

# MATH327: StatMech and Thermo, Spring 2025

## Extra practice — Gaussian integrals

As mentioned in the module's [logistical information](#), I anticipate that you have previously learned about [gaussian integrals](#). However, several students struggled with these last year, so a review may be helpful.

First show

$$\mathcal{G} \equiv \int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi}.$$

There are several ways this can be done. My favourite is to evaluate  $\mathcal{G}^2$  using polar coordinates.

Next use a change of variables to show

$$\int_{-\infty}^{\infty} e^{-a(x+b)^2} dx = \sqrt{\frac{\pi}{a}},$$

assuming  $a > 0$ .

Finally consider the gaussian probability distribution

$$p(x) = C \exp \left[ -\frac{(x - N\mu)^2}{2N\sigma^2} \right],$$

with finite  $\mu$  and  $\sigma^2 > 0$ . Based on the properties of probabilities, determine the positive coefficient  $C$  and compare your result against Eq. 10 in the lecture notes.