Thu 13 Mar   83 50 27

Plan
- Mixing entropy
  - Maxwell's demon
- Therm. cycles
  - otto cycle ~ petrol engine

Useful trick: $\Delta S_{mix} = S_c - S_0$

$$= \frac{\partial}{\partial T}\left(T \log Z_c - T \log Z_0\right)$$

$$= \frac{\partial}{\partial T}\left(T \log \frac{Z_c}{Z_0}\right)$$

$$\frac{Z_c}{Z_0} = \frac{\frac{1}{(N!)^2}\left(\frac{2V}{\lambda_{th}^3}\right)^{2N}}{\left[\frac{1}{N!}\left(\frac{V}{\lambda_{th}^3}\right)^N\right]^2} = 2^{2N} \qquad (T\text{-indep.})$$

$$\Delta S_{mix} = \log\left(\frac{Z_c}{Z_0}\right) = 2N \log 2 > 0 \quad ✓$$

same as fully dist'able!

Less info but same relative increase upon mixing

& Final entropy $S_F = (S_F - S_c) + S_c = S_c + \frac{\partial}{\partial T}\left(T \log \frac{Z_F}{Z_c}\right)$

Gibbs approx. of N particle on each side

ν red on left → $N-\nu$ blue

$N-\nu$ red & ν blue on right

$$Z_F = \sum_{\nu=0}^{N} Z_\nu = \sum_\nu \left[\frac{1}{\nu!} \left(\frac{V}{\lambda_{th}^3}\right)^{\nu} \frac{1}{(N-\nu)!} \left(\frac{V}{\lambda_{th}^3}\right)^{N-\nu} \right]^2$$

$$= \left(\frac{V}{\lambda_{th}^3}\right)^{2N} \sum_\nu \frac{1}{(\nu!)^2[(N-\nu)!]^2}$$

$$= \left(\frac{V}{\lambda_{th}^3}\right)^{2N} \frac{1}{(N!)^2} \sum_\nu \binom{N}{\nu}^2$$

$$= \left(\frac{V}{\lambda_{th}^3}\right)^{2N} \frac{1}{(N!)^2} \binom{2N}{N}$$

$$\frac{Z_F}{Z_c} = \frac{1}{2^{2N}} \binom{2N}{N}$$

$$S_F \approx S_c + \log\left(\frac{(2N)!}{(N!)^2}\right) - 2N \log 2$$

$N \gg 1$: $S_F \approx S_c + 2N\log(2N) - 2N - 2(N\log N - N) - 2N\log 2$

$= S_c$

S_0 $\quad S_F \approx S_c > S_0$ consistent w/second law ✓

Next terms: $\frac{1}{2}\log(2\pi \cdot 2N) - 2 \cdot \frac{1}{2}\log(2\pi N)$

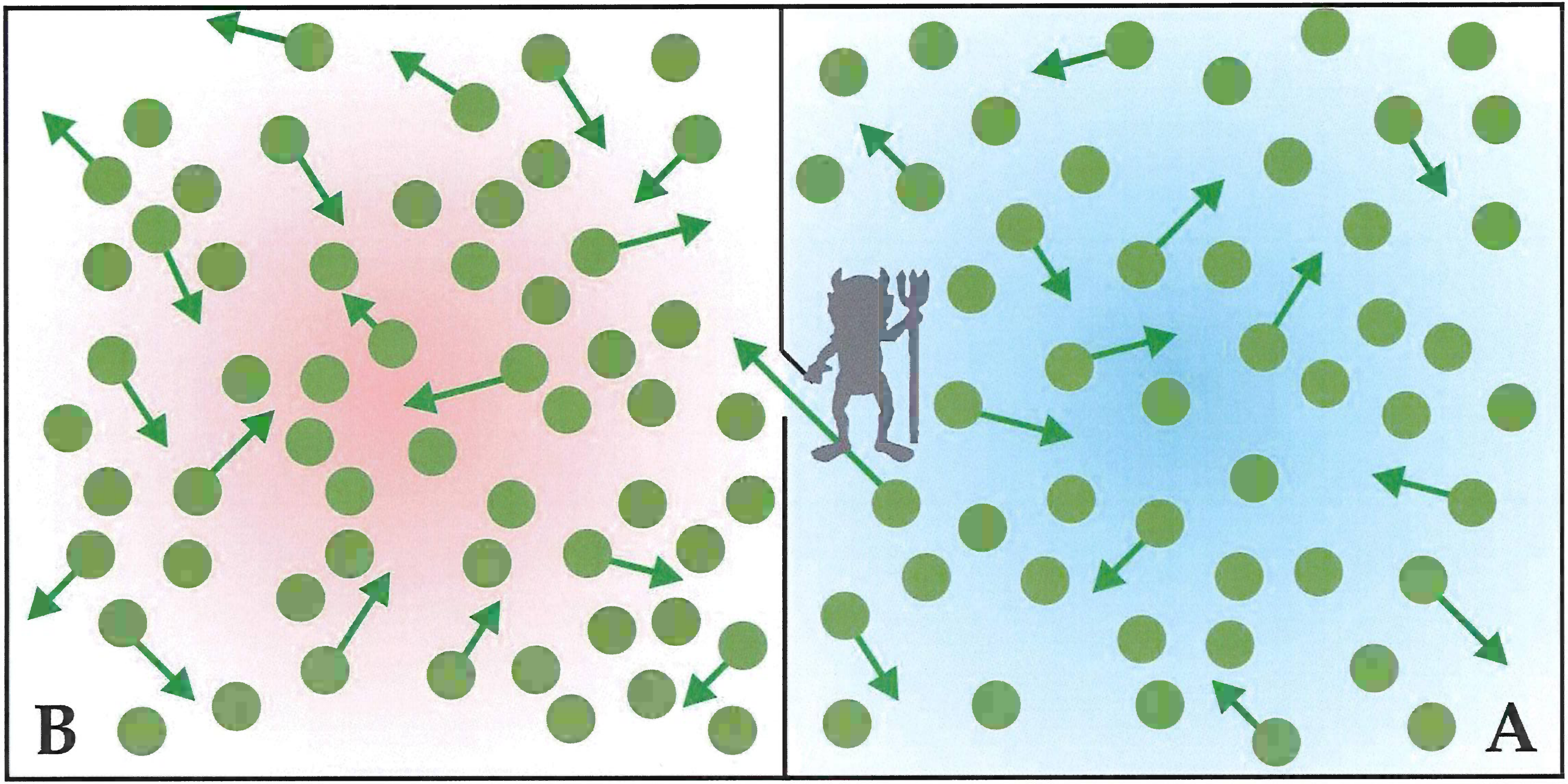
$$= \log\left(\frac{\sqrt{4\pi N}}{2\pi N}\right) = -\log(\sqrt{\pi N}) < 0$$

$\rightarrow S_F \approx S_c - \log(\sqrt{\pi N}) < S_c$ ✗

Solution: Go beyond Gibbs approx, $(N \pm h)$ to see second law obeyed

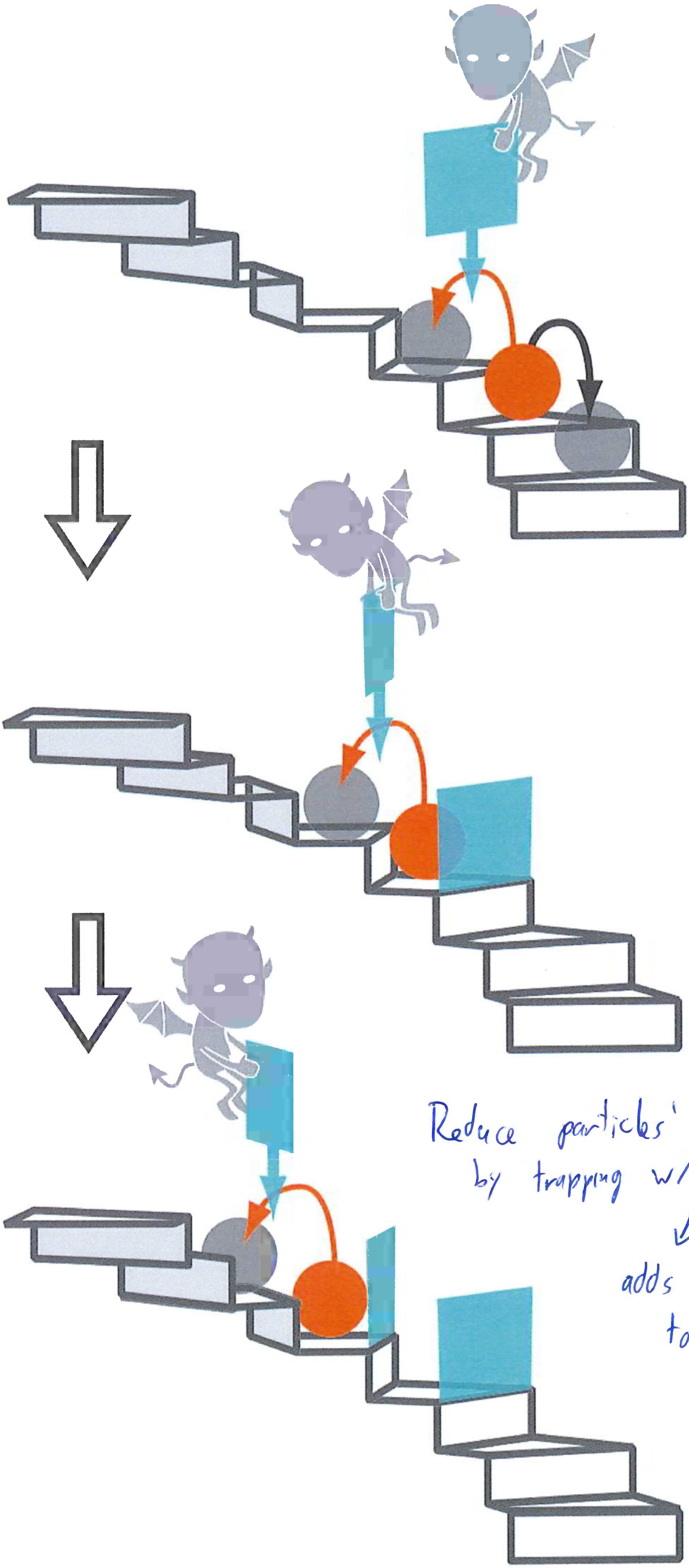
<u>Maxwell's demon</u> (1867) – experimentally tested (2010)

Conceptual argument: Demon's action add net entropy of universe



$T_B > T_A$

Toyabe et al. 2010



Reduce particles' entropy by trapping w/lasers
↓
adds entropy to universe

Otto cycle – idealized petrol engine

No slow isothermal stages

Gas is mixture of air and vaporized petrol

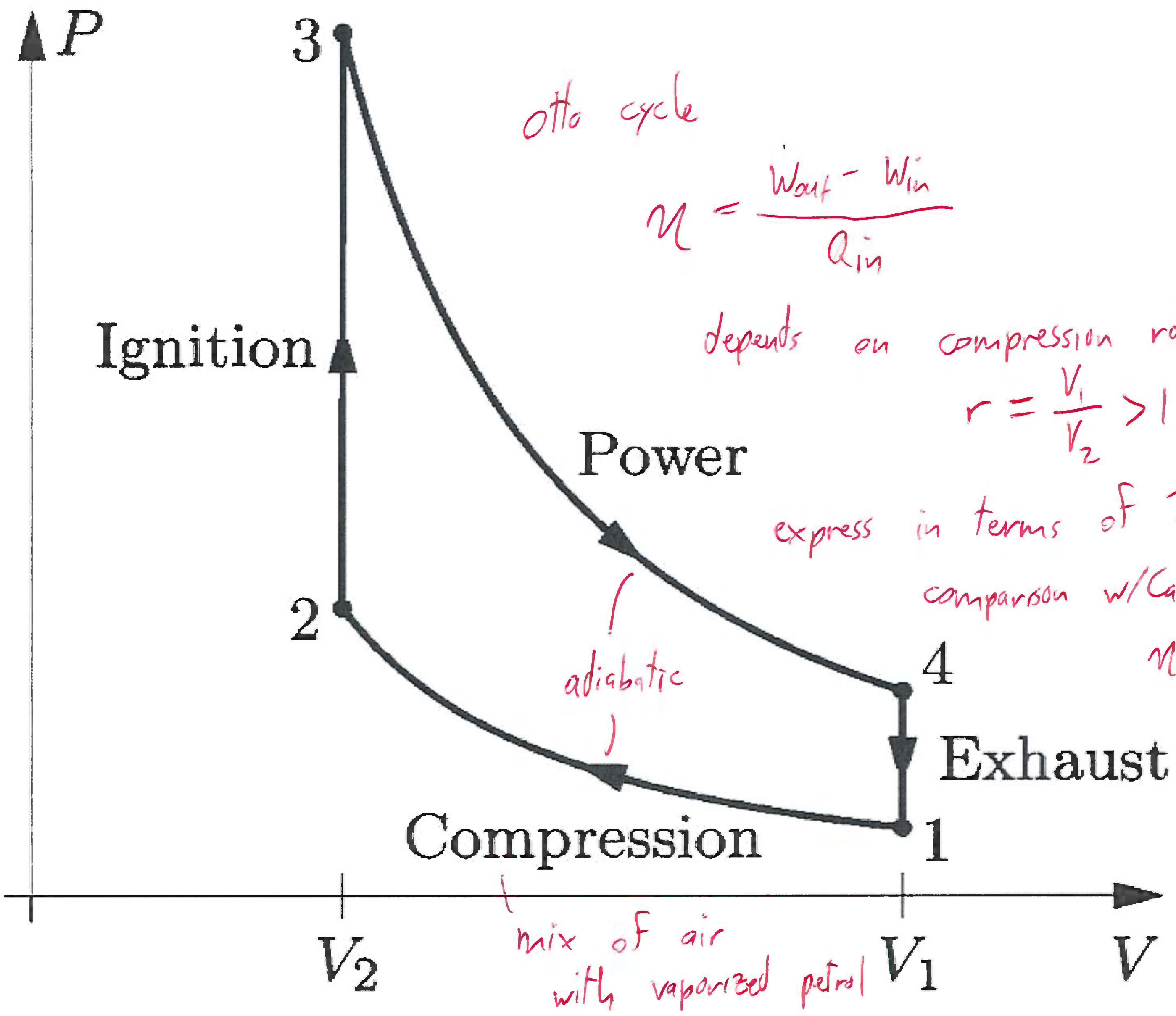
Compress to high pressure, ignite ~ hot reservoir

after power ~~stage~~ replace with fresh ~ cold reservoir

Efficiency $\eta = \frac{W_{out} - W_{in}}{Q_{in}}$ depends on compression ratio $r = \frac{V_1}{V_2} > 1$

Express in terms of T_1, T_2, T_3, T_4,

compare with Carnot $\eta_C = 1 - \frac{T_1}{T_3}$



otto cycle

$$\eta = \frac{W_{out} - W_{in}}{Q_{in}}$$

depends on compression ratio

$$r = \frac{V_1}{V_2} > 1$$

express in terms of T_1, T_2, T_3, T_4

comparison w/ Carnot

$$\eta_c = 1 - \frac{T_1}{T_3}$$