Comments on supersymmetry introduction (18 January 2013)

Algebra Srednicki's presentation seems very terse, starting with the spinor supercharges Q_a and $Q^\dagger_{\dot{a}}$. (As an aside, are larger $\mathcal{N} = 8$ or 16 possible in higher dimensions?) I'm used to greater emphasis being given to $\left\{Q_a, Q^\dagger_{\dot{a}}\right\} = -2\sigma^\mu_{a\dot{a}} P_\mu$, in relation to the Coleman–Mandula theorem that connects the conserved charges to the Poincaré generators. (As another aside, Oliver reminded me that the Coleman–Mandula theorem is based on the S matrix, so another way to get around it is to consider conformal field theory, where the S matrix is not well-defined.)

Superspace I find superspace to be very useful (and hadn't known it was introduced by Salam and Strathdee in 1974), though the anti-commuting coordinates (θ, θ^*) may seem weird at first. (Note that θ and θ^* anti-commute with Q and $Q^\dagger$ as well as with each other.) Expansions in θ and θ^* quickly truncate (they both carry spinor indices, so no more than two θs and two θ^*s can appear in any term), and provide an organizing principle. Messy expressions can often be simplified by matching powers of θ and θ^*. With $\mathcal{N} > 1$, one can either continue using $\mathcal{N} = 1$ superspace, or add more anti-commuting coordinates.

Chiral superfield Oscar motivated the constraint $\mathcal{D}^*_{\dot{a}} \Phi(x, \theta, \theta^*) = 0$ for left-handed chiral superfields by arguing that we want to minimize the number of independent components in the superfields. (Similarly, right-handed chiral superfields are annihilated by $\mathcal{D}_a \Phi^\dagger(x, \theta, \theta^*) = 0$.) I didn't recall the fact that Φ can be written solely in terms of θ and $y^\mu \equiv x^\mu - i\theta^c \sigma^\mu_{c\dot{c}} \theta^{*\dot{c}}$.

Superpotential We identify the $\theta\theta$ component of the chiral superfield (F) as a piece of the supersymmetric action, by observing that its supersymmetry transformation is a total derivative. More generally, any function $W(\Phi)$ of chiral superfields is itself a chiral superfield, and its F term is the superpotential.

Vector superfields $V(x, \theta, \theta^*)$ is defined by hermiticity, so $\Phi^\dagger\Phi$ is a vector superfield. We identify its $\theta\theta\theta^*\theta^*$ component (D) as another piece of the supersymmetric action, since its supersymmetry transformation is another total derivative. The D term of $\Phi^\dagger\Phi$ gives us kinetic terms; since the $F^\dagger F$ term has no derivatives, F is an auxiliary field that can be replaced via its equation of motion $F^\dagger_i = -\frac{\partial W(A)}{\partial A_i}$ to give

$$\mathcal{L} = -\partial^\mu A^\dagger_i \partial_\mu A_i + i\psi^\dagger_i \overline{\sigma}^\mu \partial_\mu \psi_i - \left|\frac{\partial W(A)}{\partial A_i}\right|^2 - \frac{1}{2}\left[\frac{\partial^2 W(A)}{\partial A_i \partial A_j}\psi_i\psi_j + \text{h.c.}\right],$$

where $\Phi_i = A_i + \sqrt{2}\theta\psi_i + \theta\theta F_i$. The first two terms are the kinetic terms, the third is the scalar potential $V(A)$, and the fourth $\mathcal{L}_{mY}$ collects fermionic mass and Yukawa terms.

Degrees of freedom Originally, we had two degrees of freedom in the complex scalar A, four in the complex (two-component) spinor ψ, and another two in the complex scalar auxiliary field F, which add up to four bosonic and four fermionic degrees of freedom. After applying the equations of motion, the auxiliary field drops out, but the Dirac equation kills off two degrees of freedom for ψ, leaving us with two bosonic and two fermionic degrees of freedom. In both cases, we have the proper balance between bosons and fermions required by supersymmetry.

Wess–Zumino model Defined by superpotential $W(A) = \frac{1}{2}mA^2 + \frac{1}{6}gA^3$. The structure of the lagrangian ensures that the scalar A and fermion ψ have the same mass, and their couplings are also related through the scalar potential and $\mathcal{L}_{mY}$,

$$V(A) = \left|\frac{\partial W(A)}{\partial A_i}\right|^2 = m^2 A^\dagger A + \frac{1}{2}gm\left(A^\dagger A^2 + {A^\dagger}^2 A\right) + \frac{1}{4}g^2\left(A^\dagger A\right)^2$$

$$\mathcal{L}_{mY} = -\frac{1}{2}\left[\frac{\partial^2 W(A)}{\partial A_i \partial A_j}\psi_i\psi_j + \text{h.c.}\right] = -\frac{1}{2}m\psi\psi - \frac{1}{2}gA\psi\psi + \text{h.c.}$$

This is almost what Oliver wrote out in components before we considered superfields. Unless I'm crazy, here we have a Majorana mass term.