when production starts at time $t = 0$. Once production starts, a single plane is produced every $\Delta t = 3$ weeks. Every single week there is a 50% chance of receiving a new order for a plane.

Treat the inventory as a random walk, taking one step every $\Delta t = 3$ weeks.

(a) What are the possible step sizes ΔI and the corresponding probabilities for the random walk described above? Compute the resulting mean and variance of the single-step process.

(b) What is the minimum time t_{min} in which the initial backlog could be cleared, to give $I(t_{\text{min}}) \geq 0$? What is the probability the backlog will be cleared in this time?

(c) Using the central limit theorem, what is the expected size of the inventory when $t = 78$ weeks (approximately 18 months)?

(d) Using the central limit theorem and the integral

$$\frac{1}{\sqrt{\pi}} \int_{-\infty}^{U} e^{-u^2}\, du \approx 0.9984 \qquad \text{for} \qquad U = 2.082,$$

what is the probability the backlog will be reduced, so that $I(t) > I_0$, when $t = 78$ weeks?