Gaps Filled on Friday 9 February

Page 20

$$\langle x\rangle = \sum_x x\,P(x) = \sum_{r=0}^{N} (2r-N)\,P_r = 2\langle r\rangle - N$$

$$\langle x^2\rangle = \sum_r (2r-N)^2 P_r = 4\langle r^2\rangle - 4N\langle r\rangle + N^2$$

$$\frac{d}{d\theta} T(\theta)\Big|_{\theta=0} = \sum_r \frac{d}{d\theta} e^{r\theta} P_r\Big|_{\theta=0} = \sum_r r\, \underbrace{e^{r\theta}}_{\to 1} P_r\Big|_{\theta=0} = \langle r\rangle$$

$$\frac{d^n}{d\theta^n} T(\theta)\Big|_{\theta=0} = \sum_r r^n \underbrace{e^{r\theta}}_{\to 1} P_r\Big|_{\theta=0} = \langle r^n\rangle$$

Page 21

$$\frac{d}{d\theta}\left(e^{\theta}p+q\right)^N\Big|_{\theta=0} = N\left(e^{\theta}p+q\right)^{N-1} e^{\theta}p\Big|_{\theta=0} = Np = \langle r\rangle$$

$$\frac{d^2}{d\theta^2}\left(e^{\theta}p+q\right)^N\Big|_{\theta=0} = \left[Ne^{\theta}p\left(e^{\theta}p+q\right)^{N-1} + N(N-1)e^{\theta}p\left(e^{\theta}p+q\right)^{N-2}e^{\theta}p\right]_{\theta=0}$$

$$= Np + N(N-1)p^2 = Np(1+Np-p)$$

$$= Np(Np+q) = \langle r^2\rangle$$

$$\langle x\rangle = 2\langle r\rangle - N = 2Np - N = N(2p-1)$$

$$\langle x^2\rangle - \langle x\rangle^2 = 4\langle r^2\rangle - 4\cancel{N\langle r\rangle} + \cancel{N^2} - (2\langle r\rangle - N)^2$$

$$\left(4\langle r\rangle^2 - 4\cancel{N\langle r\rangle} + \cancel{N^2}\right)$$

$$= 4\left(\langle r^2\rangle - \langle r\rangle^2\right) = 4\left(N^2p^2 + Npq - N^2p^2\right) = 4Npq$$

Page 23

$$\frac{\Delta x}{\langle x \rangle} \propto \frac{\sqrt{t}}{t} \sim \frac{1}{\sqrt{t}} \to 0 \quad \text{for large} \quad \begin{matrix} t \to \infty \\ N \to \infty \end{matrix}$$

Page 24

$$\mu = \langle x_i \rangle = \sum_{L,R} x_i P_i = (+1)p + (-1)\overset{1-p}{q} = 2p - 1 = V_{dr} \overset{t/N}{\Delta t} = \frac{V_{dr}\, t}{N}$$

$$\langle x_i^2 \rangle = (+1)^2 p + (-1)^2 q = 1$$

$$\sigma^2 = 1 - (4p^2 - 4p + 1) = 4p(1-p) = 4pq = D^2 \Delta t = \frac{D^2 t}{N}$$

General results: $N\mu = V_{dr} t = \langle x \rangle \qquad N\sigma^2 = D^2 t = (\Delta x)^2$

(p,q <u>not</u> general)

Page 26

Configuration of $N=8$ spins fixed in a line

$\uparrow\downarrow\uparrow\uparrow\uparrow\downarrow\uparrow\uparrow$

Total # of configs: $2 \cdot 2 \cdot 2 \cdots 2 = 2^N = 2^8 = 256$

Page 27

Total (internal) energy $\quad E = (-H)n_+ + (H)n_- = -H(2n_+ - N)$

$$E(\uparrow\downarrow\uparrow\uparrow\uparrow\downarrow\uparrow\uparrow) = -H(2 \cdot 6 - 8) = -4H < 0$$

With energy $E = -4H$ conserved, what fraction of states allowed?

$$\text{Fraction} = \frac{\#\text{ allowed}}{\#\text{ total}} = \frac{\binom{8}{6}}{2^8} = \frac{8 \cdot 7/2}{256} = \frac{28}{256} = \frac{7}{64} \approx 11\%$$