Thu 6 Mar   63 59 43

Plan

Entropy bounds

Stirling's formula

Mixing entropy

$$N! = N(N-1)(N-2)\cdots 1 < N\cdot N\cdot N\cdots N = N^N$$

$$e^N = \sum_{k=0}^{\infty} \frac{N^k}{k!} > \frac{N^N}{N!} \rightarrow N! > \left(\frac{N}{e}\right)^N$$

$$N\log N - N < \log(N!) < N\log N$$

$$1 - \cancel{\frac{1}{\log N}} < \frac{\log(N!)}{N\log N} < 1$$

$$0 \rightarrow \log(N!) \sim N\log N$$

asymptotically $N \gg 1$

Derivative $\frac{d}{da}\left(\int_0^\infty e^{-ax}\,dx\right) = \frac{d}{da} a^{-1}$

$$\int_0^\infty -x\, e^{-ax}\,dx = \frac{-1}{a^2} = -a^{-2}$$

$$\frac{d^N}{da^N} \longrightarrow \int_0^\infty (-x)^N e^{-ax}\,dx = \cancel{(-1)^N} N!\, a^{-(N+1)}$$

$a \to 1$

$$N! = \int_0^\infty x^N e^{-x}\,dx \qquad \square$$

Maximize $\frac{d}{dx} x^N e^{-x} = 0 = N x^{N-1} \cancel{e^{-x}} - x^N \cancel{e^{-x}}$

$x = N$

Change var. $y = x - N$ $\quad \left|\frac{y}{N}\right| \ll 1$

$$N! = \int_0^\infty \exp\left[N \log x - x\right] dx$$

$$= \int_{-N}^\infty \exp\left[N \log\left(N\left(1+\frac{y}{N}\right)\right) - y - N\right] dy$$

$$= \int_{-N}^\infty \exp\left[N \log N + N\left(\cancel{\frac{y}{N}} - \frac{y^2}{2N^2} + \mathcal{O}\left(\frac{y^3}{N^3}\right)\right) - \cancel{y} - N\right] dx$$

$$\approx N^N e^{-N} \int_{-N}^\infty \exp\left(\frac{-y^2}{2N}\right) dy = \sqrt{2\pi N} \left(\frac{N}{e}\right)^N \quad \checkmark$$

$-N \to -\infty$

$A = \frac{1}{12}$

$B = \frac{1}{288}$

$N = 100$