

New strong dynamics beyond the standard model

Lecture 8

19 December 2017

Last time

- Early-universe production of $\Omega_{DM} \approx 5\Omega_B$ relies on DM–SM interactions
Symmetric thermal relic freezes out when universe’s expansion dilutes DM,
making DM \rightarrow SM annihilation inefficient (expect $M_{DM} \sim 100$ TeV)
Dark and baryon [asymmetries](#) related through EW sphalerons ($M_{DM} \sim 1$ TeV)
or high-dimensional effective interactions ($M_{DM} \sim 5$ GeV)
- Ongoing dark matter searches constrain DM–SM interactions
Direct detection of DM scattering with SM
Indirect detection of SM products from DM annihilation or decay,
challenging to distinguish from complicated astrophysical backgrounds
Collider searches for dark-sector particles produced in high-energy collisions
- Typical composite DM candidates are analogs of glueballs, mesons or baryons
- ‘Mesonic’ DM candidates generally need extra symmetries to suppress decays
SIMP models of PNGB DM use SM-neutral fermions
coupled to massive ‘dark photon’ kinetically mixed with hypercharge
‘Quirkonium’ DM is heavy diquark stabilized by global U(1) symmetry;
production via EW sphalerons \rightarrow ruled out by direct-detection experiments

Representative models of non-mesonic composite DM

- [Glueball DM candidates](#) from pure-gauge SU(N) Yang–Mills at low energies,
with heavy particle ($M_{UV} \gg \Lambda_{DM}$) coupled to both SM and dark sector
Again analyze through EFT, decay via high-dim ($d \geq 6$) operators
 $\frac{1}{M_{UV}^2} H^\dagger H \text{Tr} [G_{\mu\nu} G^{\mu\nu}]$ and $\frac{1}{M_{UV}^4} \text{Tr} [F_{\mu\nu} F^{\mu\nu}] \text{Tr} [G_{\mu\nu} G^{\mu\nu}] \rightarrow \Gamma \propto \frac{\Lambda_{DM}^9}{M_H^4 M_{UV}^4}$ and $\frac{\Lambda_{DM}^9}{M_{UV}^8}$
Again can freeze out through $3 \rightarrow 2$ process \rightarrow sub-GeV $M_{DM} \sim \Lambda_{DM}$ possible
- [First](#) (asymmetric) ‘dark baryon’ was ‘technineutron’ of scaled-up QCD
SU(3) baryon decay through $d \geq 6$ operators \rightarrow cosmological stability
Like neutron, unsuppressed Z exchange \rightarrow direct-detection $\sigma \simeq 10^{-2}$ pb
ruled out [~30 years ago](#)
- Avoid by making lightest dark baryon be a singlet under entire SM
Like SIMP, can make elementary fermions SM singlets,
couple DM–SM through [Higgsed dark U\(1\)](#) kinetic mixing
Alternately, construct singlet dark baryon composed of non-singlet fermions
- Focus on obtaining viable DM candidate, set aside composite Higgs connections

Dark baryon direct detection via EM form factors

- **Simplest way** to obtain SM-singlet dark baryon from QCD-like theory:
make all fermions weak singlets with electric charges $Q = Y = \frac{2}{3}, -\frac{1}{3}$ for SU(3)

- Weak singlets \rightarrow no Z exchange in direct detection

Electrically charged constituents \rightarrow photon exchange through form factors

$$\langle \text{DM}(p') | \Gamma_\mu(q) | \text{DM}(p) \rangle \simeq F_1(q^2) \gamma_\mu + F_2(q^2) \frac{i \sigma_{\mu\nu} q^\nu}{2M_{DM}} \quad q = p' - p$$

- Non-relativistic $|q^2| \ll \Lambda_{DM}^2 \rightarrow$ effective interactions suppressed by $\Lambda_{DM} \sim M_{DM}$

$$\frac{1}{\Lambda_{DM}} \bar{\psi} \sigma^{\mu\nu} \psi F_{\mu\nu} \rightarrow \sigma \propto Z^2 \frac{\kappa^2}{M_{DM}^2} \quad (Z \text{ is atomic number of target})$$

where $\kappa = F_2(0) = \frac{\mu}{2M_{DM}}$ is **magnetic moment**

$$\frac{1}{\Lambda_{DM}^2} \bar{\psi} \gamma^\nu \psi \partial^\mu F_{\mu\nu} \rightarrow \sigma \propto Z^2 \frac{q^2 (M_{DM}^2 \langle r^2 \rangle)^2}{M_{DM}^4}$$

where $M_{DM}^2 \langle r^2 \rangle = \frac{3}{2} \kappa - 6M_{DM}^2 \left. \frac{dF_1(q^2)}{dq^2} \right|_{q^2=0}$ is the electric **charge radius**

- Non-perturbative form factors \rightarrow compute on lattice from three-point function

$$C_\mu(\tau, T, p, p') = \sum_{\vec{x}, \vec{y}} e^{-i\vec{x}\cdot\vec{p}'} e^{-i\vec{y}\cdot(\vec{p}'-\vec{p})} \text{Tr} \left[B(\vec{x}, T) \bar{\psi}(\vec{y}, \tau) \gamma_\mu \psi(\vec{y}, \tau) B^\dagger(\vec{0}, 0) \right]$$

Ratio w.r.t. baryon two-point correlator $C(T, p) = \sum_{\vec{x}} e^{-i\vec{x}\cdot\vec{p}} \text{Tr} \left[B(\vec{x}, T) B^\dagger(\vec{0}, 0) \right]$ is

$$R_\mu(\tau, T, p, p') \rightarrow \langle B(p') | \Gamma_\mu(q) | B(p) \rangle + \mathcal{O} \left(e^{-M\tau}, e^{-MT}, e^{-M(T-\tau)} \right)$$

- **DWF results** not very sensitive to mass or N_f
 \rightarrow **require** $M_{DM} \gtrsim 20$ TeV, mainly due to magnetic moment

Mixed fermion masses for dark baryon

- Danger: Charged dark-sector mesons might not decay by Big Bang Nucleosynthesis (or pre-BBN dim-5 meson decay \rightarrow small $\Lambda_{UV} \rightarrow$ dim-6 baryon decay)

- Switching back to weak-nonsinglet fermions would solve this problem

\rightarrow rapid Π^\pm decay to $f'\bar{f}$ just like pion, $\langle 0 | A_\mu^\pm | \Pi^\pm \rangle = i f_\Pi p^\mu \rightarrow \Gamma \propto \frac{f_\Pi^2}{v^4} m_f^2 m_\Pi$

- May reintroduce couplings to Higgs \rightarrow another direct-detection constraint

- To avoid fate of quirky DM, **demand mixture** of vector-like and Higgs mass terms

$$\Psi_A = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} \text{ and } \Psi_B = \begin{pmatrix} \psi_3 \\ \psi_4 \end{pmatrix} \text{ in } (2, 1) \text{ and } (1, 2) \text{ reps of } \text{SU}(2)_L \times \text{SU}(2)_R$$

$\rightarrow \mathcal{L} \supset m_V (\bar{\Psi}_A \Psi_A + \bar{\Psi}_B \Psi_B) + y_{A,B} \bar{\Psi}_{A,B} \cdot H \cdot \Psi_{B,A} + \text{h.c.}$ with $Q = T_L^3 + T_R^3 = \pm \frac{1}{2}$

- $\alpha = \frac{yv}{yv+m_V}$ parameterizes Higgs coupling contribution to total $m_\psi = yv + m_V$

Dark baryon direct detection via Higgs exchange

- Higgs exchange depends on **scalar** form factors (dark matter D , nucleon N)

$$\sigma_H \propto \left| \frac{\mu_{D,N}}{M_H^2} y_\psi \langle D | \bar{\psi}\psi | D \rangle y_q \langle N | \bar{q}q | N \rangle \right|^2 \quad y_\psi = \alpha \frac{m_\psi}{v}$$

Fermion-line-**disconnected diagrams** \rightarrow direct lattice calculation challenging

- Can evaluate indirectly using **Feynman–Hellman theorem** $\langle D | \bar{\psi}\psi | D \rangle = \frac{\partial M_{DM}}{\partial m_\psi}$
- Static $\langle D | H | D \rangle = M_{DM}$ while $\langle D | D \rangle$ is a constant normalization

$$\begin{aligned} \rightarrow \frac{\partial M_{DM}}{\partial m_\psi} &= \frac{\partial}{\partial m_\psi} \langle D | H | D \rangle = \left\langle \frac{\partial D}{\partial m_\psi} | H | D \right\rangle + \langle D | H | \frac{\partial D}{\partial m_\psi} \rangle + \left\langle D \left| \frac{\partial H}{\partial m_\psi} \right| D \right\rangle \\ &= M_{DM} \left\langle \frac{\partial D}{\partial m_\psi} | D \right\rangle + M_{DM} \left\langle D | \frac{\partial D}{\partial m_\psi} \right\rangle + \left\langle D \left| \frac{\partial H}{\partial m_\psi} \right| D \right\rangle \\ &= M_{DM} \frac{\partial}{\partial m_\psi} \langle D | D \rangle + \langle D | \bar{\psi}\psi | D \rangle = \langle D | \bar{\psi}\psi | D \rangle \end{aligned}$$

- Wilson-fermion **spectrum computations** \rightarrow scalar form factor \rightarrow constrain α
Smaller α required as m_ψ/Λ_{DM} (equivalently M_{PS}/M_V) increases
 BBN requires only very small $\alpha \ll 0.01 \rightarrow$ Higgs exchange can be neglected

Stealth Dark Matter

- Final feature of **Stealth DM**: SU(4) gauge group [more generally SU(2N), $N \geq 2$]
 Scalar baryon DM \rightarrow no magnetic moment
 Custodial SU(2) \rightarrow no charge radius $\alpha \ll 0.01 \rightarrow$ neglect Higgs exchange

- Electric **polarizability** C_F puts **lower bound** on direct-detection cross section

$$\frac{1}{\Lambda_{DM}^3} \bar{\psi}\psi F^{\mu\nu} F_{\mu\nu} \rightarrow \sigma \propto Z^4 \frac{\mu_{D,\text{Xe}}^2}{R_A^2} \frac{C_F^2}{M_{DM}^6} \quad \text{Note } Z^4 \text{ rather than } Z^2$$

- **Compute C_F on lattice** as Stark shift from static background electric field \mathcal{E}

$$E_{DM}(|\mathcal{E}|) = M_{DM} + \left(2C_F a^3 - \frac{\kappa^2}{8M_{DM}^3} \right) \left| \frac{\mathcal{E}}{a^2} \right|^2 + \mathcal{O} \left(\left| \frac{\mathcal{E}}{a^2} \right|^4 \right)$$

Quantized $\mathcal{E} = \frac{4\pi n}{N_t N_s}$ induces and interacts with dipole moment \rightarrow quadratic

- Comparable C_F for both SU(3) and SU(4) suggests Pauli pairing not significant

- **Xe results** require $M_{DM} \gtrsim 300$ GeV, comparable to collider constraint

Cross section below neutrino floor for $M_{DM} \gtrsim 1$ TeV

Uncertainties dominated by nuclear matrix element (two photons $\sim \beta\beta$ decay)

Collider searches for Stealth DM

- Dark baryon typically much heavier than unstable electrically charged mesons,
unlike R-symmetric susy setups where DM candidate is lightest susy particle
→ Very different collider strategy: Search for lightest ‘ Π^\pm ’ rather than missing E_T
- Π decay width $\Gamma \propto \frac{f_\Pi^2}{v^4} m_f^2 m_\Pi$ discussed above
→ dominant decays into heaviest SM fermions
($t\bar{b}$ if $M_\Pi \gtrsim 180$ GeV, otherwise $\tau\bar{\nu}_\tau$ and $c\bar{s}$)
- Wilson-fermion [spectrum computations](#) discussed above find $M_{DM}/M_\Pi \simeq \text{few}$
(we consider $m_f \simeq \Lambda_{DM}$ rather than light PNGB limit or heavy quarkonium limit)
- LEP searches for susy $\tilde{\tau} \rightarrow \tau\bar{\nu}_\tau$ → $M_\Pi \gtrsim 90$ GeV → $M_{DM} \gtrsim 200\text{--}300$ GeV
- Current work assumes Drell–Yan production proportional to electric charge of Π^\pm
Future refinement can incorporate vector form factor $F_V^{(\Pi)}(q^2)$

Course wrap-up

- **Main goal:** Introduction to large and evolving literature
on composite Higgs, composite DM, and role of lattice gauge theory
Focus on big-picture context and concepts rather than detailed calculations
- Should have foundation to better understand new results on the arXiv,
and explore more details of topics that interest you
- More experimental searches are underway and planned
hopefully leading to new discoveries about the world beyond the standard model!

Supplement: Indirect detection of Stealth DM

- Two main sources of SM cosmic rays from dark sector:
 - Baryonic $\text{DM}-\overline{\text{DM}}$ annihilation into (several) lighter Π that then decay γ -rays from radiative decays of [higher-spin baryons](#)
- Annihilation difficult on lattice
 - Need to consider all possible final states,
 - those with many particles very hard to handle
- Wilson-fermion [spectrum computations](#) discussed above
 - can predict splittings between baryons with different spins
 - Significant model dependence remains and systematic studies not yet done

Supplement: Gravitational waves from composite DM

- Two sources of potentially observable gravitational waves from dark sector
- First, glueball DM \rightarrow ‘dark stars’ could produce [binary merger](#) signals
- Second, if early-universe confinement transition is first order
 - then it could produce a [stochastic background](#) of gravitational waves (most likely not observable by LIGO, possibly observable by eLISA)
- First-order transition [expected](#) for $F \geq 3$ sufficiently light fermions
 - or any number of sufficiently heavy fermions (the [Columbia plot](#))
- Gravitational waves produced by collisions of expanding vacuum bubbles
 - \rightarrow need to determine bubble nucleation rate, expansion speed (from latent heat)

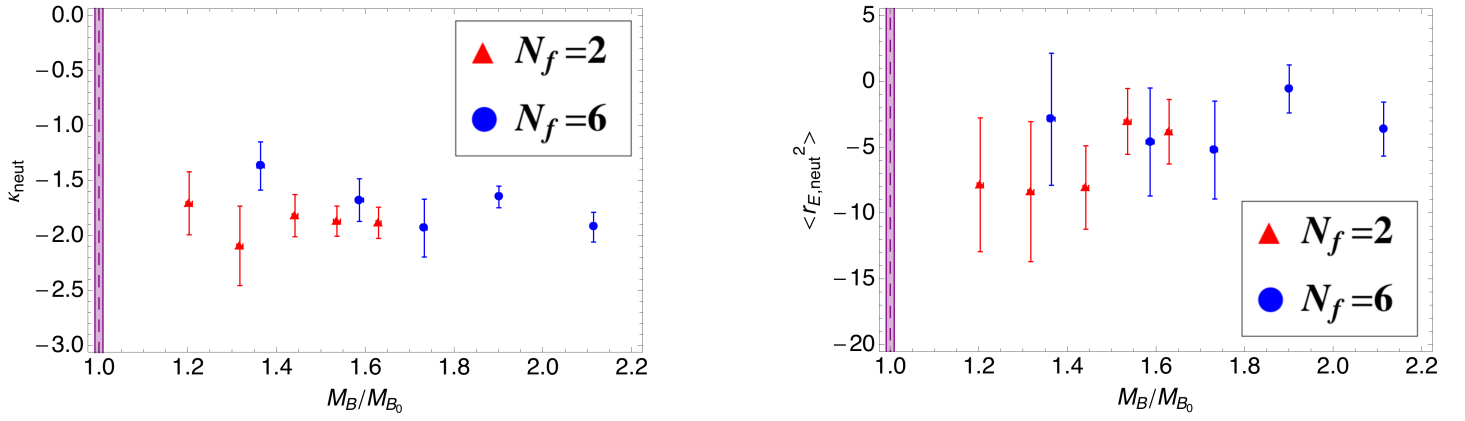


Figure 1: Domain-wall fermion lattice results (from [arXiv:1301.1693](https://arxiv.org/abs/1301.1693)) for the magnetic moment (left) and electric charge radius (right) of the dark baryon in QCD-like SU(3) gauge theories with $N_F = 2$ (red triangles) or $N_F = 6$ (blue circles) degenerate fundamental fermions. The horizontal axis is the mass of the dark baryon normalized by its value M_{B_0} in the chiral limit of vanishing fermion masses. No significant sensitivity to either the mass or N_F is visible.

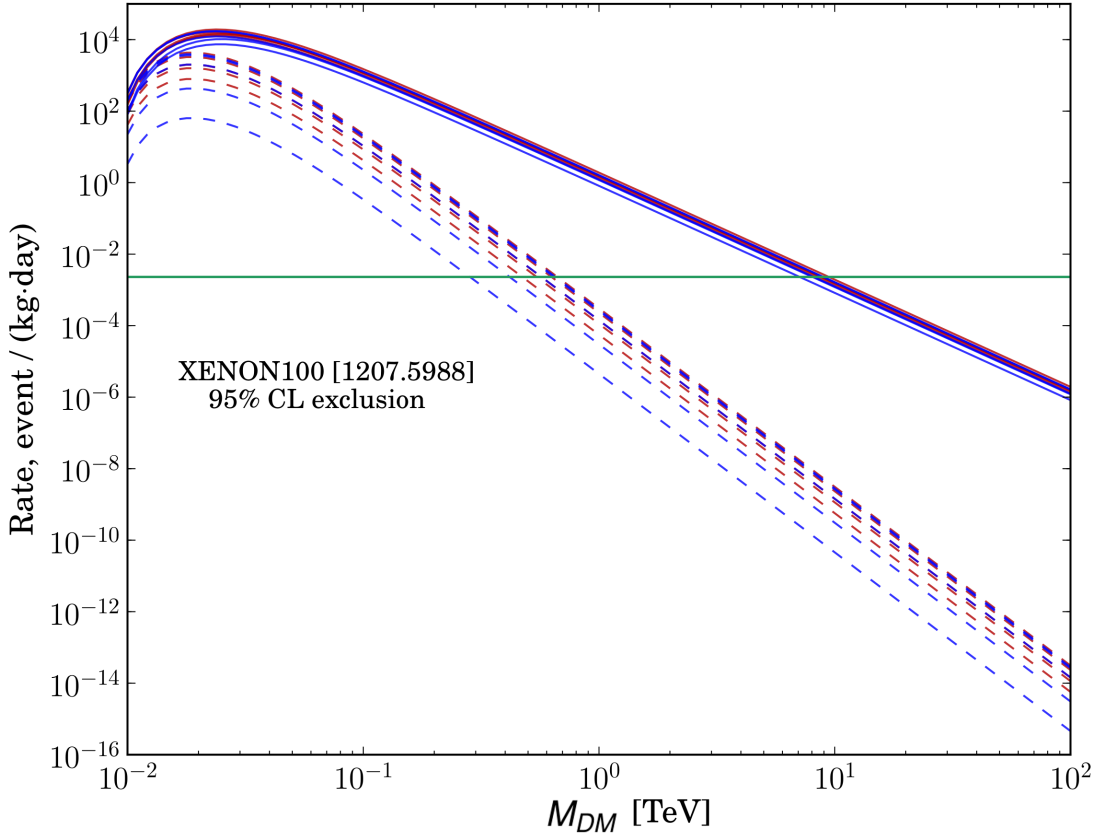


Figure 2: The direct-detection event rate in the XENON100 detector predicted (in [arXiv:1301.1693](https://arxiv.org/abs/1301.1693)) as a function of the dark baryon mass from the magnetic moment and electric charge radius results shown in Fig. 1, omitting uncertainties. The solid lines are the total event rate from both form factors, while the dashed lines are the charge radius contribution. The charge radius contribution is suppressed $\sim 1/M_{DM}^2$ compared to the magnetic moment contribution, causing the latter to dominate for $M_{DM} \gtrsim 50$ GeV. The horizontal line shows the 2012 XENON100 constraints that require $M_{DM} \gtrsim 10$ TeV, which has tightened to $M_{DM} \gtrsim 20$ TeV due to more recent experiments such as LUX.

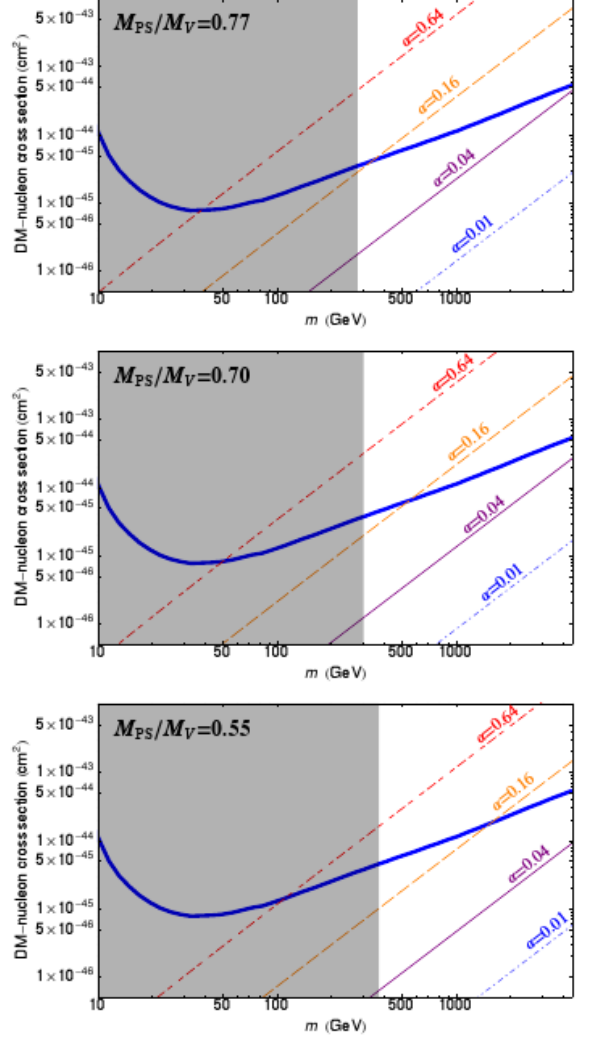
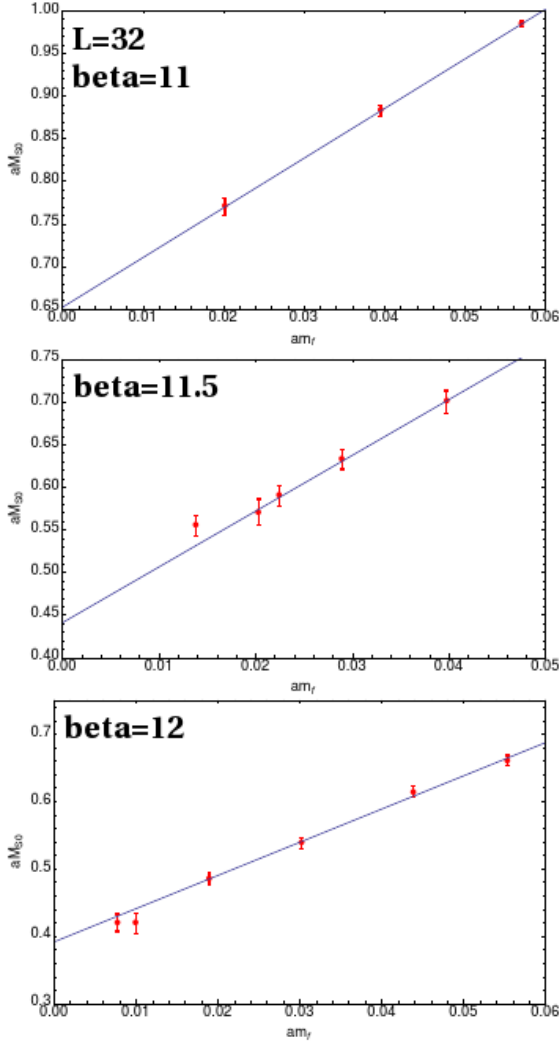


Figure 3: **Left:** Wilson fermion lattice results for the dark baryon mass vs. the renormalized fermion mass. From top to bottom the inverse bare coupling $\beta \simeq 6/g_0^2$ increases, implying a decrease in the g_0^2 that corresponds to increasing the UV cutoff $\Lambda_{UV} = a^{-1}$. The slope of each linear fit gives the scalar form factor $\langle D | \bar{\psi}\psi | D \rangle$ through the Feynman–Hellman theorem. **Right:** The scalar form factor predicts the direct-detection Higgs-exchange cross section, shown for four values of $\alpha = yv/m_\psi$ vs. the dark baryon mass. The thick blue line is the upper bound set by LUX in 2013, while the shaded region is excluded by collider searches. From top to bottom the fermion mass m_ψ decreases, corresponding to a decreasing ratio of pseudo-scalar (PNGB) and vector masses. ($M_{PS}/M_V \rightarrow 0$ in the $m_\psi \rightarrow 0$ chiral limit, and approaches unity in the heavy-mass limit $m_\psi \rightarrow \infty$.) All plots adapted from [arXiv:1402.6656](https://arxiv.org/abs/1402.6656).

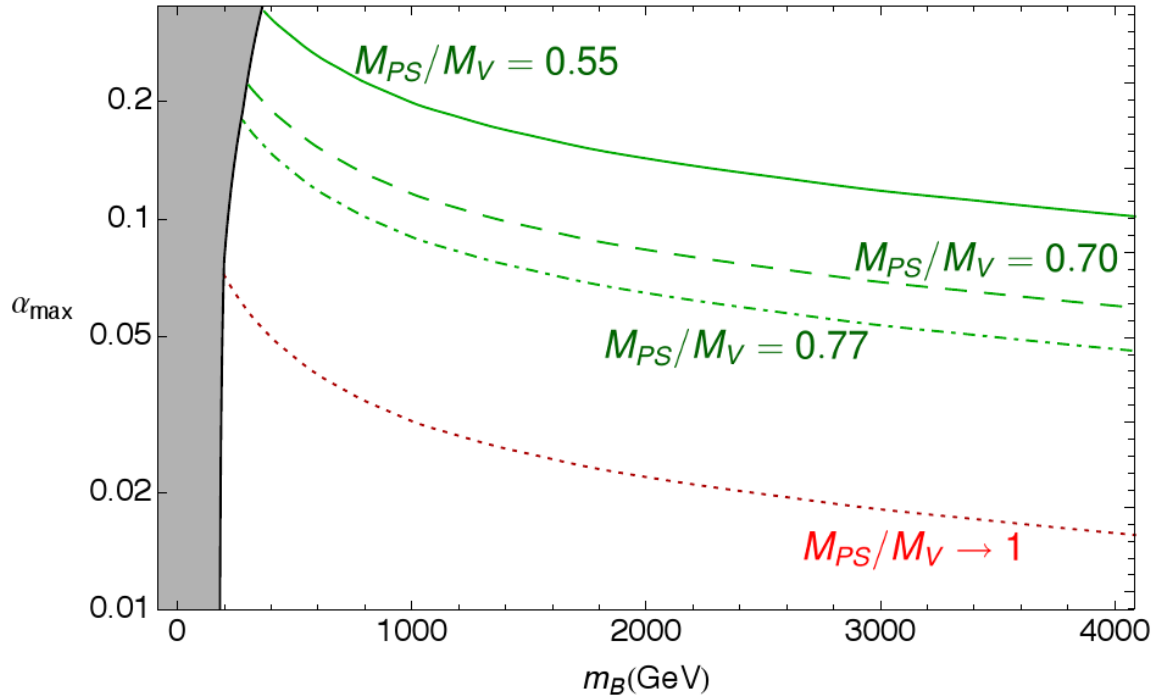


Figure 4: The three plots on the right of Fig. 3 predict the maximum $\alpha = yv/m_\psi$ allowed by LUX for a given dark baryon mass at fixed fermion mass m_ψ (represented by the ratio of pseudo-scalar (PNGB) and vector masses). In this plot (from [arXiv:1402.6656](https://arxiv.org/abs/1402.6656)) those maximum α are shown by the three green curves. As m_ψ increases M_{PS}/M_V approaches unity and the dark fermion Higgs coupling becomes more constrained, requiring smaller values of α . As in Fig. 3 the shaded region is excluded by collider searches.

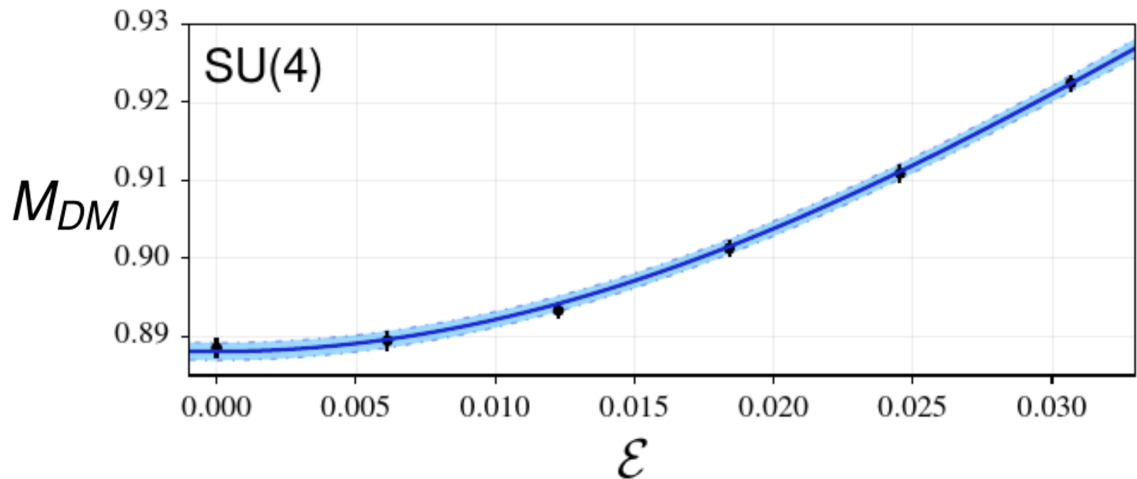


Figure 5: Wilson fermion lattice results (from [arXiv:1503.04205](https://arxiv.org/abs/1503.04205)) for the dark baryon mass as a function of the static background electric field \mathcal{E} , for $\beta = 11$ and $M_{PS}/M_V = 0.70$. The finite lattice volume $32^3 \times 64$ quantizes the accessible values of $\mathcal{E} = \frac{4\pi n}{N_t N_s}$. The coefficient of the $|\mathcal{E}|^2$ term in the blue fit is proportional to the electric polarizability C_F since the magnetic moment $\kappa = 0$ for the lightest (scalar) baryon of SU(4) gauge theory. The fit also includes an $|\mathcal{E}|^4$ term to confirm that higher-order effects are negligible.

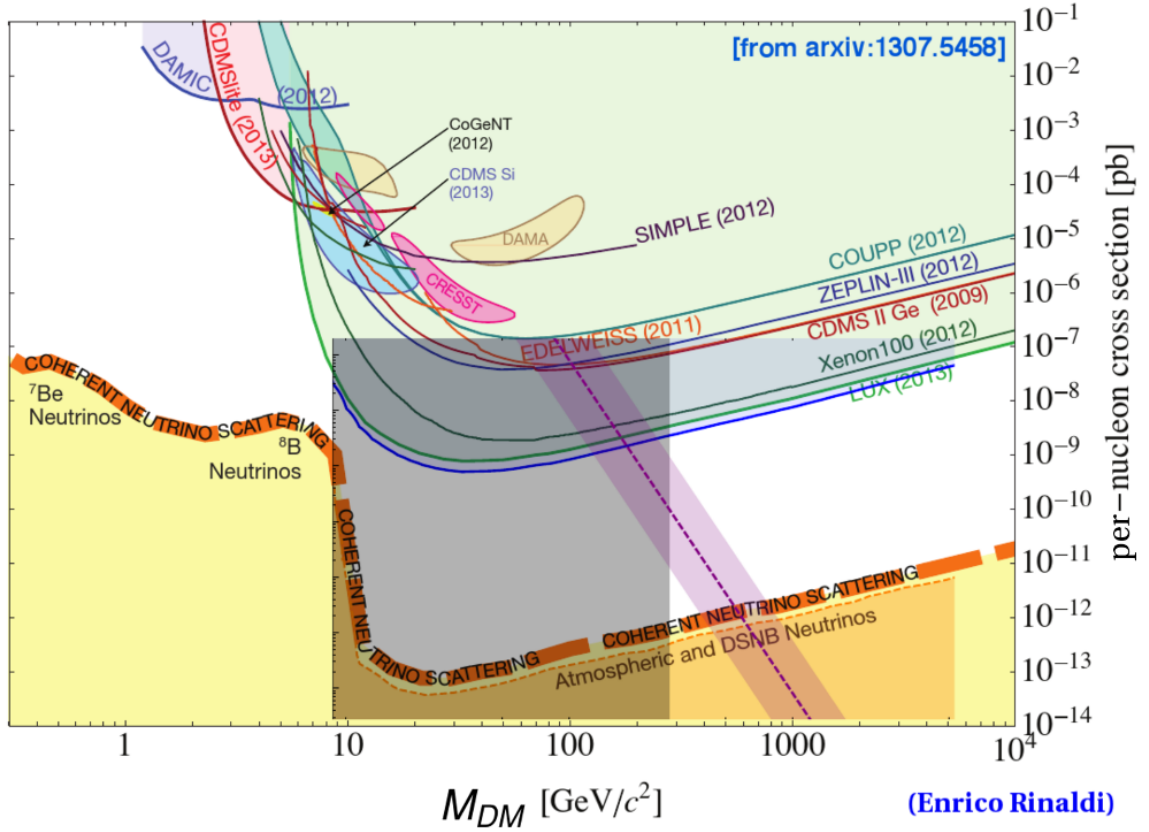


Figure 6: The polarizability computed from Fig. 5 predicts the direct-detection cross section shown by the dashed purple line. Due to the Z^4 dependence of the cross section, the results are specific to xenon. The uncertainties on the line are dominated by nuclear physics effects. (The coherent two-photon coupling involves matrix elements similar to those that arise in double- β decay.) Direct detection experiments require $M_{DM} \gtrsim 300$ GeV, but the $1/M_{DM}^6$ dependence of the polarizability cross section leads it to fall below the neutrino floor for $M_{DM} \gtrsim 1$ TeV. The shaded region is again excluded by collider searches.

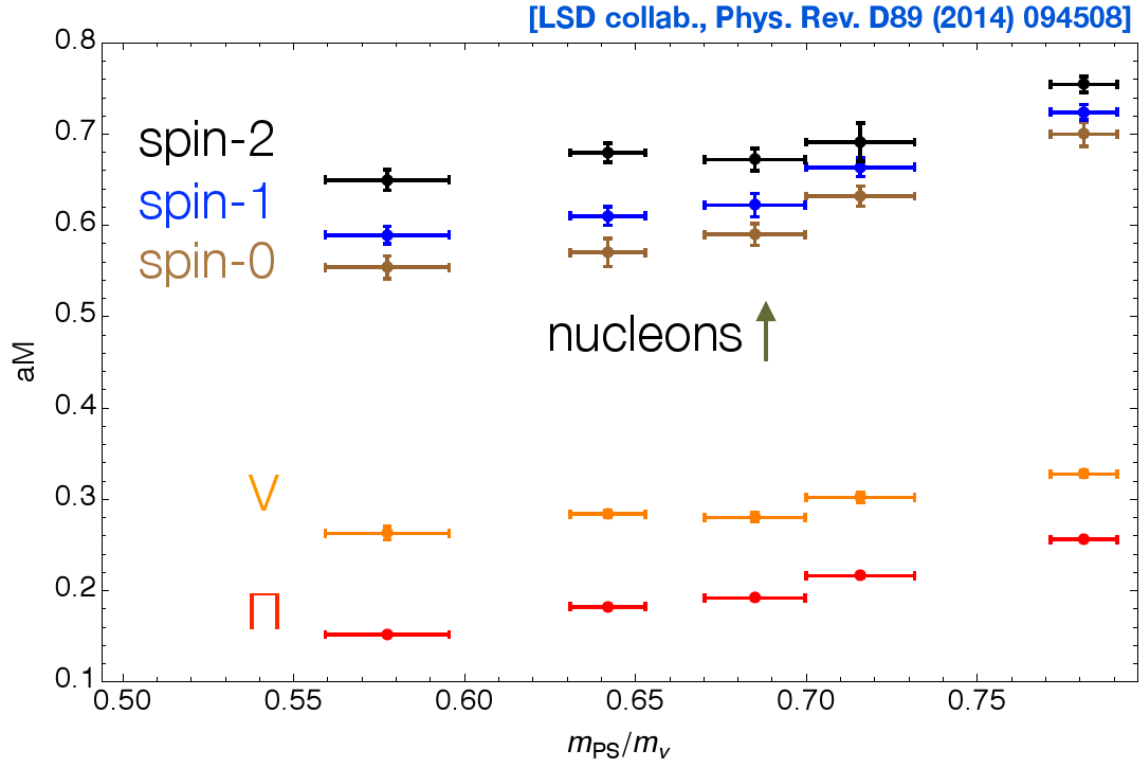


Figure 7: More Wilson fermion lattice results for dark sector masses from [arXiv:1402.6656](https://arxiv.org/abs/1402.6656), plotted against the ratio of pseudo-scalar (PNGB) and vector masses for $\beta = 11.5$ and lattice volume $32^3 \times 64$. From these results we can read off both the ratio M_{DM}/M_{Π} (for collider searches) as well as the splitting between the scalar DM candidate and higher-spin baryons (for indirect detection).

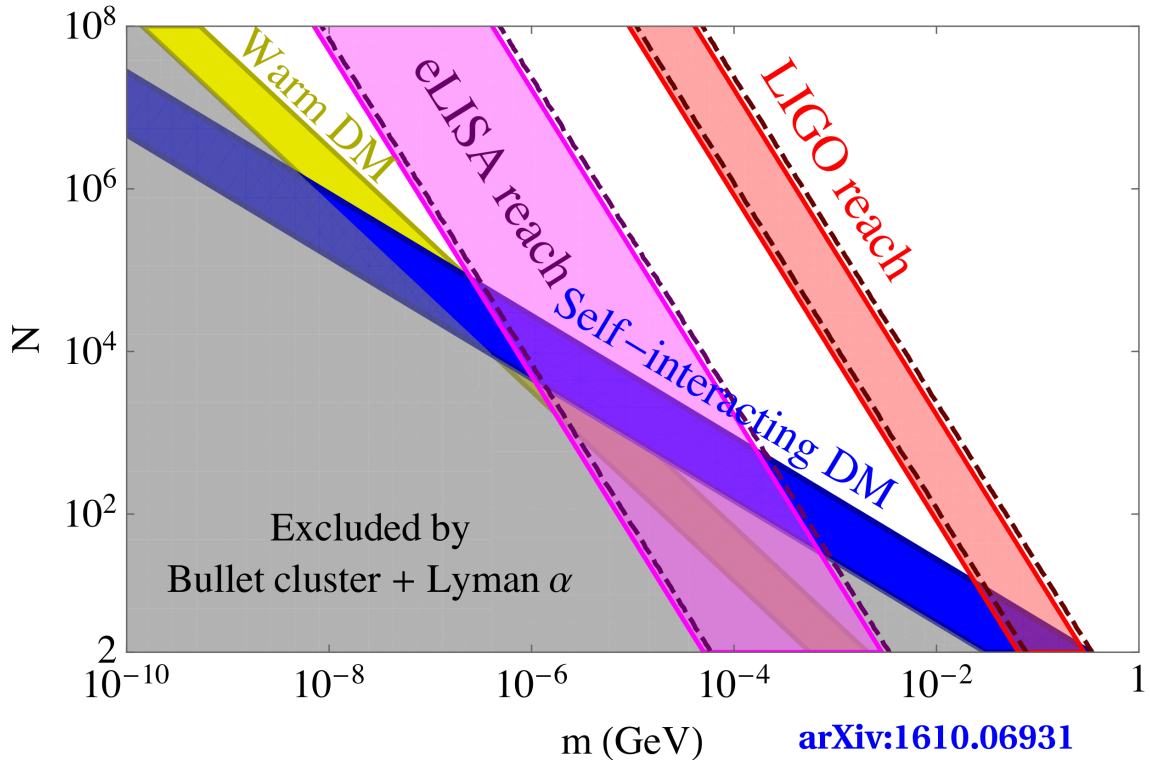


Figure 8: [arXiv:1610.06931](https://arxiv.org/abs/1610.06931) predicts that non-dissipative self-interactions of glueball composite DM lead to ‘dark stars’ that can produce binary inspiral gravitational-wave signals potentially observable by LIGO or future observatories. The signals depend on the number of ‘colors’ of the $SU(N)$ gauge group and the mass m of the glueball DM. The bands on this figure show the range of values of N and m that could produce observable gravitational-wave signals while satisfying certain other constraints.