New strong dynamics beyond the standard model

Lecture 6 5 December 2017

Last time

- Lattice discretization of spacetime regularizes strongly coupled gauge theories
  Lattice spacing $a \longrightarrow$ UV cutoff $\Lambda_{UV} = 1/a$, removed in continuum limit $a \to 0$
- Finite lattice $\longrightarrow$ observables from numerically evaluating functional integral
- Must know fundamental lagrangian at cutoff scale a^{-1} to formulate lattice theory
- Gauge fields $\longrightarrow$ gauge links $U \sim e^{-igaA}$, closed loops of links are gauge invariant
- Lattice fermion discretizations either break chiral symmetry or produce 'doublers'
  Naive: $16F$ continuum fermions, $\mathrm{U}(4F)_V \times \mathrm{U}(4F)_A$ chiral symmetry
  Staggered: $4F$ continuum fermions, $\mathrm{U}(F)_V \times \mathrm{U}(F)_A$ chiral symmetry
  Wilson: F continuum fermions, no chiral symmetry (explicitly broken)
  Domain wall: F fermions, preserves lattice 'remnant' of $\mathrm{SU}(F)_V \times \mathrm{SU}(F)_A$
- Two-point correlators $C(t) \longrightarrow$ composite particle masses and decay constants
  Representative Wilson and staggered results for some systems beyond QCD

S parameter from lattice vacuum polarization $\Pi_{V-A}(Q^2)$

- In lecture 2 we saw $S = 4\pi \left[\Pi'_{VV}(0) - \Pi'_{AA}(0)\right]_{\text{new}} = 4\pi \lim_{Q^2 \to 0} \frac{d}{dQ^2} \Pi^{(\text{new})}_{V-A}(Q^2)$
  Vacuum polarization tensor $\Pi^{\mu\nu}(x)$ is also two-point correlation function,
  but now we Fourier transform rather than projecting to $\vec{p} = 0$

$$\Pi^{\mu\nu}_{V-A}(Q) = Z \sum_n e^{iQ\cdot(n+\widehat{\mu}/2)} \mathrm{Tr}\left[\langle \mathcal{V}_\mu(n) V_\nu(0)\rangle - \langle \mathcal{A}_\mu(n) A_\nu(0)\rangle\right]$$

$$\Pi^{\mu\nu}(Q) = \left(\delta^{\mu\nu} - \frac{Q^\mu Q^\nu}{Q^2}\right)\Pi(Q^2) - \frac{Q^\mu Q^\nu}{Q^2}\Pi^L(Q^2)$$

  Smallest accessible $Q^2 = \left(\frac{2\pi n}{N_t}\right)^2, \ \left(\frac{2\pi n}{N_s}\right)^2$ with $n = 1, 2, \ldots$
- Need domain wall fermions to ensure same renormalization factor $Z_V = Z_A \equiv Z$
- DWF conserved current $\mathcal{V}$ sums over N_5 (similarly for $\mathcal{A}$, with $\mathrm{sign}[s - N_5/2]$),

$$\mathcal{V}_\mu(x) = \sum_{s=1}^{N_5} \left[\overline{\Psi}(x+\widehat{\mu}, s)\frac{1+\gamma_\mu}{2} U^\dagger_\mu(x)\Psi(x,s) - \overline{\Psi}(x,s)\frac{1-\gamma_\mu}{2} U_\mu(x)\Psi(x+\widehat{\mu}, s)\right]$$

  Ward identity would give $Z = 1$, but $V_\mu(x) = \overline{\psi}(x)\gamma_\mu\psi(0)$ on walls $\longrightarrow Z \neq 1$
- Non-perturbatively determine Z by comparing correlators involving $\mathcal{A}_\mu$ vs. A_μ

S parameter from lattice $\Pi_{V-A}(Q^2)$ (continued)

- Extract slope $\Pi'_{V-A}(Q^2=0)$ by fitting to generalized single pole dominance

$$\Pi_{V-A}(Q^2) \simeq -F_P^2 + \frac{Q^2 F_V^2}{Q^2+M_V^2} - \frac{Q^2 F_A^2}{Q^2+M_A^2} \quad \longrightarrow \quad \frac{a_0 + a_1 Q^2}{1 + b_1 Q^2 + b_2 Q^4} \tag{1}$$

  (Weinberg sum rule $F_V^2 - F_A^2 = F_P^2$ motivates dropping Q^4 term in numerator)

- Result: S decreases compared to QCD as N_F increases,
  if extra PNGBs have $M_P < M_{V0}$ but small chiral logs $\propto (N_F - 2)\log\left(\frac{M_{V0}^2}{M_P^2}\right)$
  No symmetry breaking for sufficiently large $N_F \gtrsim 10 \longrightarrow$ smaller S reasonable
  In practice, harder to reach light masses $M_P < M_{V0}$ as N_F increases

- Future: EW gauge boson contribution to Higgs potential, $c_V \propto \int dQ^2\ \Pi_{U-B}(Q^2)$
  Fermion contributions involve four-point correlators of baryonic operators

Supplement: Lattice calculations of LECs

- In lecture 2 we saw the S parameter is an EFT low-energy coefficient (LEC)
  It corresponds to ℓ_5 in the SU(2) chiral lagrangian,
  equivalently α_1 in the electroweak chiral lagrangian (without light Higgs)

- Computations of EFT low-energy coefficients are common lattice projects
  (motivating domain wall fermions with continuum-like global symmetries)

- Example: $\left\langle \overline{\psi}\psi \right\rangle / F^3 \propto B/F \longrightarrow$ chiral condensate enhancement
  probes anomalous dimension γ_m needed by bilinear fermion masses $\sim \frac{1}{\Lambda^{2-\gamma_m}} \overline{q}q\overline{Q}Q$
  (Future: Anomalous dimension $\gamma_{\mathcal{O}} = -\frac{d\log Z_{\mathcal{O}}(\mu)}{d\log\mu}$ for partial compositeness
  from lattice non-perturbative renormalization of top-partner $\mathcal{O} \sim QQQ$)

- Another example: $\pi\pi$ scattering calculations on the lattice
  $\longrightarrow$ (complicated combinations of) electroweak LECs governing WW scattering

Evidence for dark matter

- Dark matter (DM) is definite physics beyond the standard model
  Like composite Higgs, composite DM needs to explain known features of DM
  deduced from **consistent** (gravitational) evidence across all accessible scales

- Spiral galaxy rotation curves probe 'small' kiloparsec scales (1 pc $\approx$ 3.26 ly)
  Stars/gas far from center move faster than visible matter can explain
  $\longrightarrow$ local DM density ~ 0.3 GeV/cm^3

- Similarly, galaxies within clusters move faster than expected, on Mpc scales
  Lensing $\longrightarrow$ location of mass doesn't match visible matter in clusters (e.g., bullet)

- Large-scale structure (Gpc scales) only reproduced by N-body simulations w/DM

- Cosmic microwave background (CMB) is largest visible distance scale $\approx$ 13.7 Gly
  Power spectrum $\longrightarrow$ fluctuations at given angular scale (multipole),
  third peak sensitive to dark (non-ionized) matter

- Fit CMB, large-scale structure, supernovae to six-parameter 'ΛCDM' model
  $\longrightarrow$ ~5% ordinary matter, ~25% dark matter, ~70% dark energy
  (most ordinary matter is interstellar H and He gas, most of remainder is stars)

Generic features of dark matter

- **Dark:** Electrically neutral, no direct coupling to photons

- **Matter:** In addition to gravitating, evolves like matter in expanding universe
  Expansion $\longrightarrow$ metric $ds^2 = -dt^2 + a^2(t)dx_i^2$, scale factor $a(t)$ has $\frac{da}{dt} = aH(t) > 0$
  (equivalently, cosmological redshift $z \equiv \frac{a_{\text{now}}}{a_{\text{past}}} - 1 > 0$)
  Evolution $\longrightarrow$ Energy density $\rho \propto a^{-3(w+1)}$ where w is equation of state
  relating pressure and energy density, $p = w\rho$
  Matter: $w = 0 \longrightarrow \rho \propto a^{-3} \sim$ const./volume
  Radiation (photons & neutrinos): $w = 1/3 \longrightarrow \rho \propto a^{-4} \sim$ as above plus redshift
  Cosmo. constant Λ: $w = -1 \longrightarrow \rho =$ const.

- **Stable:** Same ratio $\frac{\Omega_{DM}}{\Omega_B} \approx 5$ both today and at recombination
  $\longrightarrow$ little net decay or annihilation over ~13.7 billion years

- **Cold:** Large-scale structure formation from 'bottom up' $\longrightarrow$ **non-relativistic** DM
  again rules out neutrinos, though very light axions are non-relativistic

- **Collisionless:** No (large) dissipative DM–DM ('self-')interactions,
  would allow cooling $\longrightarrow$ dark disks that are largely ruled out
  (Non-dissipative self-interactions would reduce substructure within galactic halos)

- Not clearly detected in ongoing **searches** $\longrightarrow$ at most weak DM–SM interactions

Motivations for composite dark matter

- Strong dynamics can produce stable massive particles (e.g., protons and nuclei)
- Dimensional transmutation $\longrightarrow$ natural hierarchy below Planck scale
- Possibility of joint solution to DM and EWSB from single new strong sector
  (for now we will assume an elementary SM Higgs boson)
- Strong non-dissipative self-interactions may address galactic structure issues
  ('core-vs.-cusp', 'too-big-to-fail', 'missing satellites', ...)
- Production of $\frac{\Omega_{DM}}{\Omega_B} \approx 5$ typically relies on non-gravitational DM–SM interactions, while ongoing searches limit such interactions

  Confinement of SM-charged 'dark constituents' into SM-neutral composite DM could reconcile these two features

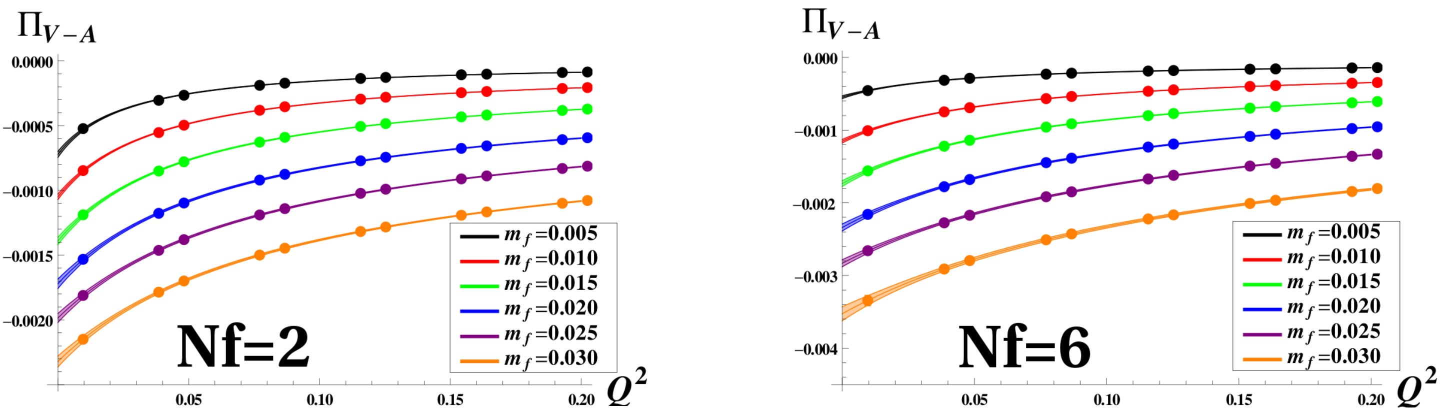

Figure 1: Fitting lattice data for the transverse vacuum polarization function $\Pi_{V-A}(Q^2)$ to Eq. 1, for SU(3) gauge theory with $N_F = 2$ (left) and 6 (right) domain wall fermions in the fundamental rep, from arXiv:0910.2224. The smallest accessible euclidean momentum transfers are $Q^2 = \left(\frac{2\pi n}{N_t}\right)^2$ and $\left(\frac{2\pi n}{N_s}\right)^2$, where $N_t = 64$ and $N_s = 32$ are the temporal and spatial extents of the lattice, respectively. The different curves come from calculations with different fermion masses m, which need to be extrapolated to the chiral limit in which there are three exactly massless NGBs and $N_F^2 - 4$ massive PNGBs.

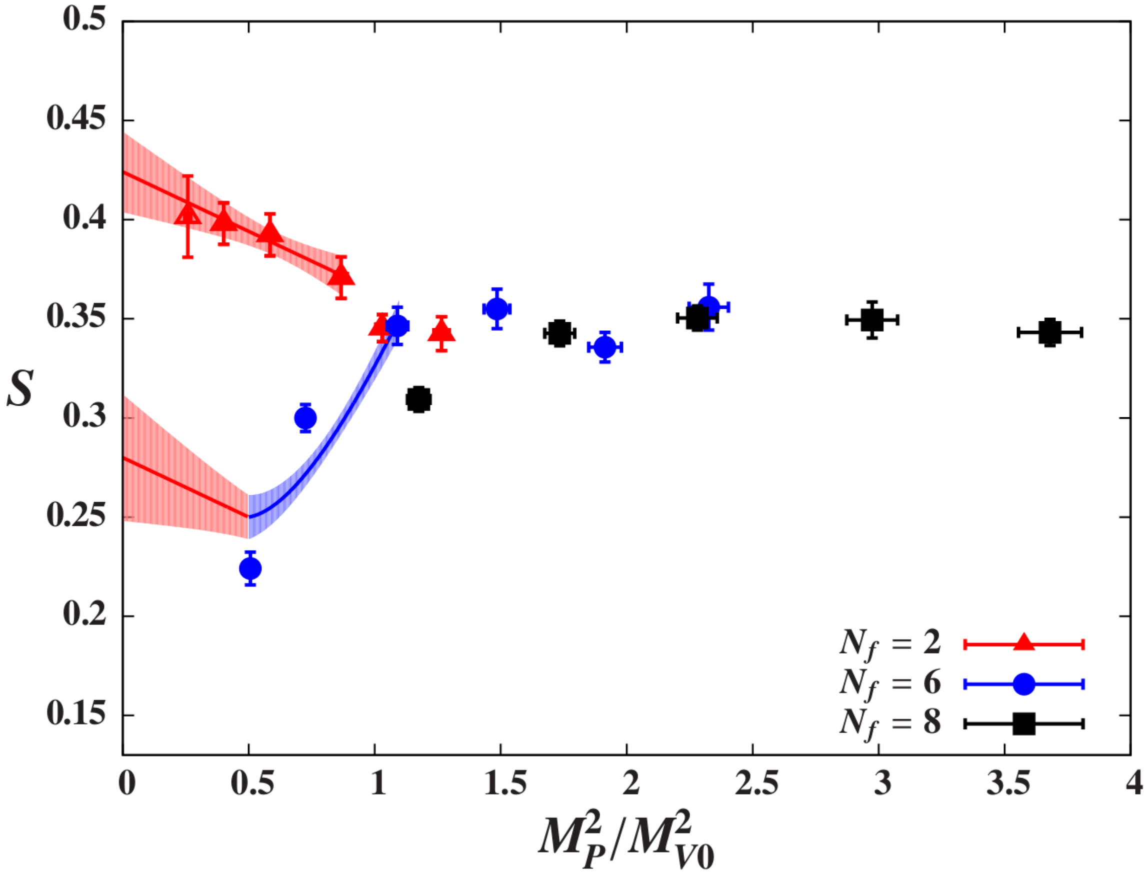

Figure 2: Results for the electroweak S parameter of SU(3) new strong dynamics with $N_F = 2$, 6 and 8 domain wall fermions in the fundamental rep, based on Fig. 11 of arXiv:1405.4752. For $N_F = 8$ the Higgs could be either a dilaton or a PNGB, while $N_F < 7$ allows only the dilaton possibility with $f = v$. The $m \to 0$ extrapolated vector mass M_{V0} is used to define the UV cutoff $1/a$, which is finite but approximately matched between the three analyses. For sufficiently large $N_F \gtrsim 10$ the theory should flow to a chirally symmetric conformal fixed point in the IR, suggesting that V–A parity doubling may appear for intermediate $N_F = 6$ or 8 at sufficiently light PNGB masses $M_P \lesssim M_{V0}$, leading to a smaller S parameter as seen in the non-perturbative numerical results. However, the extra $N_F^2 - 4$ PNGBs lead S to diverge $\sim (N_F - 2) \log\left(\frac{M_{V0}^2}{M_P^2}\right)$ as $M_P \to 0$, so these must be kept massive, with only three exactly massless NGBs eaten by electroweak symmetry breaking. Here we fix by hand $M_P^2 \approx M_{V0}^2/2$ for the $N_F^2 - 4$ massive PNGBs, in order to produce the illustrative $N_F = 6$ extrapolation. The straightforward $N_F = 2$ extrapolation produces $S = 0.42(2)$ in agreement with scaling up QCD data. Unfortunately as N_F increases it becomes harder to access small M_P^2/M_{V0}^2, further complicating the chiral extrapolations.

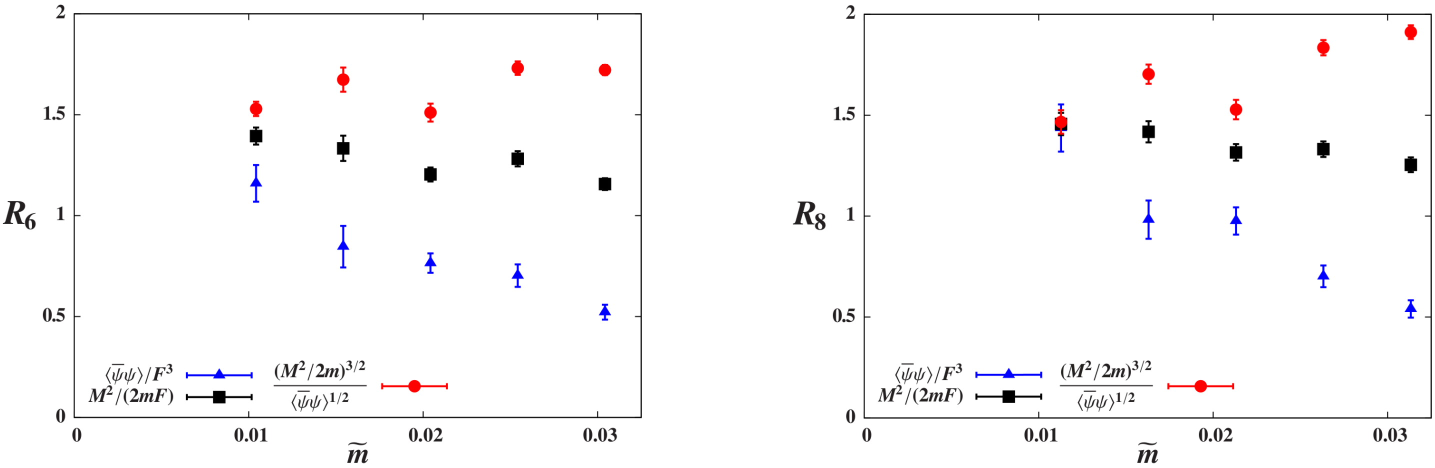

Figure 3: Enhancement of the chiral condensate in units of the decay constant $\left[\langle\overline{\psi}\psi\rangle/F^3\right]$ for SU(3) gauge theories with $N_F = 6$ (left) and 8 (right) domain wall fermions in the fundamental rep, from arXiv:1405.4752. In each case we compare three quantities that reduce to $\langle\overline{\psi}\psi\rangle/F^3$ in the chiral limit $m \to 0$, each normalized by the corresponding $N_F = 2$ value computed with approximately matched M_{V0} standing in for the UV cutoff as above. The enhancement $R_F = \exp\left(2\int \frac{d\mu}{\mu}\left[\gamma_m^{(F)}(\mu) - \gamma_m^{(2)}(\mu)\right]\right) > 1$ implies a larger mass anomalous dimension $\gamma_m^{(F)}$ for F flavors compared to $N_F = 2$.

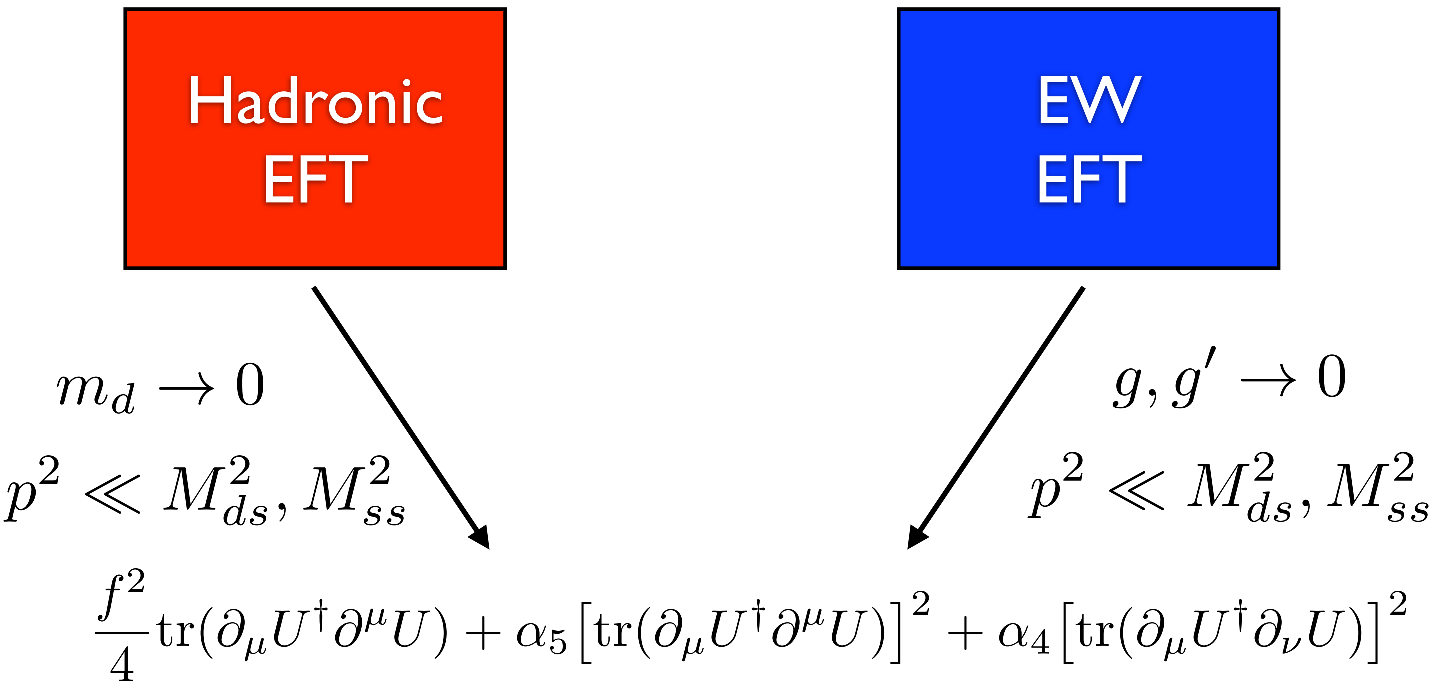

Figure 4: An illustration of relations between low-energy coefficients (LECs) of hadronic and electroweak EFTs (without including a light Higgs boson in the latter), focusing on the two electroweak LECs α_4 and α_5 that govern the scattering of like-sign WW bosons.

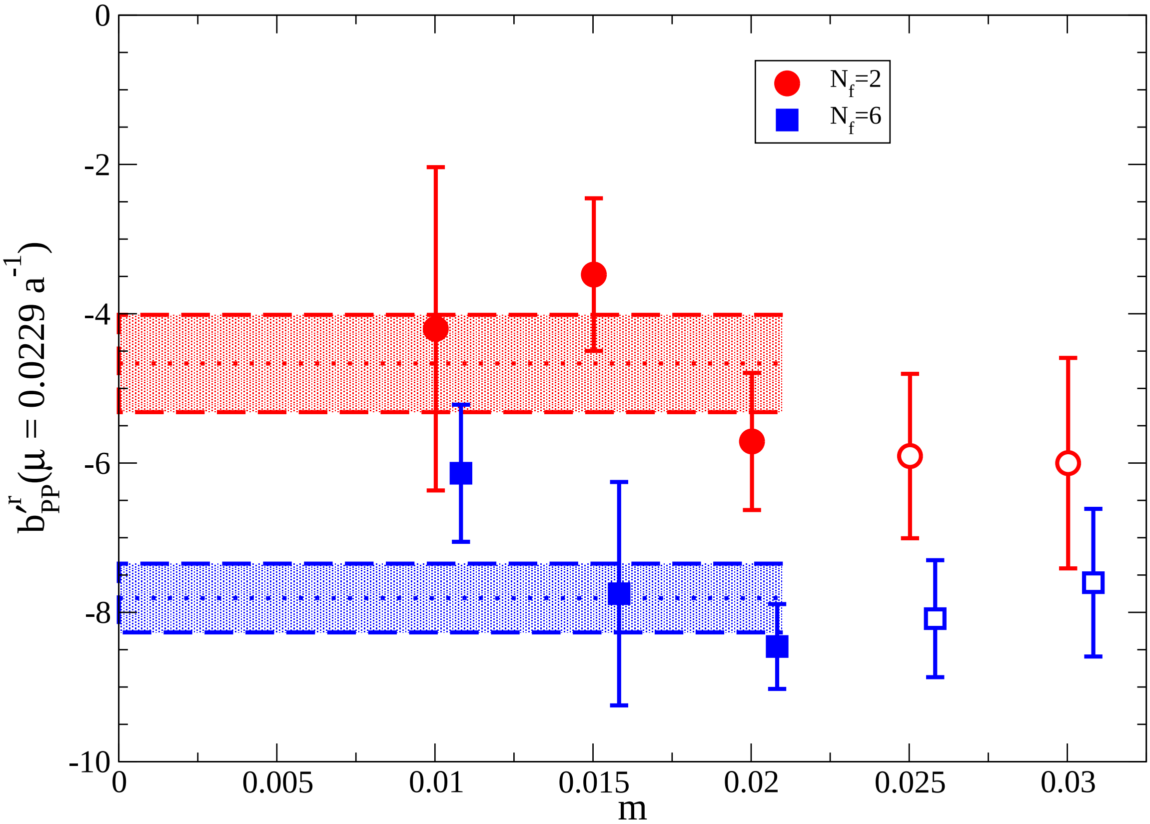

Figure 5: Results from arXiv:1201.3977 for $b'_{PP} = -256\pi^2\,[L_0 + 2L_1 + 2L_2 + L_3 - 2L_4 - L_5 + 2L_6 + L_8]$ (the linear combination of SU(3) chiral LECs L_i probed by maximal-isospin s-wave $\pi\pi$ scattering on the lattice), considering SU(3) gauge theories with $N_F = 2$ and 6 domain wall fermions in the fundamental rep. The electroweak LEC α_4 corresponds to the SU(2) chiral LEC ℓ_2 that depends on L_0 and L_2; similarly α_5 corresponds to ℓ_1 that depends on L_0, L_1 and L_2. Measurements of more processes (e.g., d-wave scattering or non-maximal isospin channels) would be needed to isolate these LECs. The more negative $N_F = 6$ results for b'_{PP} correspond to a slightly smaller $\pi\pi$ scattering length compared to $N_F = 2$.