

Eight light flavors on large lattice volumes — — — a USQCD BSM project — — —

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Lattice 2013, Mainz, 29 July



bsm.physics.yale.edu

USBSM participants

Members of USQCD using leadership computing resources
to study strongly-coupled physics beyond the standard model

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Yale T. Appelquist, G. Fleming, G. Voronov



www.usqcd.org

USBSM projects

(cf. USQCD white paper)

- **SU(3) with $N_F = 8$ fundamental**
- Pseudo-dilaton in SU(3) with $N_F = 2$ sextet
- Scalar pseudo-Goldstones in SU(2) with $N_F = 2$ fundamental
- Lattice supersymmetry ($\mathcal{N} = 1$ SYM; $\mathcal{N} = 4$ SYM; $\mathcal{N} = 1$ SQCD)

Argonne Leadership Computing Facility



SU(3) with $N_F = 8$ fundamental

2008–2010: Deuzeman, Lombardo & Pallante; Jin & Mawhinney;
Fodor, Holland, Kuti, Nogradi & Schroeder; Hasenfratz
Boulder, [arXiv:1301.1355](#) – large mass anomalous dimension
 $\gamma_m \sim 1$ across wide range of energy scales
LatKMI, [arXiv:1302.6859](#) – chirally broken with $\gamma_m \sim 1$

Goal: Large-volume p -regime lattice ensembles for community use
Pursue every possible analysis!

This talk (after overview of lattice generation):

- Initial results for the hadron spectrum
- Chiral condensate, GMOR relation, Dirac eigenvalues
- Finite-size scaling
- **Time permitting:** Valence domain wall measurements
- **Backup:** Thermalization, autocorrelations, topological suscept.

Eight-flavor lattice generation strategy

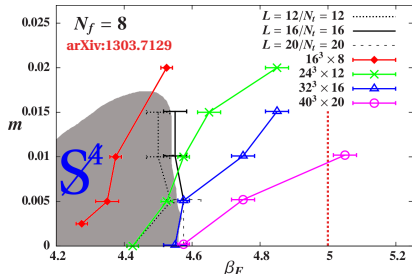
Goal: Large-volume p -regime lattice ensembles for community use

- Fix gauge coupling at relatively strong value
 - Compensate for effects of many light fermions
 - Be cautious of strong-coupling lattice artifacts

nHYP-smearred staggered lattice action

(new since May)

- Fundamental-plaquette $\beta_F = 5.0$, adjoint-plaquette $\beta_A = -1.25$
- Implemented in QHMC/FUEL (“Framework for Unified Evolution of Lattices”)



Lattice phase diagram
already explored independently

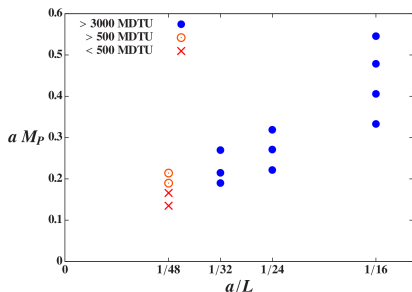
$N_T \leq 20$ thermal transitions
hit bulk transition around $\beta_F \approx 4.6$

Chiral limit requires large volumes

Eight-flavor lattice generation strategy

Goal: Large-volume p -regime lattice ensembles for community use

- Push towards chiral limit on largest possible volumes
 - χ PT radius of convergence shrinks with N_F ([arXiv:1002.3777](https://arxiv.org/abs/1002.3777))
 - Monitor finite-volume effects from overlapping ranges of masses



Ensembles up to $32^3 \times 64$ complete,
 $48^3 \times 96$ in production

Fermion mass $0.008 \leq m \leq 0.05$,
0.004 and 0.006 in production

Pseudoscalar mass $0.19 \leq M_P \leq 0.55$, with $M_P \approx 0.135$ in production
($5.3 \leq M_P L \leq 10.3$)

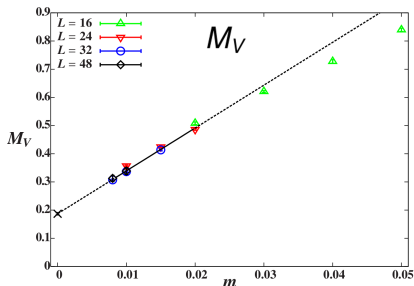
Initial results for the hadron spectrum

Linear fits mainly to guide the eye

($M_P L \geq 6$)

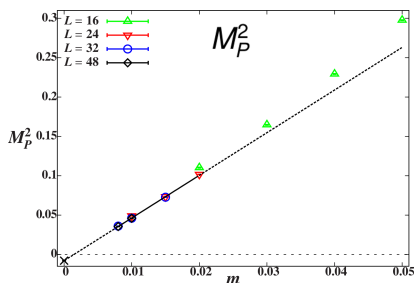
Always fit only $L \geq 24$ and $m \leq 0.02$,

omitting $L = 24$ with $m = 0.01$



Intercept: 0.187(4)

χ^2/dof : 18/5

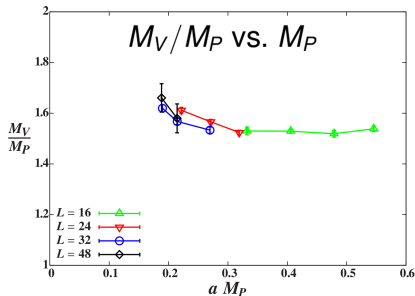
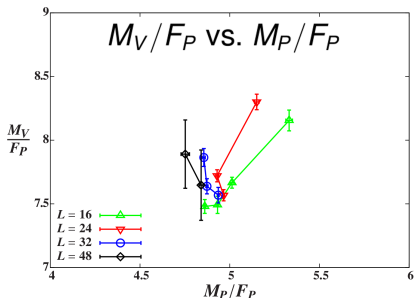


Intercept: $-0.00793(11)$

χ^2/dof : 272/5

- Clear deviations from linearity, especially as m increases
- Seem unlikely to be due to finite-volume effects or chiral logs...

More fun with the hadron spectrum



Left: Finite-volume effects increase M , decrease F_P

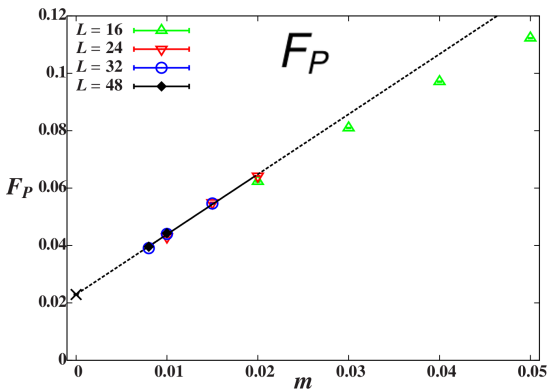
Clearly visible in lightest points for $L = 16$ and 24 (both $M_P L = 5.3$), all other points essentially on top of each other ($\sim 4\%$ variation)

Right: In chirally broken systems, $M_V/M_P \rightarrow \infty$ as $M_P \rightarrow 0$

Ratio may be starting to turn up, but not significantly ($\sim 5\%$ variation)

→ Need $m \leq 0.006$ ensembles to probe spontaneous χ SB

Initial results for pseudoscalar decay constant



$$1 \leq F_P L \leq 2.1$$

Fitted points: $F_P L \geq 1.25$

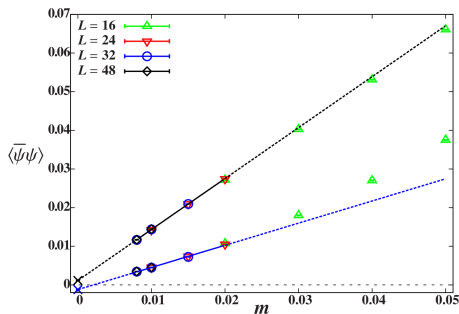
Intercept: 0.0229(7)
 χ^2/dof : 391/5

As for M_P^2 , large χ^2 from $m \leq 0.02$ linear fit,
with additional deviation as m increases

Motivates investigation of chiral condensate...

Initial results for the chiral condensate

- In chiral limit, order parameter of spontaneous χ SB
- Direct measurements sensitive to valence mass in term $\propto m_v/a^2$
- Leading-order χ PT (GMOR relation): $\langle \bar{\psi}\psi \rangle = M_P^2 F_P^2 / 2m$



$$\langle \bar{\psi}\psi \rangle \text{ and } M_P^2 F_P^2 / 2m$$

Intercepts: 0.001146(18)
and -0.001223(20)

χ^2/dof : 51/5 and 285/5

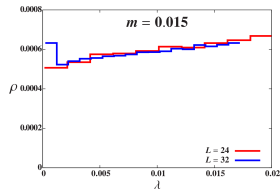
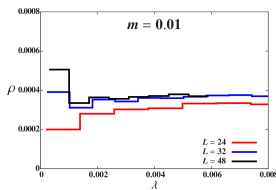
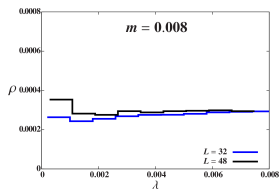
- Direct measurements require order-of-magnitude extrapolation
- GMOR predictions less sensitive to m_v , fit has larger χ^2
- Third option: Dirac eigenvalue spectrum $\rho(\lambda \rightarrow 0)$

Chiral condensate from Dirac eigenmode number

Address valence mass effects in $\langle \bar{\psi}\psi \rangle$

by analyzing the eigenvalues of the massless Dirac operator

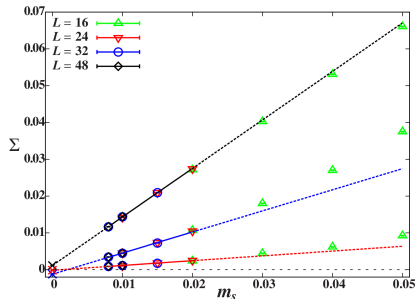
Compare $\rho(\lambda)$ on different volumes with fixed sea mass:



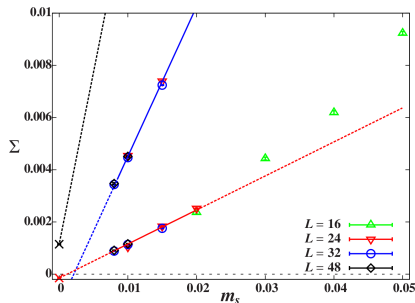
Good agreement up to expected finite-volume effects,
and topological zero-mode effects in first bin

Extract $\Sigma_{m_s} \equiv \pi \rho(\lambda \rightarrow 0)$ from derivative of mode number $\nu \sim \int \rho d\lambda$

Chiral condensate from all three approaches



Recall: $0.001146(18)$
and $-0.001223(20)$
 χ^2/dof : 51/5 and 285/5



Zoom in on Σ
Intercept: $-0.000151(16)$
 χ^2/dof : 32/5

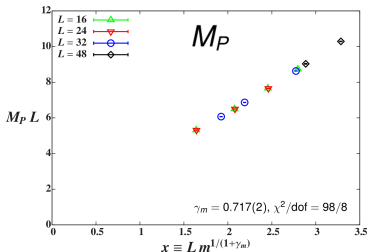
- Σ from eigenvalues seems to be the best controlled
- $\lim_{m_s \rightarrow 0} \langle \bar{\psi}\psi \rangle$ is small or vanishing — motivates finite-size scaling...

(Future fun with the eigenmode number:

extract running mass anomalous dimension – Anqi Cheng, 11:40 Weds.)

Initial results for finite-size scaling

- IR conformality $\implies ML = f(x)$ with scaling variable $x \equiv Lm^{1/(1+\gamma_m)}$
- Search for anomalous dimension γ_m that optimizes curve collapse
- I use method of Houdayer and Hartmann, [cond-mat/0402036](https://arxiv.org/abs/cond-mat/0402036)



Relatively small number of points for FSS

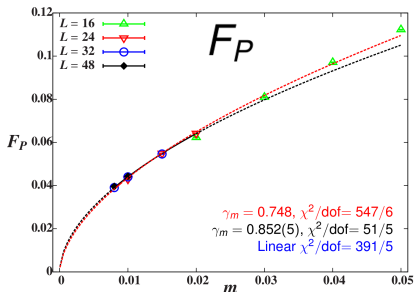
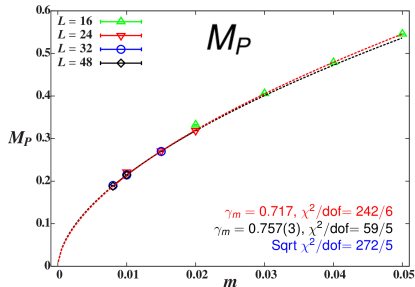
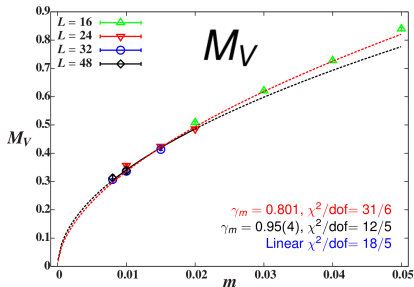
Obs.	γ_m	χ^2/dof
M_P	0.717(2)	98/8
M_V	0.801(14)	40/8
F_P	0.748(2)	1444/8

- Roughly consistent $\gamma_m \sim 0.75$, though widely-varying quality
- Try using these γ_m in power-law fits to hadron spectrum...

(Future fun with finite-size scaling:

account for nearly-marginal gauge coupling – Anna Hasenfratz, 14:20 Tues.)

Revisit hadron spectrum: Power-law fits



Same seven points in fits

Red fits fix γ_m from FSS

Black fits let γ_m float

(γ_m always increases)

χ^2/dof smaller than linear fits,

$\gamma_m \gtrsim 0.75$ preferred

Motivation for valence domain wall analyses

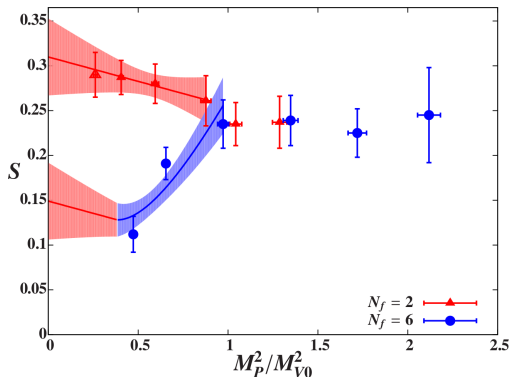
Experiment requires small electroweak S parameter,

$$S = 0.03(10) \text{ with } M_H = 125 \text{ GeV}$$

Lattice Strong Dynamics Collaboration found reduction in S for $N_F = 6$

(PRL **106**:231601, 2011)

Important observable to explore for $N_F = 8$



LSD analysis: $32^3 \times 64$
domain wall fermions
($M_H^{(ref)} \sim 1 \text{ TeV}$)

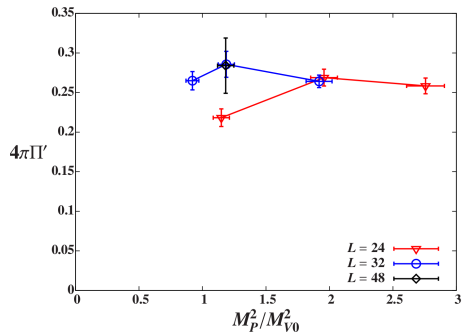
Want to make sure
decrease is not
finite-volume effect

Initial results for $V-A$ vacuum polarization

S parameter depends on $Q^2 \rightarrow 0$ slope of transverse Π_{V-A}

$$S = 4\pi N_D \lim_{Q^2 \rightarrow 0} \frac{d}{dQ^2} \Pi_{V-A}(Q^2) - \Delta S_{SM}(M_H)$$

$$\Pi_{V-A}^{\mu\nu}(Q) = Z \sum_X e^{iQ \cdot (x + \hat{\mu}/2)} \text{Tr} \left[\langle \mathcal{V}^{\mu a}(x) \mathcal{V}^{\nu b}(0) \rangle - \langle \mathcal{A}^{\mu a}(x) \mathcal{A}^{\nu b}(0) \rangle \right]$$



We've already seen
clear finite-volume effects
in lightest $24^3 \times 48$ point

$N_F = 6$ reduction began
only for $M_P^2 \lesssim M_{V0}^2$

Looking forward to smaller masses on $48^3 \times 96$!

Recapitulation: SU(3) with $N_F = 8$ fundamental

This USBSM project is off to a good start!

Goal: Large-volume p -regime lattice ensembles for community use

- Ensembles up to $48^3 \times 96$ becoming available
- Initial analyses suggest small or vanishing chiral condensate
- Finite-size scaling prefers large $\gamma_m \gtrsim 0.7$
- Prospects for S parameter from valence domain wall

Looking forward to more fun in the future

- Running mass anomalous dimension from eigenmode number
- Unitary staggered analysis of vacuum polarization
- Other USBSM projects: light scalars; lattice supersymmetry



Thank you!

Thank you!

Contributors to this talk

George Fleming, Anna Hasenfratz, Meifeng Lin, Ethan Neil, James Osborn

The rest of the USBSM community

Tom Appelquist, Rich Brower, Mike Buchoff, Simon Catterall, Michael Cheng, Joel Giedt, Kieran Holland, Joe Kiskis, Julius Kuti, Heechang Na, Gregory Petropoulos, Claudio Rebbi, Chris Schroeder, Don Sinclair, Gennady Voronov, Pavlos Vranas, Oliver Witzel



Backup: Status of ensemble generation

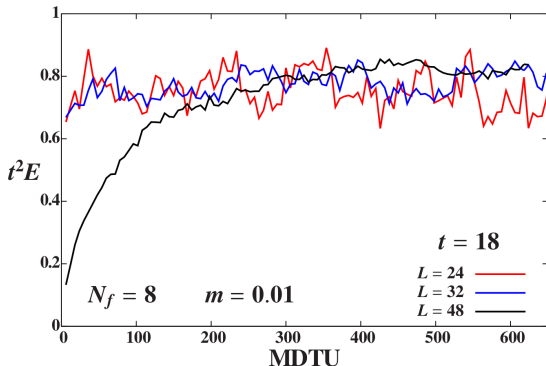
Many-flavor lattice systems may have long autocorrelations

Expect observables related to topology to be sensitive (arXiv:1204.6000)

Wilson flow to monitor thermalization, autocorrelations

Integrate infinitesimal stout smearing steps out to flow time t

with $\sqrt{8t}$ comparable to $L/2$



$$E(t) = -\frac{1}{2} \text{ReTr} F_{\mu\nu} F^{\mu\nu}(t)$$

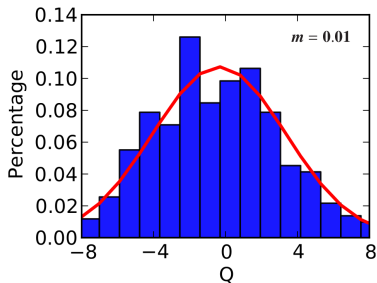
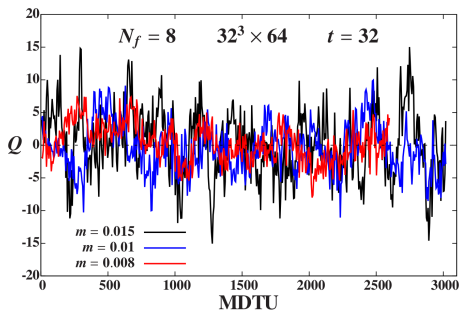
$$\sqrt{8t}/24 = 0.5$$

$$\sqrt{8t}/32 = 0.375$$

$$\sqrt{8t}/48 = 0.25$$

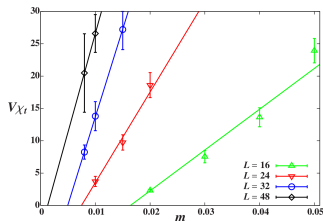
Backup: Topo. charge and χ_t from the Wilson flow

$$32\pi^2 Q = \text{ReTr}[\epsilon_{\mu\nu\sigma\tau} F_{\mu\nu} F^{\sigma\tau}] \text{ after flowing to } \sqrt{8t} = L/2$$



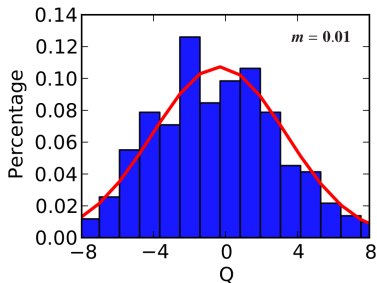
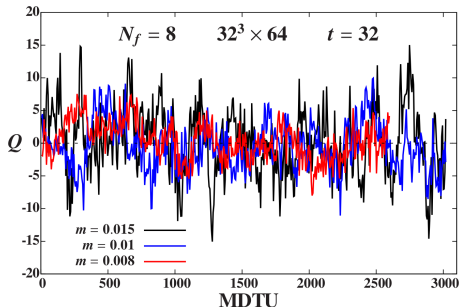
m -dependence of topological suscept.

Linear $m \rightarrow 0$ extrapolations go negative



Backup: Topo. charge and χ_t from the Wilson flow

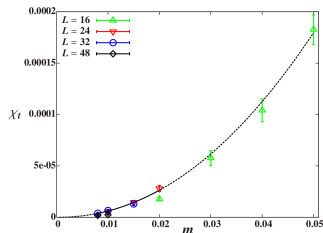
$$32\pi^2 Q = \text{ReTr}[\epsilon_{\mu\nu\sigma\tau} F_{\mu\nu} F^{\sigma\tau}] \text{ after flowing to } \sqrt{8t} = L/2$$



m -dependence of topological suscept.

For IR-conformal system, $\chi_t \propto m^{4/(1+\gamma_m)}$

Neglecting poorly-determined $L = 48$,
 $\gamma_m = 0.91(16)$ with $\chi^2/\text{dof} = 1.5/3$

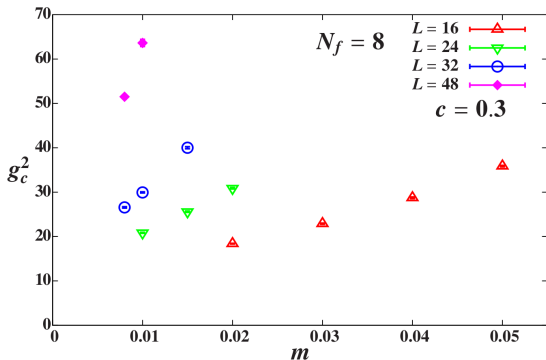


Backup: Wilson flow running coupling (arXiv:1208.1051)

Z. Fodor, K. Holland, J. Kuti, D. Negradi and C. H. Wong

define $SU(N)$ running coupling from Wilson flow $\langle t^2 E(t) \rangle$:

$$g_c^2(L) = \frac{128\pi^2 \langle t^2 E(t) \rangle}{3(N^2 - 1)(1 + \delta_c)}$$



$$c \equiv \sqrt{8t}/L = 0.3$$

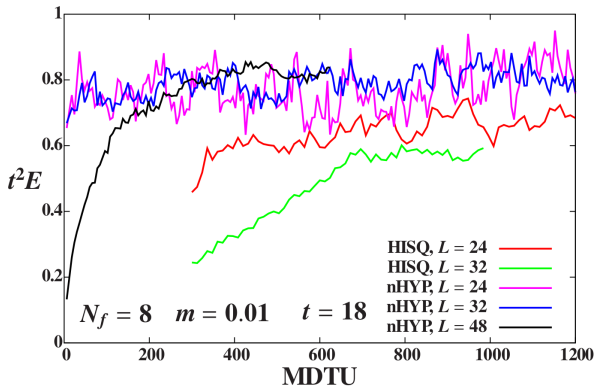
$$1 + \delta_c \approx 0.97$$

$$g_c^2 \sim \mathcal{O}(10)$$

Sensitive to non-zero fermion mass as well as lattice volume

Backup: Trouble with HISQ at strong coupling

Our new nHYP action is **orders of magnitude** faster than HISQ
at comparably strong couplings



As above,
Wilson flow $t^2 E$
as measure of
thermalization

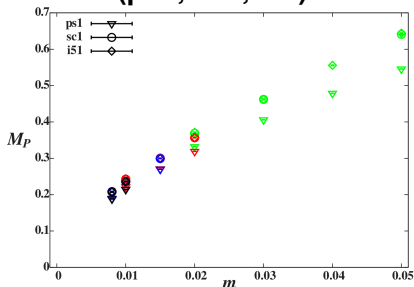
HISQ strong-coupling numerical instabilities \longrightarrow short trajectories
 \longrightarrow long thermalization/autocorrelation times

Backup: nHYP taste splitting

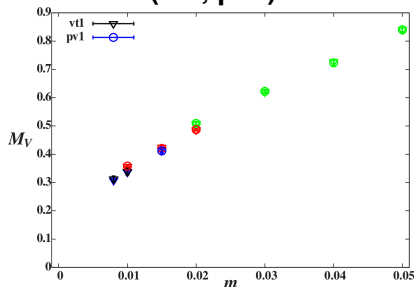
The nHYP-smeared action

exhibits excellent control over staggered taste splitting

**Pseudoscalar states
(ps1, sc1, i51)**



**Vector states
(vt1, pv1)**



These are very preliminary results;

pv1 masses are not yet determined for some $L = 32$ and 48 runs

Backup: Valence domain wall procedure (LHPC, arXiv:0705.4295)

- HYP smear to reduce m_{res} and get renormalization factors $Z \sim 1$
- Tune domain wall height M_5 and length L_s of fifth direction so that residual chiral symmetry breaking $m_{res} \ll m$
- Tune bare valence mass m_f so that M_P matches unitary value

$$M_5 = 1.8 \text{ and } L_s = 16$$

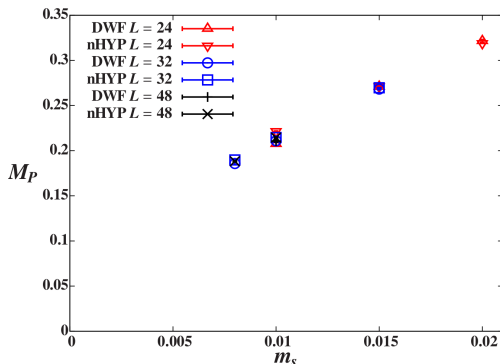
$$\longrightarrow Z_V \approx Z_A \approx 1.08$$

$$\longrightarrow m_{res} \approx 0.001 \lesssim m_f/13$$

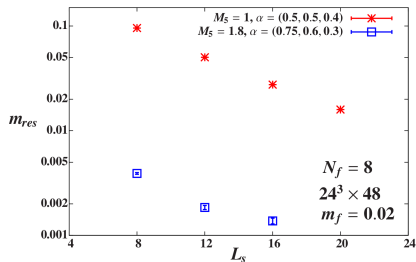
Need $m_f > m_s$ to match M_P :

$$1.7 \lesssim m/m_s \lesssim 2.05$$

where $m \equiv m_f + m_{res}$

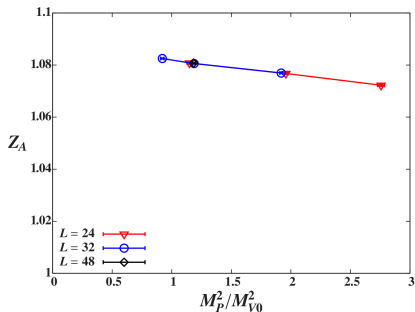
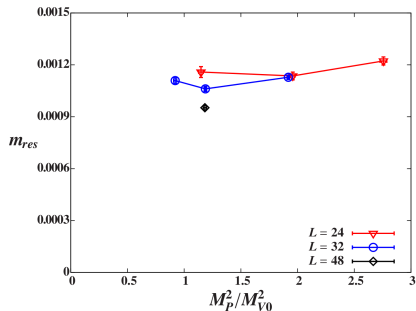


Backup: Valence domain wall m_{res} and Z_A



• Want $m_{res} \ll m$

• Want $Z_A \sim 1$



The S parameter

(Peskin and Takeuchi)

Constrain the physics of electroweak symmetry breaking
from its effects on vacuum polarizations $\Pi(Q)$ of EW gauge bosons



(independent of flavor physics/ETC)

$$S = 4\pi N_D \lim_{Q^2 \rightarrow 0} \frac{d}{dQ^2} \Pi_{V-A}(Q^2) - \Delta S_{SM}(M_H)$$

$$\textcircled{1} \quad \Pi_{V-A}^{\mu\nu}(Q) = Z \sum_x e^{iQ \cdot (x + \hat{\mu}/2)} \text{Tr} \left[\langle \mathcal{V}^{\mu a}(x) \mathcal{V}^{\nu b}(0) \rangle - \langle \mathcal{A}^{\mu a}(x) \mathcal{A}^{\nu b}(0) \rangle \right]$$

$\textcircled{2} \quad N_D \geq 1$ is the number of doublets with chiral electroweak couplings

$\textcircled{3} \quad \Delta S_{SM}(M_H)$ subtracted so that $S = 0$ in the standard model

Removes three eaten modes, depends on Higgs mass