Comments on gauged spontaneous symmetry breaking (28 Sept. 2012)

Breaking An important result in these chapters (which first appears in the context of global symmetry breaking in chapter 32) is that a generator T is broken if $T_{ij}v_j \neq 0$ and unbroken if $T_{ij}v_j = 0$, where v_i is the vev of field ϕ_i. This bears some resemblance to our discussion of BRST symmetry last week, in which the physical states were annihilated by the BRST charge due to their gauge invariance. In this case the state of interest is the vacuum, which any unbroken generators T (corresponding to preserved symmetries) must annihilate. (I'm fairly confident it is still correct to refer to the vacuum as a closed state in the kernel of these T even if the T are not nilpotent.)

Goldstones Another important result that first appeared in chapter 32 is that each broken generator results in a massless (Nambu–)Goldstone boson. Goldstone, Salam and Weinberg presented three proofs of this statement in 1962, one perturbative and two more general. Srednicki goes through (what I find) the simplest of these proofs by deriving

$$\frac{\partial^2 V}{\partial \phi_i \partial \phi_j} T_{jk}\phi_k + \frac{\partial V}{\partial \phi_j} T_{ji} = 0$$

for the potential V and generator T. The vacuum $\phi_i = v_i$ minimizes V, $\left.\frac{\partial V}{\partial \phi_j}\right|_{\vec{\phi}=\vec{v}} = 0$, while $\left.\frac{\partial^2 V}{\partial \phi_i \partial \phi_j}\right|_{\vec{\phi}=\vec{v}} = (m^2)_{ij}$, so $(m^2)_{ij}\, T_{jk}v_k = 0$ for any generator. If the corresponding symmetry is unbroken $T_{jk}v_k = 0$ and we're done. If it's broken, then $T_{jk}v_k \neq 0$ is an eigenvector of the mass-squared matrix with eigenvalue zero, corresponding to a massless particle. There are of course some caveats (in particular, we need to establish that this classical effective potential obeys the same symmetries as the quantum action). (Peskin & Schroeder present the current-conservation proof, also in a way that looks much simpler than my notes from Iain's Weinberg-based class. Weinberg2 goes through both of these in section 19.2.)

Eating When the broken symmetry that would produce a Goldstone is gauged, the gauge-covariant derivatives in the Higgs field kinetic term produce a mass term for the gauge field, as well as a cross term between the gauge field and the derivative of the would-be-Goldstone field. Fixing to R_ξ gauge cancels that cross term, and gives the would-be Goldstone an unphysical ξ-dependent mass ($= gv\sqrt{\xi}$), so that the would-be Goldstone vanishes in unitary gauge $\xi \to \infty$. Because we're theorists, we just say that the gauge field "eats" the would-be Goldstone to become massive (and gain a longitudinal degree of freedom). This gives us the same number of degrees of freedom with or without spontaneous symmetry breaking.

Components Although the massive vector field now has three degrees of freedom, we are still packing it into a four-component object. Oliver pointed out that the fourth degree of freedom is now killed off by the equation of motion. In the abelian case, the equation of motion gives $M^2\partial^\nu A_\nu = 0$, which provides a single constraint if the vector field mass $M^2 = g^2v^2 \neq 0$, and no constraint if $M^2 = 0$.

Subgroups There is a straightforward but somewhat messy procedure to determine how a given representation of a given group transforms under a given subgroup, which probably is not worth going through in complete detail (though some rules for manipulating Young tableaux may be useful). Sometimes the result is "intuitively obvious", as in the case Oliver put on the board: $\mathbf{5} \to (\mathbf{3}, \mathbf{1}, -\frac{1}{3}) \oplus (\mathbf{1}, \mathbf{2}, \frac{1}{2})$ for SU(5) $\to$ SU(3)×SU(2)×U(1) by the vev $\langle\phi\rangle = \mathrm{diag}(-\frac{1}{3}, -\frac{1}{3}, -\frac{1}{3}, \frac{1}{2}, \frac{1}{2})$. For a simple but fun example of something you might not find intuitively obvious, I posted solutions to a pair of group theory problems showing that the fundamental and adjoint representations of SU(3) each transform differently under two different SU(2) subgroups of SU(3).

Unitary gauge I liked Srednicki's explanation in chapter 85 for the difficulty of establishing renormalizability in unitary gauge: the absence of ghost kinetic terms and the projector $\left(g^{\mu\nu} + \frac{k^\mu k^\nu}{M^2}\right)$ in the massive vector field propagator result in both ghost and vector propagators scaling like M^{-2} at large momenta, so that simple power-counting arguments for renormalizability would blow up. I had also forgotten that unitary gauge is equivalent to R_ξ gauge with $\xi \to \infty$.

Gauges Oliver argued that these examples illustrate why it can be useful to study multiple choices of gauge: unitary gauge makes it easier to identify the physical degrees of freedom but harder to carry out loop calculations, and vice-versa in R_ξ gauge with $\xi = 1$.