Comments on supersymmetry and the MSSM (25 January 2013)

Review A thorough review of the topics we covered last week seemed beneficial. I appreciated being reminded of the intuitive way of thinking about susy transformations as "rotations" in superspace, in the sense that they relate the different components of superfields in a way analogous to how the components of ordinary vectors behave under rotations. I also hadn't thought about generic superfields as reducible representations of the susy algebra, which become irreps (vector or chiral superfields) after imposing either hermiticity or $\mathcal{D}^*_{\dot{a}}\Phi = 0$. This still isn't clear to me, but seems like an interesting perspective to think about.

Berezin integral Oliver also explained how integration over the anti-commuting components of superspace can pick out the F and D terms that appear in the action. With

$$\int d\theta = 0 \qquad\qquad \int d\theta^* = 0$$
$$\int \theta d\theta = 1 \qquad\qquad \int \theta^* d\theta^* = 1$$
$$d^2\theta \equiv d\theta_a d\theta_b \qquad\qquad d^4\theta \equiv d\theta_a d\theta_b d\theta^*_{\dot{a}} d\theta^*_{\dot{b}},$$

we can write the lagrangian as

$$\mathcal{L} = \int d^4\theta K(\Phi^\dagger\Phi) + \int d^2\theta W(\Phi),$$

where $W(\Phi)$ is the superpotential and $K(\Phi^\dagger\Phi)$ is the Kähler potential (last week we considered the simplest case $K(\Phi^\dagger\Phi) = \Phi^\dagger\Phi$).

~~**susy**~~ An important result that Srednicki leaves to the problems is the proof that

$$\langle\psi|4H|\psi\rangle = \langle\psi|\mathrm{Tr}\left\{Q_a, Q^\dagger_{\dot{a}}\right\}|\psi\rangle = \mathrm{Tr}\left[\langle\psi|Q_a Q^\dagger_{\dot{a}}|\psi\rangle + \langle\psi|Q^\dagger_{\dot{a}} Q_a|\psi\rangle\right]$$
$$= ||Q^\dagger|\psi\rangle||^2 + ||Q|\psi\rangle||^2 \geq 0.$$

If the vacuum $|0\rangle$ is supersymmetric, then $Q_a|0\rangle = 0$ and $Q^\dagger_{\dot{a}}|0\rangle = 0 \Rightarrow \langle 0|H|0\rangle = 0$. (Perturbatively, we can picture perfect cancellation between bubble diagrams involving the bosonic and fermionic fields related by susy.) Since $\langle H\rangle \geq 0$ for any susy lagrangian, susy is spontaneously broken if and only if $\langle 0|H|0\rangle > 0$ (that is, if and only if the set of equations $\langle 0|H|0\rangle = 0$ has no solution).

Supergauge transformation Write it as $V \to V + i\left(\Xi^\dagger - \Xi\right)$, where V is a vector superfield and Ξ is a chiral superfield (so that $i\left(\Xi^\dagger - \Xi\right)$ is another vector superfield). Considering V as an enlarged gauge field, this amounts to an enlarged gauge invariance that can kill off the extra degrees of freedom, leaving only $V = (\theta\sigma^\mu\theta^*)\, v_\mu + \theta\theta\theta^*\lambda^\dagger + \theta^*\theta^*\theta\lambda + \frac{1}{2}\theta\theta\theta^*\theta^* D$. Here v_μ is the usual gauge field with abelian gauge transformation $v_\mu \to v_\mu - \partial_\mu b$, λ is the gaugino, and D is an auxiliary field.

Charged chiral superfields With gauge coupling g and charge $q = 1$, take the chiral superfield's supergauge transformation to be $\Phi \to e^{-2ig\Xi}\Phi$ and $\Phi^\dagger \to \Phi^\dagger e^{2ig\Xi^\dagger}$, so that kinetic term $\Phi^\dagger e^{-2gV}\Phi$ is invariant. The anti-commuting coordinates make it easy to the expand the exponential in θ and θ^*.

Superfield strength To determine the vector superfield kinetic term, we define $W_a = \frac{1}{4}\mathcal{D}^*_{\dot{a}}\mathcal{D}^{*\dot{a}}\mathcal{D}_a V$, which is a supergauge-invariant left-handed superfield, and the first fermionic superfield we've seen. The two $\mathcal{D}^*_{\dot{a}}$ pick out the component of the $\theta^*\theta^*$ term in $\mathcal{D}_a V$, $W_a = \lambda_a + \theta_a D - (S_L^{\mu\nu})_a{}^c\theta_c F_{\mu\nu} + i\theta\theta\sigma^\mu_{a\dot{a}}\partial_\mu\lambda^{\dagger\dot{a}}$, where $F_{\mu\nu} = \partial_\mu v_\nu - \partial_\nu v_\mu$ is the gauge-invariant field strength of v_μ. The F term of $W^a W_a$ can appear in the action,

$$\mathcal{L}_{kin} = \frac{1}{4}W^a W_a\bigg|_F + \text{h.c.} = i\lambda^\dagger\overline{\sigma}^\mu\partial_\mu\lambda - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}D^2.$$

θ **angle** Oliver pointed out that $i\widetilde{F}^{\mu\nu}F_{\mu\nu}$ also appears in $W^a W_a|_F$, and informed us that the corresponding θ parameter (not to be confused with the superspace coordinate) can be treated as the imaginary component of a complexified gauge coupling.

SYM When generalizing to non-abelian supersymmetric gauge theories (such as susy Yang–Mills), we have to keep track of the order in the supergauge transformation of $e^{-2gV} \to e^{-2ig\Xi^\dagger}e^{-2gV}e^{2ig\Xi}$. Similarly, we replace derivatives ∂_μ with covariant derivatives D_μ, and the field-strength superfield becomes $W_a = -\frac{1}{8g}\mathcal{D}^*_{\dot{a}}\mathcal{D}^{*\dot{a}}e^{2gV}\mathcal{D}_a V e^{-2gV}$, with supergauge transformation $W_a \to e^{-2ig\Xi}W_a e^{2ig\Xi}$. While chiral superfields can be in any representation, vector superfields (and therefore their gaugino components) must transform in the adjoint.

MSSM Srednicki does this even more quickly than usual: just promote all the standard model fields to separate superfields, add a second Higgs superfield $\overline{H}$ with opposite weak hypercharge, and impose R parity to forbid $\overline{H}\cdot L$ terms (which I need to think about a bit more). There are multiple reasons why we need two Higgs superfields. First, if we only had $H \ni h$, then we wouldn't be able to get both $h\cdot Q\cdot\overline{d}$ and $h^\dagger\cdot Q\cdot\overline{u}$ terms out of the superpotential $W(H)$, in which $H^\dagger$ cannot appear. In addition, the higgsino $\widetilde{h}$ transforms in a complex representation of the standard model gauge group, so a $\widetilde{\overline{h}}$ with the opposite weak hypercharge is required to cancel gauge anomalies.

~~susy~~ Phenomenologically, susy must be spontaneously broken, which requires additional "hidden sector" fields. Srednicki says no more about them, just parameterizing their effects through a spurion analysis: coupling a constant chiral superfield $S = m_S^2\theta\theta$ to the other chiral superfields (including the W_a built from the gauge superfields) via either D or F terms.