Tue 25 Apr 11 56 07

Interacting systems – much harder to analyze
needed to describe phenomena
like phase transitions

Same set of particles → very different emergent behaviour
ice vs. water vs. steam from H_2O
nuclei vs. quark-gluon plasma in early Universe
electrons in bilayers graphene
insulating → superconducting at magic $\theta = 1.1^\circ$
no energy loss!
need $T \lesssim 1.7\,K$

What precisely distinguishes interacting or not?
Recall non-interacting spin systems

$$E_i = -H \sum_{n=1}^{N} s_n = \sum_{n=1}^{N} \varepsilon_n \qquad s_n = \pm 1$$

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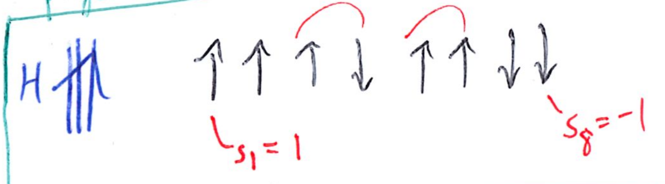

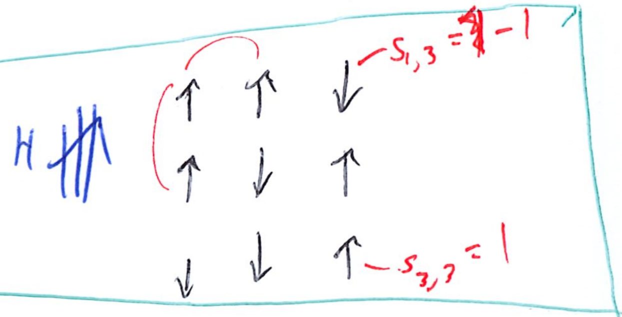

Fix spins in d-dim'l lattice
Micro-state ω_i given by $\{s_n\}$
Factorization → $Z_N = Z_1^N = (2\cosh(\beta H))^N$ extremely simple

Definition:
Consider change ΔE_j from alteration to jth particle
Non-interacting iff. ΔE_j independent of all particles $k \neq j$

Flip spin $s_j \to -s_j$

$$E = -H\Big(s_j + \cancel{\sum_{k\neq j}} s_k\Big) \longrightarrow -H\Big(-s_j + \cancel{\sum_{k\neq j}} s_k\Big)$$

$\Delta E_j = 2Hs_j$
indep. of s_k $k \neq j$
non-interacting ✓

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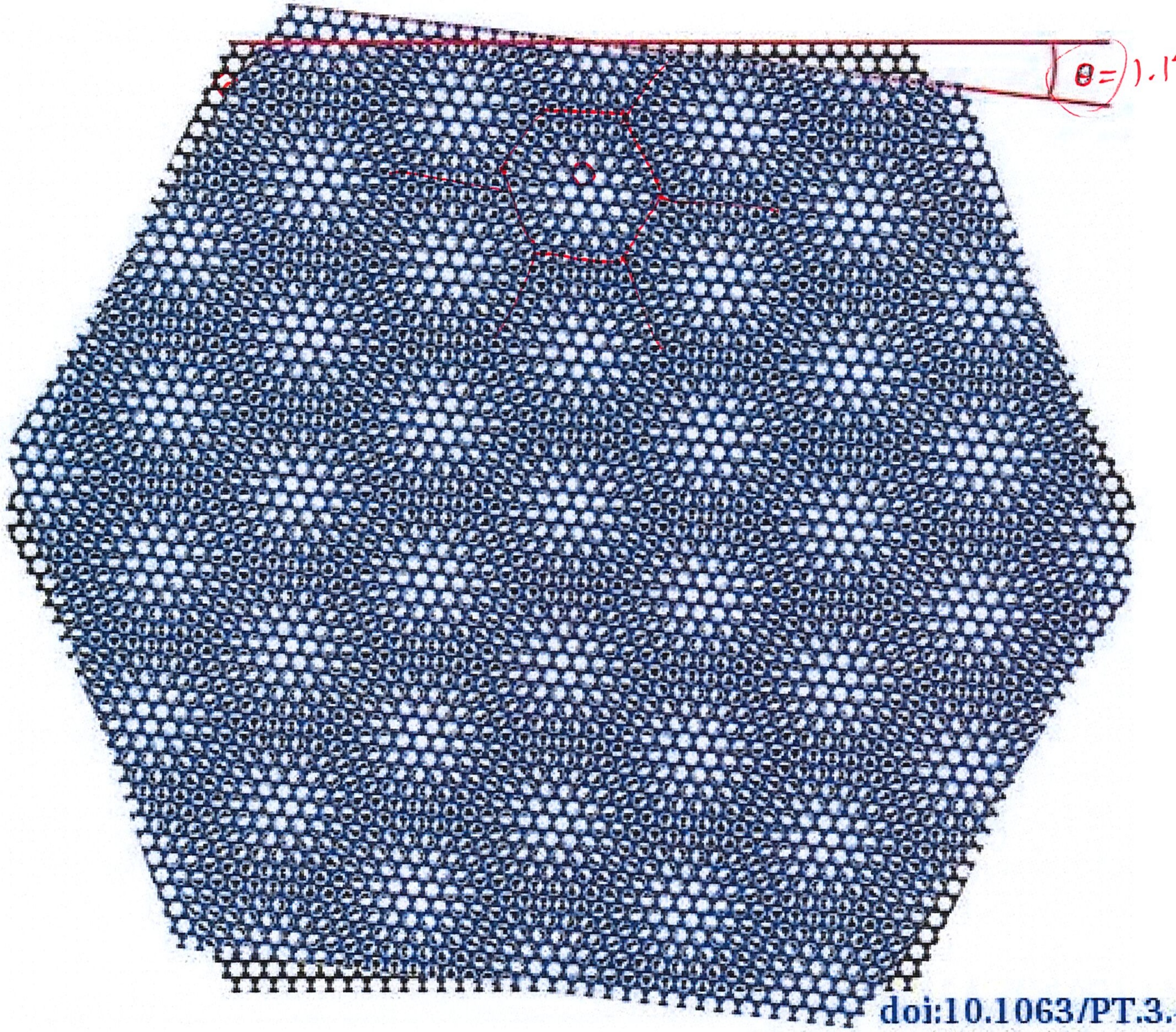

doi:10.1063/PT.3.4384

More interesting:

$$E_i = -\sum_{(j,k)} s_j s_k - H \sum_n s_n$$

all pairs of nearest-neighbour (n.n.) spins

Again flip $s_j \to -s_j$

$$E = -s_j \sum_{k\in(jk)} s_k - \cancel{\sum_{(mk)\not\ni j} s_m s_k} - H\Big(s_j + \cancel{\sum_{k\neq j} s_k}\Big)$$

$$\to s_j \sum_{k\in(jk)} s_k - \cancel{\sum_{(mk)\not\ni j} s_m s_k} - H\Big(-s_j + \cancel{\sum_{k\neq j} s_k}\Big)$$

$$\Delta E_j = 2 s_j \Big(H + \sum_{k\in(jk)} s_k\Big)$$

Depends on s_k with $k \neq j$ → interacting

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n.n. pairs depend on lattice structure

$E(s_n)$ & lattice → Ising model (by Lenz)

d-dim'l cubic lattice

<u>Sites</u> where spins are located

<u>Links</u> correspond n.n. pairs

Simplify lattice by wrapping it into a d-dim'l torus

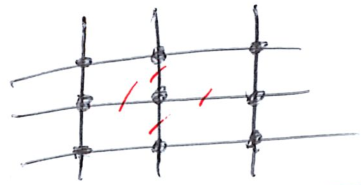
periodic boundary conditions (PBC)

Flat space, constant distance along links

Count links per site

1d: 

2 links (n.n.) per site each shared by 2 sites $\Big\}$ $\#l = \frac{2N}{2} = N$

2d: 

$\#l = \frac{4N}{2} = 2N$

General d-dim'l hypercubic lattice: $\#l = \frac{2d \cdot N}{2} = d \cdot N$ total links

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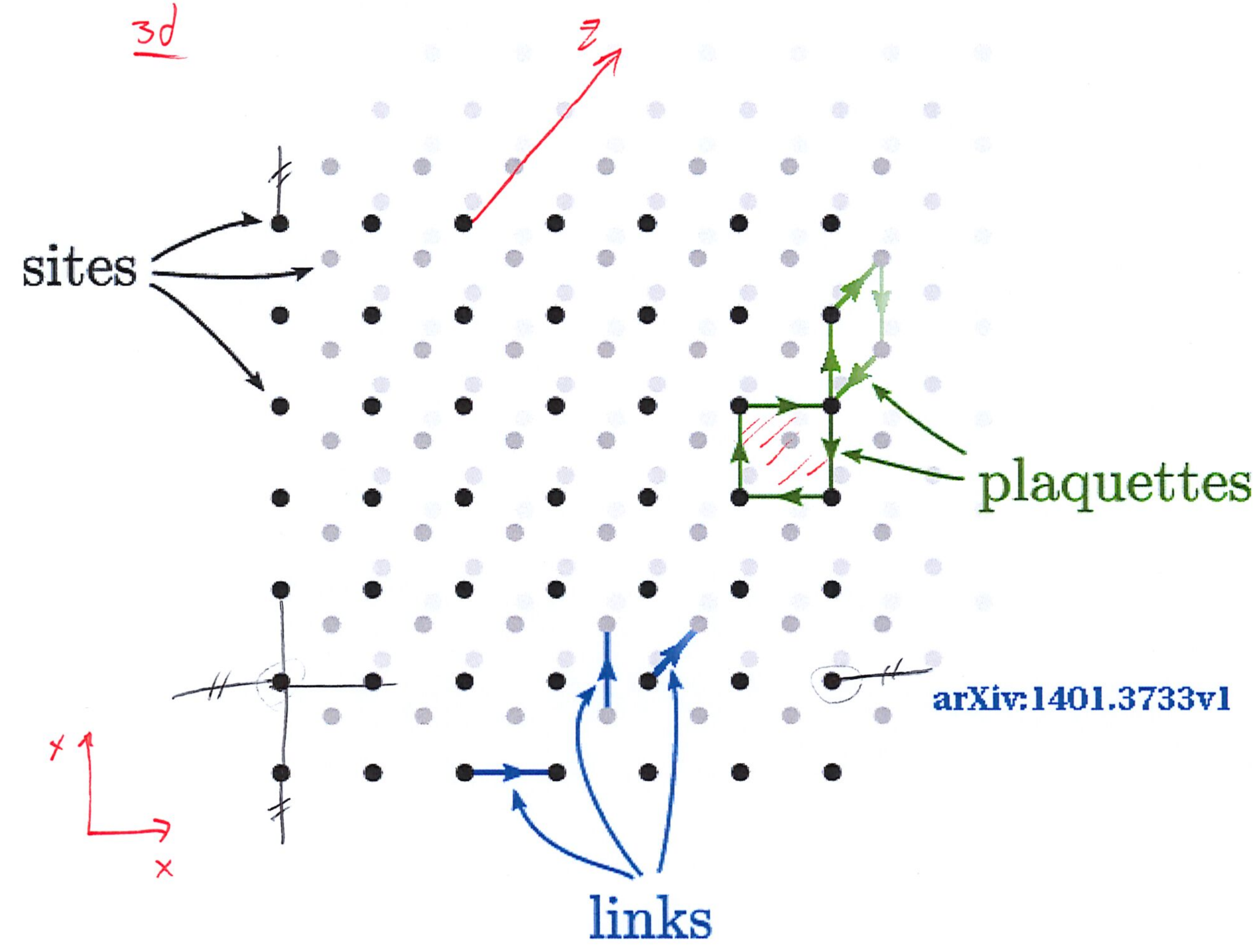

3d
z
sites
plaquettes
links
x
y
arXiv:1401.3733v1

Can we solve Ising model canon. part. func.?

$$Z(\beta, N, H) = \underbrace{\sum_{\{s_n\}}}_{2^N \text{ terms}} \exp\Big[\beta \underbrace{\sum_{\langle j,k\rangle} s_j s_k}_{d\cdot N \text{ terms}} + \beta H \underbrace{\sum_n s_n}_{N \text{ terms}}\Big]$$

Interacting → no factorization

$d=1$: Exact solution by Ising (1924)

$d=2$: Exact solution for $H=0$ by Onsager (1944)

$3 \le d < \infty$ no known exact solution

Brute-force numerical solution impractical

Tiny 10×10 → $N=100$

→ $2^{100} \times (300) \sim 10^{32}$ terms

Billion terms per sec → 10^{23} sec $\sim 10^{15}$ years

$\sim 500{,}000 \times$ age of Universe

Plan: High-T & low-T limits

Simple approximation

Smarter computing