

Plan

Ising model ground states vs. lattice structure

Exam prep

Ground states

$$H=0 \rightarrow E = -J \sum_{\langle i,k \rangle} s_i s_k$$

$J > 0$ :  $E = -d \cdot N$  for  $d$ -dim'l cubic lattice

( $J=1$ ) all spins aligned (degen. 2)

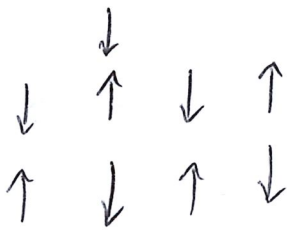
ordered phase (ferromagnetic)

order parameter magnetization  $|m|=1$

All links interchangeable - lattice structure doesn't matter

$J < 0$ :

Aligned spins increases energy  $\rightarrow$  opposing spins in ground state  
 maximize



Fully alternating ground state (degen 2)

anti-ferromagnetic

$$m = \frac{n_+ - n_-}{n_+ + n_-} = 0 \text{ like disordered phase}$$

But clear pattern/order

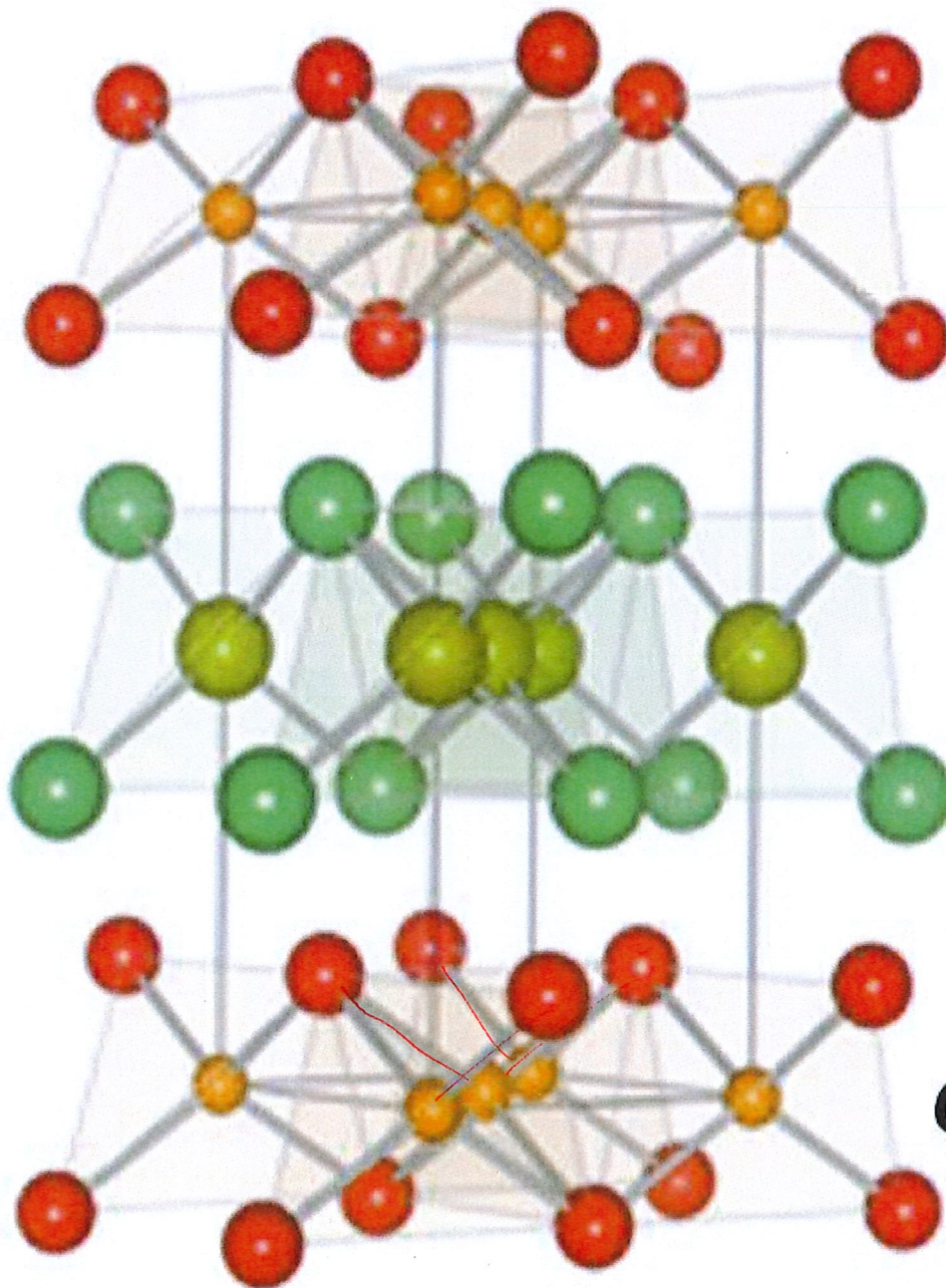
Parameter:  $\frac{1}{N} \sum_n (-1)^n s_n = m_s$  defines "staggered magnetization"

$|m_s|=1$  vs.  $m_s=0$  in disordered phase

Relate to free energy with "staggered magnetic field"

$$E = -J \sum_{\langle i,k \rangle} s_i s_k - H_s \sum_n (-1)^n s_n \rightarrow \langle m_s \rangle \propto \frac{\partial}{\partial H_s} F$$

set  $H_s=0$



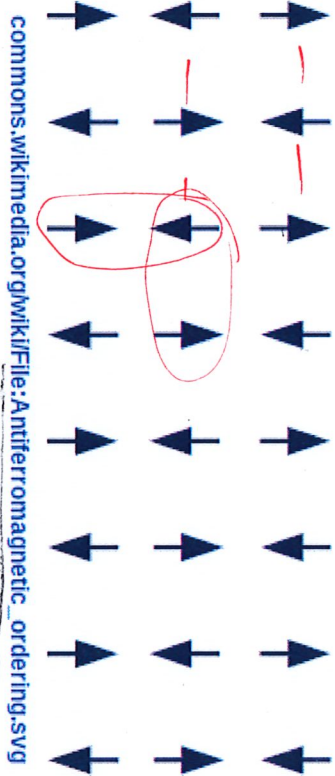
High  $T_c \sim 50K$

**Pn** <sup>nitrogen</sup>

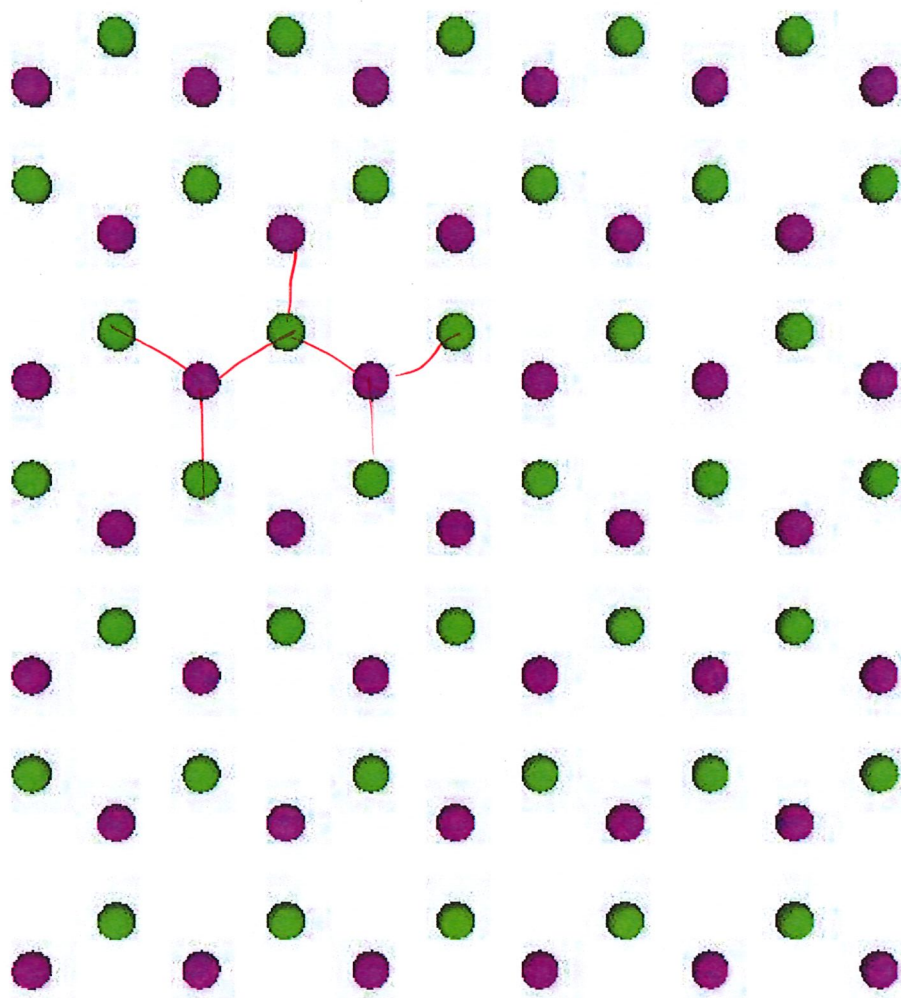
**Fe**

**Ln**

**O,F**



[commons.wikimedia.org/wiki/File:Antiferromagnetic\\_ordering.svg](https://commons.wikimedia.org/wiki/File:Antiferromagnetic_ordering.svg)



# Triangular lattice $J < 0$



Cannot order into fully alternating ground state

"Frustrated antiferromagnet"  
(non-bipartite)

6 degen. configs with  $E = -J(-2+1) = J < 0$

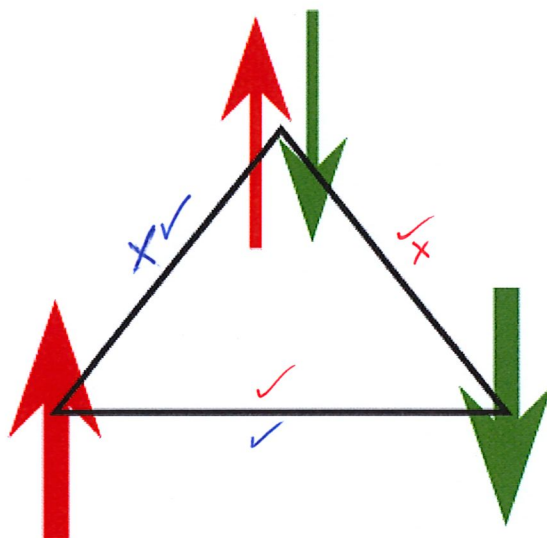
$N \gg 1$  has no order, no order param

many degenerate micro-states

$\rightarrow S > 0$  even at absolute zero

Finding ground state is NP-hard  $\rightarrow$  quantum computing

?



cond-mat/0408370



# MATH327: Statistical Physics

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Spring Term 2023

## LECTURE NOTES

Last modified 11 May 2023

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