

Thu 6 Mar

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Plan

Entropy bounds

Stirling's formula

Mixing entropy

$$N! = N(N-1)(N-2)\dots 1 < N \cdot N \cdot N \dots N = N^N$$

$$e^N = \sum_{k=0}^{\infty} \frac{N^k}{k!} > \frac{N^N}{N!} \rightarrow N! > \left(\frac{N}{e}\right)^N$$

$$N \log N - N < \log(N!) < N \log N$$

$$1 - \frac{1}{\log N} < \frac{\log(N!)}{N \log N} < 1$$

$$\rightarrow \log(N!) \sim N \log N$$

asymptotically $N \gg 1$

Derivative $\frac{d}{da} \left(\int_0^{\infty} e^{-ax} dx \right) = \frac{d}{da} a^{-1}$

$$\int_0^{\infty} -x e^{-ax} dx = \frac{-1}{a^2} = a^{-2}$$

$$\frac{d^N}{da^N} \rightarrow \int_0^{\infty} (-x)^N e^{-ax} dx = (-1)^N N! a^{-(N+1)}$$

$$N! = \int_0^{\infty} x^N e^{-x} dx \quad \square$$

$$\text{Maximize } \frac{d}{dx} x^N e^{-x} = 0 = N x^{N-1} e^{-x} - x^N e^{-x}$$

$$x = N$$

$$\text{Change var. } y = x - N \quad \left| \frac{y}{N} \right| \ll 1$$

$$N! = \int_0^{\infty} \exp[N \log x - x] dx$$

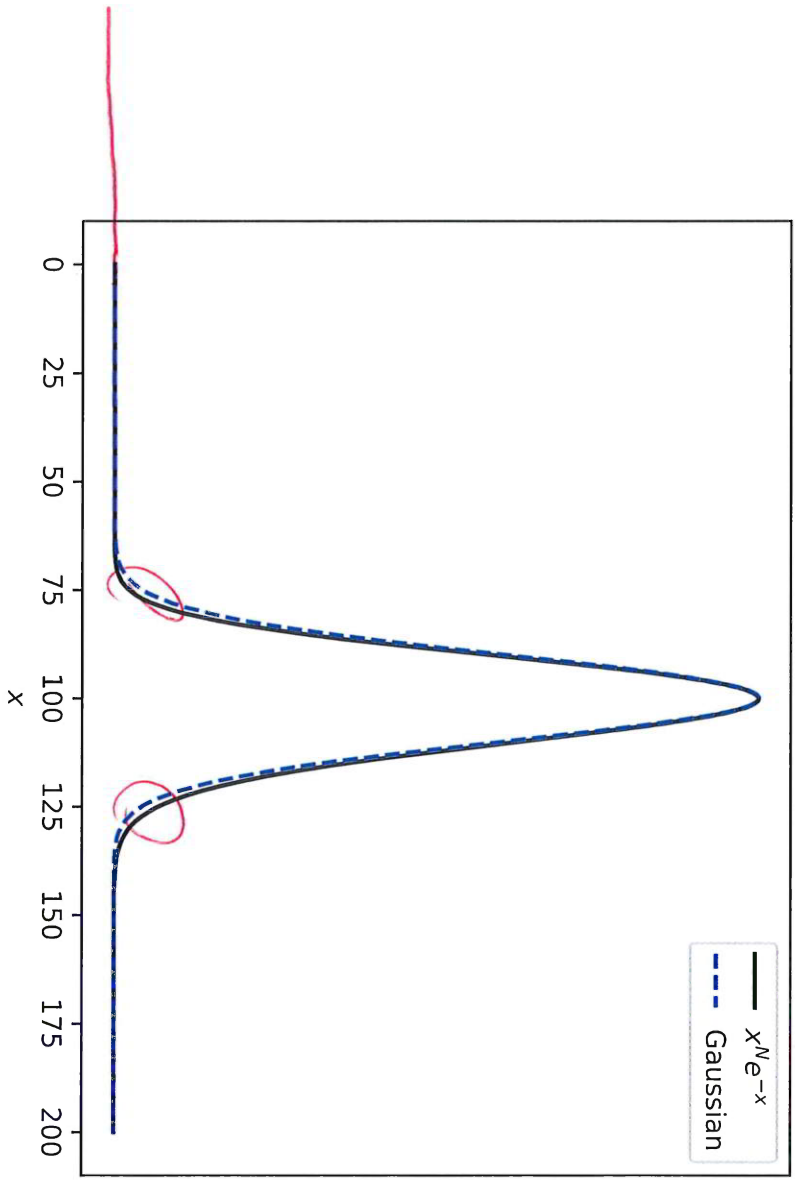
$$= \int_{-N}^{\infty} \exp\left[N \log\left(N\left(1 + \frac{y}{N}\right)\right) - y - N\right] dy$$

$$= \int_{-N}^{\infty} \exp\left[N \log N + N\left(\frac{y}{N} - \frac{y^2}{2N^2} + \mathcal{O}\left(\frac{y^3}{N^3}\right)\right) - y - N\right] dy$$

$$\approx N^N e^{-N} \int_{-N}^{\infty} \exp\left(\frac{-y^2}{2N}\right) dy = \sqrt{2\pi N} \left(\frac{N}{e}\right)^N \checkmark$$

$$A = \frac{1}{12}$$

$$B = \frac{1}{288}$$



$N = 100$