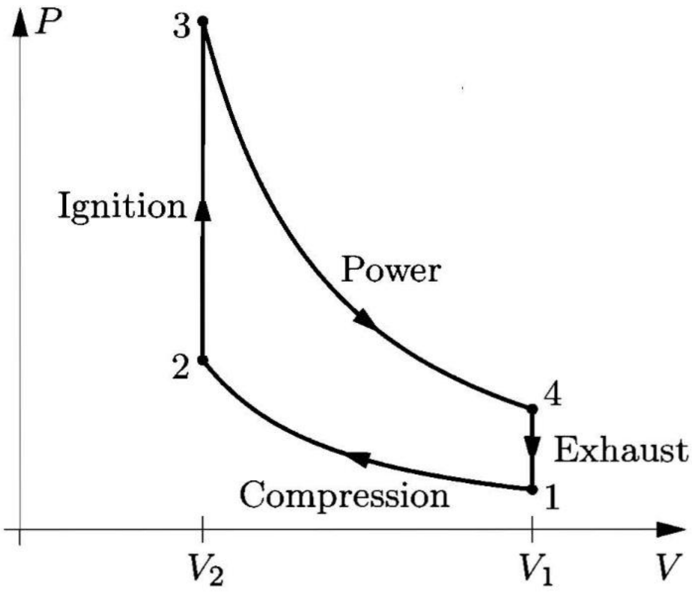
MATH327: Statistical Physics, Spring 2024

Tutorial activity — Otto cycle

This activity will be introduced in our 14 March tutorial, and you'll have the week until our next tutorial on 21 March to work on it. In particular, in the next lecture we'll define the **efficiency** of a thermodynamic engine as the amount of work it performs divided by the amount of input heat it needs. Knowing this, you can start working on this activity by considering the work and heat corresponding to each of the four processes forming the 'Otto cycle' shown below, which describes an idealized petrol engine:

- Fast (adiabatic) compression increases the pressure of the gas (a mixture of air and vaporized petrol), until a spark ignites it.
- This ignition introduces lots of heat almost instantaneously, while the volume is fixed at V_2. Even though the gas itself is burning, we can interpret this heat as coming from energy exchange with a hot thermal reservoir.
- The gas then does work by adiabatically expanding back to volume $V_1 > V_2$.
- Finally, heat is expelled at fixed volume V_1 by swapping the hot exhaust for an equal amount of cooler, fresh gas ready to be burned.



The efficiency η of the Otto cycle depends on the **compression ratio**

$$r \equiv \frac{V_1}{V_2} > 1.$$

What is this efficiency? How does it compare to the efficiency of the Carnot cycle? How should V_1 and V_2 be chosen to maximize the efficiency?

Hint: Given the labels in the PV diagram above, T_1 is the low temperature of the cold reservoir while T_3 is the high temperature of the hot reservoir. The corresponding Carnot cycle efficiency is therefore $\eta_C = 1 - \frac{T_1}{T_3}$, and the comparison is easiest if the Otto cycle efficiency is expressed in terms of temperatures rather than volumes.