

8.325 Homework 1

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Due: In lecture Feb. 21.

Problem 1) Practice with Local and Global NonAbelian Symmetry

- a) Consider a field $\xi(x) = \xi^A(x)T^A$ in the adjoint representation of SU(3). Derive a result for the action of the covariant derivative on $\xi(x)$ by insisting that the chain rule is satisfied, $D_\mu(\xi\psi) = (D_\mu\xi)\psi + \xi(D_\mu\psi)$, where ψ is a color triplet fermion.
- b) Consider the QCD gauge transformations, where $A_\mu \rightarrow UA_\mu U^{-1} + i/g U\partial_\mu U^{-1}$. Let the covariant derivative be $D_\mu = \partial_\mu + ig'A_\mu^A T^A$ and the field strength be $F_{\mu\nu}^A = \partial_\mu A_\nu^A - \partial_\nu A_\mu^A + g'' f^{ABC} A_\mu^B A_\nu^C$. Denoting $F_{\mu\nu} = F_{\mu\nu}^A T^A$, show that the QCD Lagrangian

$$\mathcal{L}_{\text{QCD}} = \bar{\psi} i \not{D} \psi - \frac{1}{2} \text{tr}[F_{\mu\nu} F^{\mu\nu}] \quad (1)$$

is only gauge invariant if $g' = g$ and $g'' = g$.

- c) Consider a global color SU(3) transformation and derive the Noether current for the pure Yang Mills theory (this theory is Eq. (1) without the fermion term). Check that your current is conserved (you may lookup the nonabelian equation of motion for the gluon, and use it). [For extra practice: extend your results to include the fermion too, but I'm not asking you to turn this in.]

Problem 2) Practice with Fadeev-Popov in QCD and QED

In Peskin & Schroeder you will find the Fadeev-Popov procedure applied to QCD in generalized Lorentz gauge $\partial^\mu A_\mu^a = 0$ with gauge-fixing parameter ξ . In the first part of this problem you will repeat the steps but use a generalized axial gauge, $n^\mu A_\mu^a = 0$ with $n^2 < 0$ and gauge-fixing parameter ξ .

- a) Derive the generalized axial-gauge QCD Lagrangian including gauge-fixing and ghost terms.
- b) Usually “axial-gauge” is reserved for the case $n \cdot A = 0$ which can be obtained from the $\xi \rightarrow 0$ limit. Derive the form of the gluon-propagator in this limit and show that the ghost fields from part a) decouple.

Usually one does not worry about ghost interactions in QED. However, this does depend on the gauge choice.

- c) Consider the gauge choice $G(A) = \partial_\mu A^\mu + \lambda A_\mu A^\mu$ in QED (notation as in Peskin). Derive the Feynman rule for the ghost-photon interaction.

Problem 3) Positronium Decay

Positronium is a non-relativistic bound state of an e^- and an e^+ . The S-wave bound state has spin $S = 0$ (para-positronium) or $S = 1$ (ortho-positronium).

a) Use charge conjugation to show that para-positronium decays to either two or three photons, and that ortho-positronium does the opposite.

b) The probability that the e^+ and e^- are at the same spacetime point is determined by the non-relativistic wavefunction for the bound state, $\psi(\vec{x})$. Derive a formula for the decay rate Γ for positronium to two-photons. Your result should involve ψ , and \mathcal{M} , where \mathcal{M} is the appropriate $e^+e^- \rightarrow \gamma\gamma$ amplitude. (The discussion in Peskin around eq.(5.43) might be useful.)

c) Compute $|\mathcal{M}|^2$ at lowest order and thus determine Γ in terms of m_e and the QED coupling α .