

Comments on anomalies (12 October 2012)

One-loop exact Anomalies are responsible for the non-existence of otherwise-consistent chiral gauge theories. After mentioning that the ultimate origin of anomalies is that the functional measure for the fermion path integral is not gauge invariant, Srednicki...calculates some one-loop diagrams! In particular, he calculates the famous triangle diagrams, which end up violating a Ward identity, that is, gauge invariance expressed in terms of current conservation. I didn't previously appreciate how neat (and potentially unique) it is that this one-loop result is exact, and receives no further corrections from higher orders in perturbation theory. Srednicki derives this from the path integral in chapter 77, but mentions that it was originally "established by a careful study of Feynman diagrams."

Tricks By embedding Weyl fields into Dirac (or Majorana) fields by means of chiral projectors, Srednicki is able to re-use earlier results. The main difference ends up being extra factors of $\frac{1}{2}$ and γ_5 from these projectors, which can be combined within any given trace. Oscar also complimented Srednicki's analysis of the Lorentz structure of integrals, which often allowed very simple arguments to establish that certain terms must vanish. I forgot to bring up the cute free-field mode expansion that Srednicki uses to check the number of degrees of freedom in the projected Dirac field, and to determine the helicities of the corresponding particles.

Regularization I was familiar (probably from [David Kaplan's Les Houches lectures](#)) with the subtlety of γ_5 in dimensional regularization, but had forgotten about the more straightforward problem with Pauli-Villars regularization of chiral gauge theories, which forbid fermion masses. Peskin & Schroeder use dimensional regularization anyway, which Srednicki describes as "workable, but cumbersome". Srednicki also states that these difficulties regulating chiral gauge theories are "a hint that they may not make sense", which I thought was a bit too glib.

ABJ Peskin & Schroeder present three "way[s] of understanding" the Adler-Bell-Jackiw anomaly

$$\partial_\mu j_A^\mu = -\frac{e^2}{16\pi^2} \epsilon^{\alpha\beta\mu\nu} F_{\alpha\beta} F_{\mu\nu},$$

of which I was most familiar with the triangle diagram calculation where the anomaly appears from shifting the loop momentum in a linearly-divergent integral. (Srednicki goes through similar calculations in the context of chiral gauge theories in chapter 75.) Another argument is based on moving the two fermion fields in the axial current infinitesimally close together, which I believe was meant to remind the reader of the operator product expansion (OPE) that Peskin & Schroeder cover in chapter 18 but we didn't go through. (It also reminded me of the "differential regularization" that Oliver was telling me about, but this may be completely unrelated.) The third approach is the functional integral method that Srednicki covers in chapter 77, where the anomaly appears from the Jacobian of the axial U(1) transformation.

SU(2) Srednicki mentions that a more subtle global anomaly forbids gauge theories with an odd number of Weyl fields in a pseudo-real rep, such as the fundamental rep of SU(2). Several of us are currently working on SU(2) gauge theories with fundamental fermions, but we're using Dirac fields, so we're safe from this. We're also safe from gauge anomalies since the anomaly coefficient $A(R) = 0$ for all SU(2) reps.

String Theory A connection that Oliver explained is how anomalies restrict the number of dimensions in which string theories are consistent. Specifically, the trace anomaly T^μ_μ of the energy-momentum tensor would break conformal invariance on the two-dimensional world-sheet swept out by the string. This anomaly cancels in 26 dimensions for bosonic string theory, or 10 dimensions for supersymmetric string theory.

Green–Schwarz A popular way of describing anomalies is that a symmetry present in the classical action is broken by quantum effects from the functional integral. I mention this now because Oliver sketched how the [Green–Schwarz mechanism](#) (originally found in ten dimensions) works with an action that is not gauge invariant: hexagonal loop diagrams that would have introduced an anomaly (had the action been gauge invariant) instead cancel problematic tree diagrams.

One-loop exact To end with the same heading as I began, Oliver also mentioned that exact supersymmetric β functions arise because supersymmetry relates the energy-momentum tensor $T^{\mu\nu}$ to the axial current j_A^μ whose divergence $\partial_\mu j_A^\mu$ is one-loop exact. (The rest of this supersymmetric multiplet is filled by the gravitino, $\gamma_\mu \psi_{3/2}^\mu$.)