Physics 8.324, Fall 2007

Homework #5

Due **Wednesday, December 12** by 4:00 PM in the 8.323 homework box.

1. Peskin and Schroeder problem 15.1 (parts c, d)

2. Peskin and Schroeder problem 15.2

3. Peskin and Schroeder problem 15.3

4. Peskin and Schroeder problem 15.5

5. Fun with SU(3) representations:
In this problem we use the methods developed in class to look further into some SU(3) representations.

a) Given a highest weight state $|\mu\rangle$ for an irreducible representation of SU(3) with $\mu_\beta = 1, \mu_{-\gamma} = 1/2$, determine the weights appearing in this irreducible representation by completing each irrep of the SU(2) subgroups generated by $e_{\pm\alpha}, e_{\pm\beta}, e_{\pm\gamma}$.

b) Determine the multiplicities at each weight for the irrep of part a). Hint: you can do this by first constructing all states at a given weight which can be realized by a product of "lowering" operators (those corresponding to the adjoints $e_\gamma, e_{-\beta}$ of the generators associated with the simple roots $\beta, -\gamma$) on the highest weight state. Then determine the degeneracy at this level by computing the matrix of norms for these states, using $\langle\mu|\mu\rangle = 1$ and the commutation relations between the raising and lowering operators.

c) The tensor product of two representations R, R' is given by taking all states $|\nu, \nu'\rangle$ where $|\nu\rangle$ is a state in R and $|\nu'\rangle$ is a state in R', acted on by the generators $T_{R\otimes R'} = T_R \otimes 1 + 1 \otimes T_{R'}$. The weight of the tensor product state is thus just the sum of weights $\nu + \nu'$. Consider the tensor product of the fundamental and adjoint SU(2) representations. Construct the set of weights and associated multiplicities. Decompose into a sum of irreducible representations by subtracting the irrep associated with the highest weight in the tensor product iteratively until all multiplicities of each weight are incorporated into a component irrep.