New strong dynamics beyond the standard model

Lecture 2 9 November 2017

Last time

- The SM is an EFT valid up to some UV scale Λ_{SM}
- Dimensional analysis suggests high UV scales $\Lambda_i \gtrsim 10^{13}$ GeV
  for rare processes like lepton and baryon number violation
  Dimensional analysis suggests a low UV scale $\Lambda_{SM} \sim 1$–10 TeV for EWSB
- A new strong interaction can dynamically generate the EW scale $v = 246$ GeV,
  and produce a natural hierarchy via near-conformality and dim'l transmutation
- A "scaled-up" copy of $N_F = 2$ QCD with chiral symm. breaking at $f = 246$ GeV
  predicts EWSB with correct $M_W = M_Z \cos\theta_W$ (due to SU(2) custodial symmetry)
- Scaled-up QCD has three big problems:
  —Expect a scalar Higgs boson mass $m_\sigma \sim 5f \sim 1$ TeV
  —Fermion masses from higher-dimensional operators $\sim \frac{1}{\Lambda_{UV}^2}(\overline{q}_R q_L)(\overline{Q}_R Q_L)$
  in tension with flavor-changing neutral currents $\sim \frac{1}{\Lambda_{UV}^2}(\overline{q}_R q_L)(\overline{q}_R q_L)$
  —Electroweak precision observables (especially the S parameter)...

A few more details about the S parameter

- Consider transverse vacuum polarization functions of W^a and photon

$$\int d^4x\; e^{-iq\cdot x}\,\langle J_X^\mu(x) J_Y^\nu(0)\rangle = i\Pi_{XY}^{\mu\nu}(q) = i\eta^{\mu\nu}\Pi_{XY}(q^2) + (q^\mu q^\nu \text{ terms})$$

- Expand four independent $\Pi_{XY}(q^2)$ for $q^2 \ll M_Z \ll \Lambda$ (using EM Ward identity):

$$\begin{aligned}
\Pi_{ee}(q^2) &= q^2\Pi'_{ee}(0) + \mathcal{O}(q^4/\Lambda^2)\\
\Pi_{3e}(q^2) &= q^2\Pi'_{3e}(0) + \mathcal{O}(q^4/\Lambda^2)\\
\Pi_{11}(q^2) &= \Pi_{11}(0) + q^2\Pi'_{11}(0) + \mathcal{O}(q^4/\Lambda^2)\\
\Pi_{33}(q^2) &= \Pi_{33}(0) + q^2\Pi'_{33}(0) + \mathcal{O}(q^4/\Lambda^2)
\end{aligned}$$

  Fix three of six parameters through α_{em}, G_F and M_Z (equivalently g_1, g_2 and v)
- Traditional parameterization of remaining three is

$$\begin{aligned}
S &= 16\pi\left[\Pi'_{33}(0) - \Pi'_{3e}(0)\right]_{\text{new}}\\
T &= \frac{4\pi}{\sin^2\theta_W\cos^2\theta_W M_Z^2}\left[\Pi_{11}(0) - \Pi_{33}(0)\right]_{\text{new}}\\
U &= 16\pi\left[\Pi'_{33}(0) - \Pi'_{11}(0)\right]_{\text{new}}
\end{aligned}$$

- Subtract SM contributions so that non-zero values $\longleftrightarrow$ BSM physics

A few more details about the S parameter (continued)

- Caveat: Some $\Pi''(0)$ combinations (Y and W) can be more important than $\Pi'(0)$
  In SM-like EFTs, $\{S, T, Y, W\}$ correspond to dimension-6 operators

$$S \sim \frac{1}{\Lambda_{SM}^2} \left(H^\dagger \sigma_a H\right) W_{\mu\nu}^a B_{\mu\nu} \qquad T \sim \frac{1}{\Lambda_{SM}^2} \left|H^\dagger D_\mu H\right|^2$$
$$Y \sim \frac{1}{\Lambda_{SM}^2} \left(\partial_\rho B_{\mu\nu}\right)^2 \qquad W \sim \frac{1}{\Lambda_{SM}^2} \left(D_\rho W_{\mu\nu}^a\right)^2$$

  The dimension-8 operator $U \sim \frac{1}{\Lambda_{SM}^4} \left(H^\dagger W_{\mu\nu}^a H\right)^2$ is generally much smaller

- Global EW fit with fixed $U = 0$ gives $S = 0.07(8)$ and $T = 0.10(7)$

- Can rewrite $T \propto \left[\frac{\Pi_{WW}(0)}{M_W^2} - \frac{\Pi_{ZZ}(0)}{M_Z^2}\right]_{\text{new}} \longrightarrow$ measures custodial symmetry violation

- S measures 'size' of EWSB sector. Rewrite as $S = 4\pi \left[\Pi'_{VV}(0) - \Pi'_{AA}(0)\right]$

$$\Longrightarrow S = 4 \int \frac{ds}{s} \text{Im} \left[\Pi'_{VV}(s) - \Pi'_{AA}(s)\right] = \frac{1}{3\pi} \int \frac{ds}{s} \left[R_V(s) - R_A(s)\right]$$

  Can compute QCD $R_V(s)$ and $R_A(s)$ from $e^+e^- \to$ hadrons data
  Appropriately 'scaling up' the results gives $S \simeq 0.4$, strongly ruled out

- Note added: Need to subtract $\Delta S \sim \log\left(M_H^2 / M_\pi^2\right)$ from eaten $M_\pi = 0$ NGBs
  Can be made more systematic through SU(2) chiral perturbation theory,
  $S = \frac{1}{12\pi} \left[-192\pi^2 \ell_5^r(\mu) + \log\left(\mu^2/m_H^2\right) - \frac{1}{6}\right]$ with μ rescaled by v_{EW}/f_π
  The FLAG review reports $\ell_5^r(M_\rho) \approx -0.005 \Longrightarrow S \approx 0.4$

Pseudo-NGB composite Higgs

- SM does not suffer from these problems
  $\Longrightarrow$ It should help to keep composite Higgs light while making all else heavier

- **In principle** we can obtain such a hierarchy
  by making the composite Higgs a pseudo-NGB of an approximate symmetry

- Won't discuss case of 'dilaton' Higgs from approximate conformal symmetry
  A focus of my current lattice research, but harder to handle analytically

- Instead focus on composite Higgs as PNGB of *internal* global symmetry

Effective lagrangians for PNGBs (CCWZ construction)

- Callan, Coleman, Wess and Zumino (1969 & 1969) give general construction of effective lagrangians for (P)NGBs from $\mathcal{G} \to \mathcal{H}$ symmetry breaking at scale f
- NGBs Π^a contained in the coset $\mathcal{G}/\mathcal{H}$, transform in reps of $\mathcal{H}$
  Parameterize as fluctuations around symmetry-breaking vacuum Σ_0,
  $\Sigma = \exp\left[i\sqrt{2}T_a\Pi^a(x)/f\right]\Sigma_0$ where T_a are **broken** generators of $\mathcal{G}$
- For scaled-up QCD, $\mathcal{G} = \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{U}(1)_B \longrightarrow \mathcal{H} = \mathrm{SU}(2)_V \times \mathrm{U}(1)_B$
  Since we identify weak $\mathrm{SU}(2)_L \subset \mathcal{G}$ while $\mathrm{U}(1)_Y \subset \mathrm{SU}(2)_R$ and $\mathrm{U}(1)_{em} \subset \mathrm{SU}(2)_V$
  $G \to H$ breaks electroweak symmetry with $f = v$ and $m_\sigma/f \simeq 5$
- The alternative is for the **strong** dynamics to preserve $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \subseteq \mathcal{H}$
  To protect $T = 0$, helps to preserve larger custodial $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \subseteq \mathcal{H}$
- We want the NBGs in $\mathcal{G}/\mathcal{H}$ to include a SM-like Higgs doublet
  Complex Higgs doublet in $2_{1/2}$ rep of $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$
  corresponds to real $(H, i\sigma_2 H^*)$ in **bidoublet** $(2,2)$ rep of $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$
- $\Longrightarrow$ $\mathcal{G}$ has at least four more generators than $\mathcal{H}$
  $\Longrightarrow$ Gauging only $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \subseteq \mathcal{H}$ explicitly (but **weakly**) breaks $\mathcal{G}$
  $\Longrightarrow$ Potential (including mass) for all but the three NGBs eaten through EWSB
- **Need to check** that PNGB Higgs potential $\longrightarrow$ EWSB with correct v and M_H
  $\Longrightarrow$ Radiative corrections from SM fields **misalign vacuum** away from Σ_0

(Too-)Minimal (custodial) Composite Higgs Model

- Smallest custodial $\mathcal{H} = \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \sim \mathrm{SO}(4)$ with six generators
  Smallest coset $\mathcal{G}/\mathcal{H} \ni$ four NGBs in $(2,2)$ rep of $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \to$ **4** of $\mathrm{SO}(4)$
  $\Longrightarrow$ Minimal possibility is $\mathcal{G} = \mathrm{SO}(5)$ with ten generators
- CCWZ parameterization of four NGBs h^a is $\quad \Sigma(x) = \exp\left[i\sqrt{2}T_a h^a(x)/f\right]\left(\vec{0}, 1\right)^T$
  Broken generators T^a rotate between SO(4) subgroup and fifth component:
$$\Sigma = \begin{pmatrix} \cdots & \frac{h^a}{h}\sin\left(\frac{h}{f}\right) \\ -\frac{h^a}{h}\sin\left(\frac{h}{f}\right) & \cos\left(\frac{h}{f}\right) \end{pmatrix} \begin{pmatrix} \vec{0} \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{h^a}{h}\sin\left(\frac{h}{f}\right) \\ \cos\left(\frac{h}{f}\right) \end{pmatrix} \qquad h \equiv \sqrt{h^a h^a}$$
  Can check $\lim_{h\to 0}\Sigma = \Sigma_0$ or explicitly use fund.-rep $\quad T^a_{IJ} = -\frac{i}{\sqrt{2}}\left(\delta^a_I \delta^5_J - \delta^a_J \delta^5_I\right)$
- h^a in vector rep of $\mathrm{SO}(4) \longrightarrow \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ bidoublet $\quad \frac{1}{h}\begin{pmatrix} h^1 - ih^2 \\ h^3 - ih^4 \end{pmatrix} \quad \checkmark$

Explicit $\mathcal{G}$ breaking

- CCWZ provides procedure to incorporate explicit $\mathcal{G}$ breaking discussed above
  Spurion trick: Gauge full $\mathcal{G}$, decompose into reps of $\mathcal{H}$, set to physical value
- For NGBs, decomposition yields expected SO(4)-vector part of Σ:
  $$\widehat{\Sigma} = \frac{h^a}{h}\sin\left(\frac{h}{f}\right) \longrightarrow \sin\left(\frac{h}{f}\right)\frac{1}{h}\begin{pmatrix} h^1 - ih^2 \\ h^3 - ih^4 \end{pmatrix}$$
- For now, **assume** non-zero vev that we can rotate to be
  $$\langle h^a \rangle = (0, 0, \langle h \rangle, 0)^T \qquad \Longrightarrow \left\langle \widehat{\Sigma} \right\rangle = \sin\left(\frac{\langle h \rangle}{f}\right)\begin{pmatrix} 0 \\ 1 \end{pmatrix}$$
- Then usual $\mathcal{L}_2 = \frac{f^2}{4}\text{Tr}\left[D_\mu \Sigma^T D^\mu \Sigma\right] \quad \Longrightarrow \quad M_W = \frac{1}{2}g_2 f \sin\left(\frac{\langle h \rangle}{f}\right)$ and $M_Z = \frac{M_W}{\cos\theta_W}$
  $\Longrightarrow v = f\sin\left(\frac{\langle h \rangle}{f}\right) \leq f$ of scaled-up QCD

Decoupling limit

- In **decoupling limit** $f \to \infty$ we have $v = \langle h \rangle + \mathcal{O}\left(\langle h \rangle^3 / f^2\right)$ and **recover SM!**
  $\Longrightarrow$ All deviations from SM should depend on $\xi \equiv \dfrac{v^2}{f^2} = \sin^2\left(\dfrac{\langle h \rangle}{f}\right) \leq 1$
  (no decoupling limit for scaled-up QCD or dilatonic Higgs)
- Easy to confirm for Higgs couplings to $V = W$, Z that also come from $\mathcal{L}_2$
  Expanding $h = (0, 0, \langle h \rangle + \phi, 0)^T$ around vev,
  $$f^2\sin^2\left(\frac{h}{f}\right) = f^2\sin^2\left(\frac{\langle h \rangle}{f}\right) + 2f\sin\left(\frac{\langle h \rangle}{f}\right)\cos\left(\frac{\langle h \rangle}{f}\right)\phi + \left[1 - 2\sin^2\left(\frac{\langle h \rangle}{f}\right)\right]\phi^2 + \mathcal{O}\left(\phi^3/f\right)$$
  $$= v^2 + 2v\sqrt{1-\xi}\,\phi + (1-2\xi)\,\phi^2 + \mathcal{O}\left(\phi^3/f\right)$$
  $\Longrightarrow$ Couplings between vector bosons and one or two Higgs bosons modified by
  $$\kappa_{VVh} \equiv \frac{g_{VVh}}{g_{VVh}^{\text{SM}}} = \sqrt{1-\xi} \qquad\qquad \kappa_{VVhh} \equiv \frac{g_{VVhh}}{g_{VVhh}^{\text{SM}}} = 1 - 2\xi$$
  LHC phenomenology $\Longrightarrow$ need $\xi \lesssim 0.1$ (PDG 2016)
- Generality of CCWZ construction $\Longrightarrow$ decoupling is '**universal**' result
  Generic feature of dozens of $\mathcal{G} \to \mathcal{H}$ composite Higgs models studied in literature

Radiative EWSB via vacuum misalignment

- Still need to ensure EWSB via **vacuum misalignment** $\longrightarrow \langle h \rangle \neq 0$
  $\Longrightarrow$ Higgs potential should be periodic function of "misalignment angle" h/f

- Higgs potential $V(H^\dagger H)$ corresponds to one-loop Coleman–Weinberg potential
  resumming diagrams with n insertions of $VVhh$ vertex $\Gamma_{\mu\nu}$

$$\Gamma_{\mu\nu}(q^2) = \frac{1}{4}\left(\eta^{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)\Pi_{U-B}(q^2)\sin^2\left(\frac{h}{f}\right)$$

(Note added: CW potential is sum of all 1PI diagrams with n external $H^\dagger H$ lines)

- $\Pi_{U-B}(q^2)$ is difference between two-point functions
  along unbroken ("U") and broken ("B") directions
  (for QCD-like symmetry breaking $U = V$ are vector generators and $B = A$ are axial)

  Depends on strong dynamics $\longrightarrow$ model-dependent and not analytically calculable
  Should vanish at high energies $Q^2 \gg f^2 \longrightarrow$ reasonable for integral to be finite

- Resumming produces log, which we expand to leading order: $(Q^2 = -q^2)$

$$V(h) = \frac{1}{2}\int \frac{d^4Q}{(2\pi)^4}\log\left[1 + \frac{3g_2^2 + g_1^2}{4Q^2}\Pi_{U-B}(Q^2)\sin^2\left(\frac{h}{f}\right)\right] \simeq c_V \sin^2\left(\frac{h}{f}\right)$$

$$c_V = \frac{3g_2^2 + g_1^2}{8(16\pi^2)}\int_0^\infty dQ^2\ \Pi_{U-B}(Q^2)$$

(factors of 2 questionable but irrelevant)

- Witten's inequality implies integral is generically **positive**
  $\Longrightarrow$ Vacuum alignment: potential minimized at $\langle h \rangle = 0$, no EWSB

- Even if sign were negative, minimum would be at $\xi = \sin^2\left(\frac{\langle h \rangle}{f}\right) = 1 \longrightarrow$ ruled out

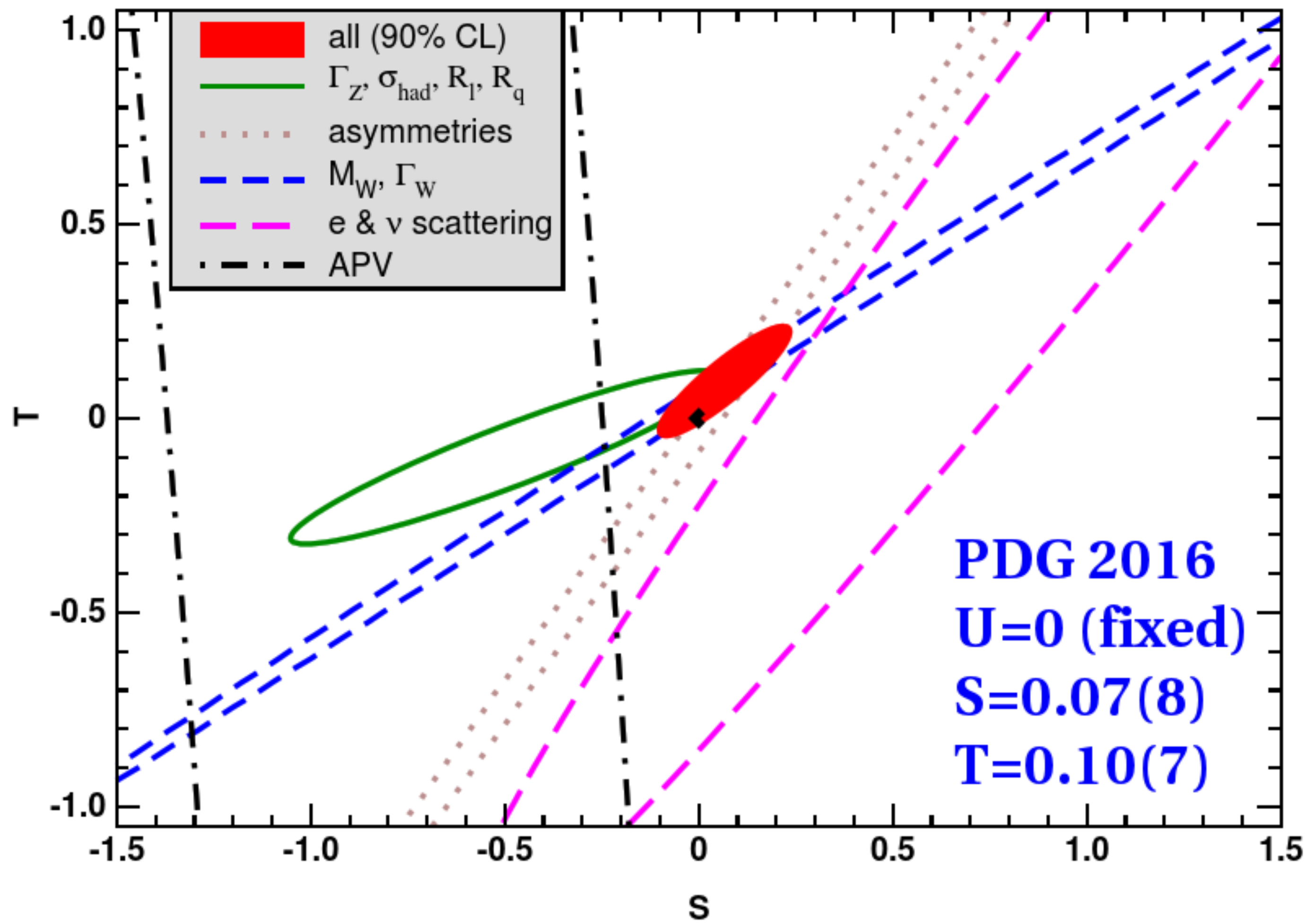

Figure 1: Current experimental constraints on the electroweak S and T parameters, from a global electroweak fit (with fixed $U = 0$) reported in the 2016 Review of Particle Physics.

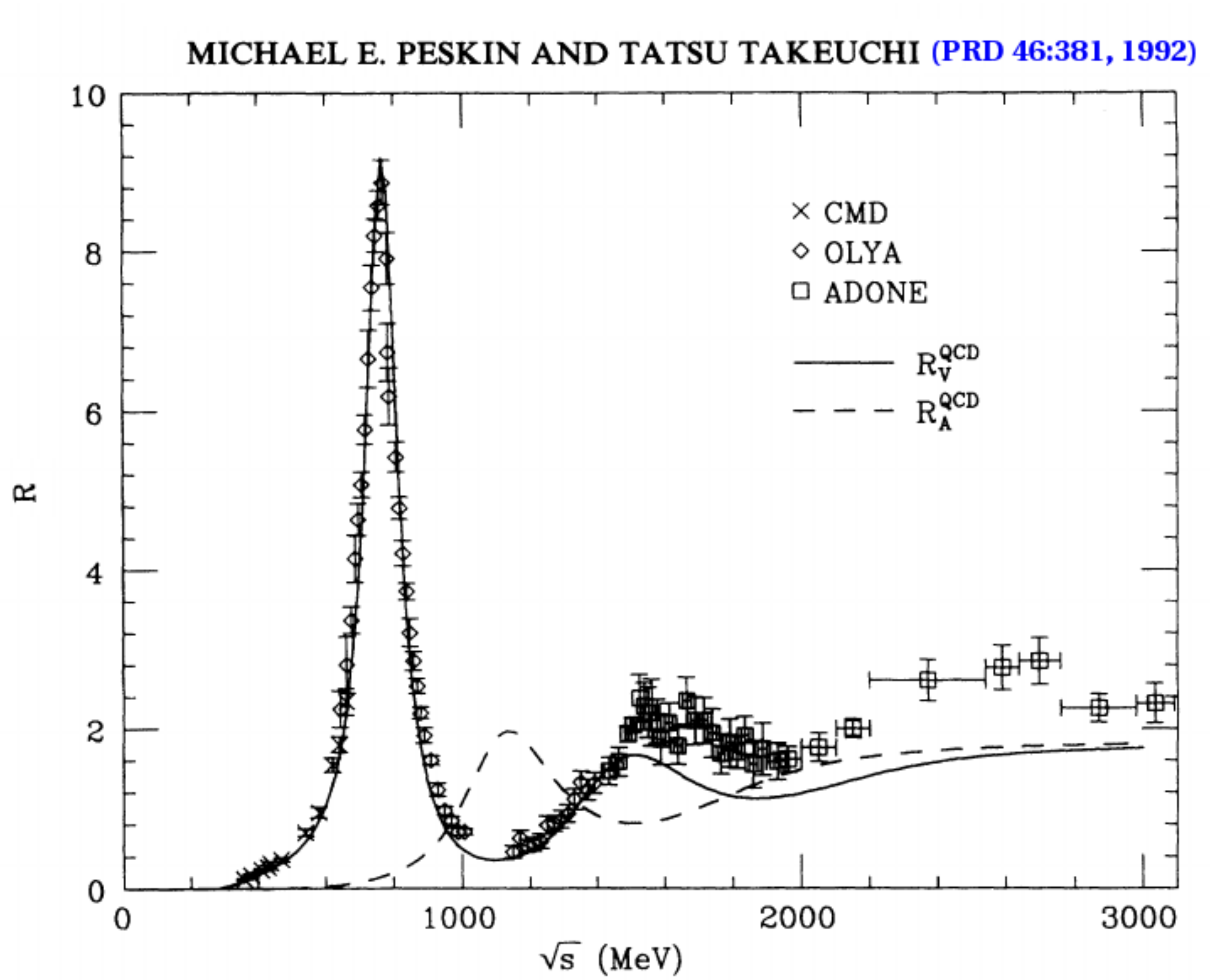

Figure 2: Vector and axial-vector spectral functions ($R_V(s)$ and $R_A(s)$, respectively) for QCD, from Peskin & Takeuchi, 1992. R_V is determined by fitting experimental data for the production of even numbers of pions in e^+e^- annihilation. For $\sqrt{s} \lesssim M_\tau \approx 1.8$ GeV R_A is determined by fitting experimental data for τ decays to odd numbers of pions; it is then extrapolated to higher energies. The data points are for the total $R(s) = R_V(s) + \frac{1}{4}\Pi'_{YY}(s)$. As expected $R_V(s) - R_A(s) \to 0$ as $\sqrt{s}$ increases.