Comments on classical non-abelian gauge theory (7 September 2012)

Gauging Both Peskin & Schroeder as well as Srednicki motivate gauge transformations by appealing to sightings of gauge invariance in their earlier discussions of QED (section 4.1 and chapter 58, respectively). I was more comfortable with the way Peskin & Schroeder introduce the gauge field through a parallel transporter/comparator/connection/whatever, as opposed to Srednicki's comments on "local symmetries", which is a term I try to avoid[1] in order to distinguish between symmetry and gauge invariance. As we discussed, gauge invariance introduces a level of redundancy so that we can treat massless vector fields (with two transverse degrees of freedom) using four-component relativistic notation.[2] Changing the dimension of spacetime also changes the number of transverse degrees of freedom, which may come up again in the context of anomalies or chiral symmetry.

Minimal coupling As in general relativity, replace $\partial_\mu \to D_\mu$, where the latter are gauge-covariant derivatives. Since $D_\mu\psi$ then transforms like ψ, both $\overline{\psi}D_\mu\psi$ and $\overline{\psi}\psi$ terms are gauge invariant (the gauge transformation is unitary), and can appear in the lagrangian.

Reminiscence When I saw this stuff at MIT, Wati talked a lot about groups and representations (including roots, weights, Dynkin diagrams, the Killing–Cartan classification), which can be fun but doesn't seem particularly relevant for our purposes. When Wati eventually got back to physics, he spoke about vector bundles, considering each point in spacetime to possess a complex vector space (just a complex plane in the abelian case) with the gauge fields serving to connect the vector spaces at different spacetime locations.

Neat trick I liked recalling the properties of group and algebra elements by thinking about their eigenvalues: elements T of unitary groups have eigenvalues that are pure phases, implying that elements t of the corresponding algebra ($T = e^{i\alpha t}$) have real eigenvalues – that is, they are hermitian. Similarly, the same thought process relates $\det T = 1$ to $\mathrm{tr}\,[t] = 0$. I'm sure this can be (and should be) formulated more carefully, but I've probably seen and forgotten that formulation, while I'm more likely to retain this mnemonic.

Wilson loops I sometimes get sloppy and gloss over the fact that the non-abelian Wilson loop (which Srednicki deals with in chapter 82) has to be defined with a trace in order to be gauge invariant. Similarly, the non-abelian field strength is not gauge-invariant, so $\mathrm{Tr}\,[F_{\mu\nu}F^{\mu\nu}]$ appears in the lagrangian.

Symmetrization When Srednicki talked about the invariance of $\phi^{\dagger i}\phi_i$ below Eqn. 70.19, I immediately pictured the SU(N) Young diagrams for $N \otimes \overline{N} = 1 \oplus A$, and wondered what had happened to the adjoint rep. Of course, it's not there because Srednicki is summing over i to pick out the singlet, and I enjoyed his subsequent derivation of $N \otimes \overline{N} = 1 \oplus A$.

Anomalies I don't recall working with the d^{abc} or the anomaly coefficient that Srednicki defines in Eqn. 70.33; I presume we'll see this again when we get to anomalies.

[1] Except in the context of the spurion trick.

[2] I suspect the books discuss this issue earlier, in the context of QED; I am most familiar with the discussion in chapter 5 of Preskill's lecture notes.