Thu 3 Apr 44 04 55

Plan

HW1

Phonons gas $\rightarrow C_V \sim T^3$

Electron gas $\rightarrow C_V \sim T$

(Sommerfeld expansion)

Phonon gas: Non-interaction quasi-particles $\rightarrow \mu = 0$ Planck spectrum

$$\langle E \rangle \propto \int_0^{\omega_{max}} \frac{\omega^3}{e^{\beta\hbar\omega} - 1} d\omega \qquad x = \beta\hbar\omega = \frac{\hbar\omega}{T}$$

$$\propto \left(\frac{T}{\hbar}\right)^4 \int_0^{T_D/T} \frac{x^3}{e^x - 1} dx \qquad \text{Debye temp. } T_D \propto \sqrt[3]{N/V}$$

High T: $\frac{T_D}{T} \ll 1 \rightarrow 0 \leq x \ll 1 \rightarrow e^x - 1 \simeq x$

$$\langle E \rangle \propto T^4 \int_0^{T_D/T} \frac{x^3}{x} dx \propto T^4 \left(\frac{T_D}{T}\right)^3 \propto T$$

$C_V = \frac{\partial}{\partial T} \langle E \rangle = \text{const.}$ <u>same</u> as Einstein solid

Low T: $\frac{T_D}{T} \gg 1 \qquad \int_0^{T_D/T} \frac{x^3}{e^x - 1} dx \approx \Gamma(4)\zeta(4) = \text{const.}$

$$\langle E \rangle \propto T^4 \rightarrow C_V \propto T^3$$

Electron gas: $\frac{\langle N \rangle_F}{g_0} \simeq \frac{2}{3}\mu^{3/2} \underset{1}{I_0} + T\mu^{1/2} \underset{0}{I_1} + \frac{1}{4}T^2\mu^{-1/2} I_2$

$$I_n = \int_{-\infty}^{\infty} \frac{x^n e^x}{(e^x + 1)^2} dx$$

$$I_2 = 2\int_0^\infty \frac{x^2 e^x}{(e^x+1)^2}\,dx \qquad u = x^2 \qquad dv = \frac{e^x}{(e^x+1)^2}\,dx \rightarrow v = \frac{-1}{e^x+1}$$

$$= -2\underbrace{\frac{x^2}{e^x+1}\bigg|_0^\infty}_{0} + 2\int_0^\infty \frac{2x}{e^x+1}\,dx$$

$$= 4\left(1-\frac{1}{2}\right)\Gamma(2)\,\zeta(2) = \frac{\pi^2}{3}$$

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$$\langle N\rangle_F \simeq g_0\left[\frac{2}{3}\mu^{3/2} + \frac{\pi^2}{12\mu^{1/2}}T^2\right]$$

g_0 relate to $E_F = \frac{\hbar^2}{2m}\left(3\pi^2\rho_F\right)^{2/3}$

$$E_F^{3/2} = \frac{3\pi^2\hbar^3}{2\sqrt{2m^3}}\,\frac{\langle N\rangle_F}{V} = \frac{3}{2}\,\frac{\langle N\rangle_F}{g_0}$$

$$\cancel{\langle N\rangle_F} \approx \frac{3}{2}\,\frac{\cancel{\langle N\rangle_F}}{E_F^{3/2}}\left[\frac{2}{3}\mu^{3/2} + \frac{\pi^2}{12\mu^{1/2}}T^2\right]$$

$$1 \approx \left(\frac{\mu}{E_F}\right)^{3/2} + \frac{\pi^2T^2}{8E_F^{3/2}\mu^{1/2}}$$

$$\frac{\mu}{E_F} \approx \left[1 - \frac{\pi^2T^2}{8E_F^{3/2}\mu^{1/2}}\right]^{2/3} \approx 1 - \frac{\pi^2T^2}{12E_F^2}$$

$E_F \approx \mu$

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$$\frac{\langle E\rangle_f}{g_0} = \int_0^\infty E^{3/2} F(E)\,dE \approx \frac{2}{5}\int_{-\infty}^{\infty} \frac{e^x}{(e^x+1)^2}\, E^{5/2}\,dx$$

$$\approx \frac{2}{5}\mu^{5/2}\underset{1}{\cancel{I_0}} + T\mu^{3/2}\underset{0}{\cancel{I_1}} + \frac{3}{4}T^2\mu^{1/2}\underset{\pi^2/3}{\cancel{I_2}}$$

$$\langle E\rangle_f \approx \frac{3}{2}\frac{\langle N\rangle_f}{E_F^{3/2}}\left[\frac{2}{5}\mu^{5/2} + \frac{\pi^2}{4}\mu^{1/2}T^2\right]$$

$$\mu^{5/2} \approx E_F^{5/2} - \frac{5}{24}\pi^2 T^2 E_F^{1/2}$$

$$\boxed{\langle E\rangle_f = \frac{3}{5}\langle N\rangle_f E_F + \frac{\pi^2}{4}\langle N\rangle_f\frac{T^2}{E_F} + \mathcal{O}\left(\frac{T^4}{E_F^3}\right)}$$

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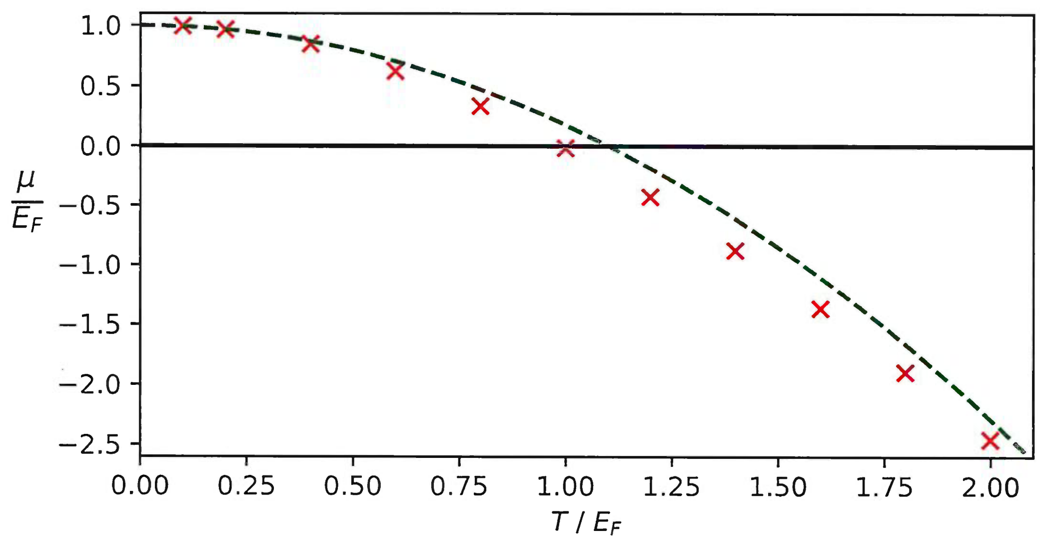
$$C_V = \frac{\pi^2}{2}\frac{\langle N\rangle_f}{E_F}\,T \quad ✓$$

$$\alpha = \frac{\pi^2}{2}\frac{\langle N\rangle_f}{E_F}$$



1.0
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