$(g-2)_\mu$ FAQ

David Schaich

20 September 2012

Broad Outline

- Status: standard model vs. experiment
- Hadronic contributions and role of lattice QCD
- Future prospects

Inspired by talks by Tom Blum and Brendan Casey

What is $(g-2)$?

Magnetic moment $\vec{\mu}$

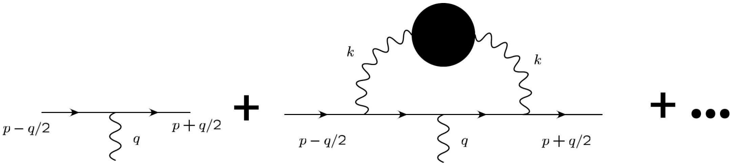
governs interaction of spin $\vec{S}$ with static magnetic field $\vec{B}(\vec{x})$

$$V(\vec{x}) = -\vec{\mu}\cdot\vec{B}(\vec{x}) = -\frac{ge}{2m}\vec{S}\cdot\vec{B}(\vec{x})$$

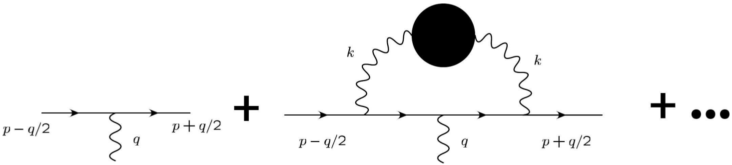
Free Dirac equation predicts $g=2$ for elementary spin-1/2 particles

$\Longrightarrow a \equiv (g-2)/2$ is **anomalous magnetic moment**

$(g-2) \neq 0$ due to quantum effects



What is $(g-2)$?



Quantum effects produce vertex function

$$\gamma_\mu \longrightarrow \Gamma_\mu(q^2) = \gamma_\mu F_1(q^2) + \frac{i\sigma_{\mu\nu}q^\nu}{2m} F_2(q^2)$$

General structure due to Lorentz invariance (with on-shell lepton) and current conservation (Ward identity $q^\mu \Gamma_\mu(q^2) = 0$)

Form factors as $q^2 \to 0$:

- $F_1(0) = 1$ (electric charge of lepton in units of e)
- $g = 2F_1(0) + 2F_2(0) \Longrightarrow F_2(0) = (g-2)/2 = a$

What's the point?

Precise comparisons of theory vs. experiment

Agreement Confirms theory to available level of precision
Example: electron $(g-2)_e$ match to $\sim$1 pp**b** (below)
"Crowning achievement of QED"

Discrepancy Implies new physics beyond the "standard model"
Example: deviations in orbit of Uranus
used to discover Neptune in 1846

For $(g-2)_e$, we have agreement to $\sim 10^{-12}$ (arXiv:1205.5368)

Experiment: $a_e \times 10^{12} = 1\ 159\ 652\ 180.73(28)$

Standard model: $a_e \times 10^{12} = 1\ 159\ 652\ 181.78(77)$

Discrepancy: $\Delta a_e \times 10^{12} = -1.06(82)$

What's the point?

Precise comparisons of theory vs. experiment

Agreement Confirms theory to available level of precision
Example: electron $(g-2)_e$ match to $\sim$1 pp**b**
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Discrepancy Implies new physics beyond the "standard model"
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For $(g-2)_\mu$, we seem to have a 3.6σ discrepancy!

Experiment:	$a_\mu \times 10^{11} = 116\,592\,089(63)$
Standard model:	$a_\mu \times 10^{11} = 116\,591\,802(49)$
Discrepancy:	$\Delta a_\mu \times 10^{11} = 287(80)$

(all errors added in quadrature)

Does $(g-2)_\mu$ imply new physics?

For $(g-2)_\mu$, we seem to have a 3.6σ discrepancy!

Experiment:	$a_\mu \times 10^{11} = 116\,592\,089(63)$
Standard model:	$a_\mu \times 10^{11} = 116\,591\,802(49)$
Discrepancy:	$\Delta a_\mu \times 10^{11} = 287(80)$

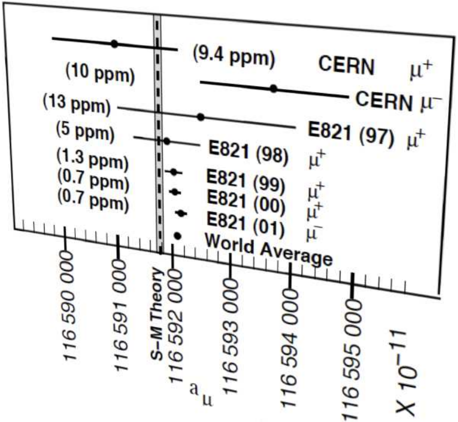
"Basically useless because no **solid** prediction" (emphasis added)

$\sim 3\sigma$ discrepancy has been claimed for around a decade
and may well remain for another decade

Before speculating about new physics scenarios,
let's review where these two values come from

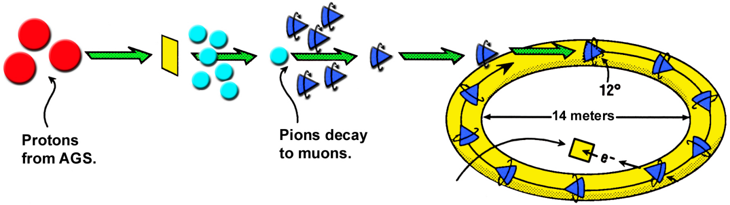
Where does the experimental $(g-2)_\mu$ come from?

Current experimental result dominated by E821 at Brookhaven
(final results published 2004)



Not yet confirmed by any other experiment…

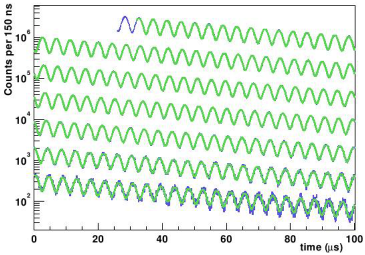
How did E821 work?



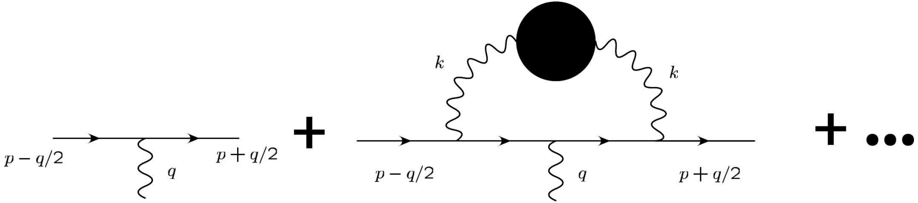
Circulate muons (either μ^+ or μ^-) in a confining storage ring
Measure precession in uniform 1.45T magnetic field

Focus with static electric field,
"magic" $p = 3.09$ GeV cancels $\vec{v} \times \vec{E}$

$(g-2) \neq 0$ produces the wiggles:
cyclotron period 0.15 μs
precession period 4.37 μs
muon lifetime 64.4 μs



Where does the standard model $(g-2)_\mu$ come from?



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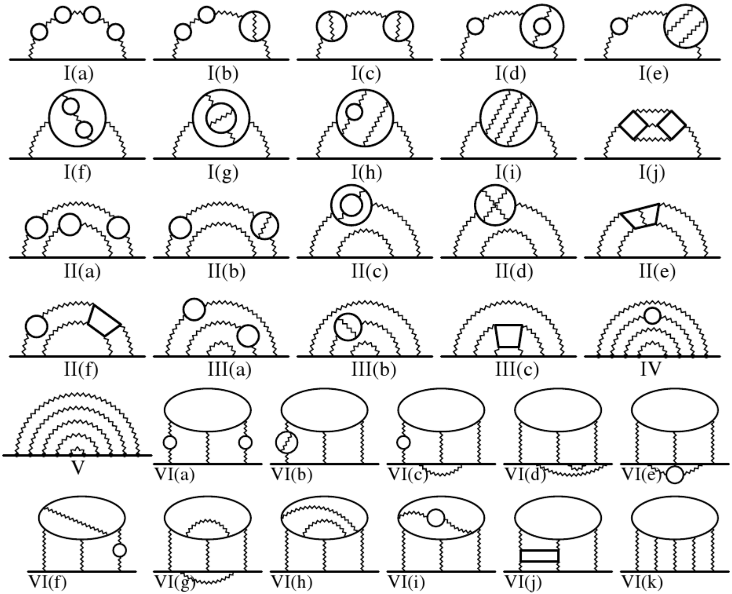


Where does the standard model $(g-2)_\mu$ come from?

...and that's just **part** of the picture!

Again adding all errors δa_μ in quadrature, we have:

Contribution	$a_\mu \times 10^{11}$	$\delta a_\mu \times 10^{11}$
QED*	116 584 718	0.2
LO hadronic vacuum polarization	6 923	42
Electroweak ($\sim \Delta a_\mu/2\,!$)	154	$2^\dagger$
Hadronic light-by-light	105	26
Higher-order hadronic vacuum pol.	-98	1
Total	116 591 802	49

Result is dominated by Schwinger term $\frac{\alpha}{2\pi} = 0.00\,116\,14...$

Uncertainty is dominated by hadronic contributions

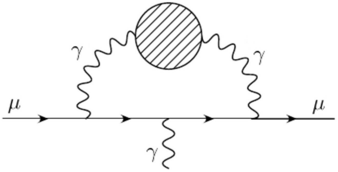
$\sim 17,000$ times smaller

*Does not yet include tenth-order contributions on previous page

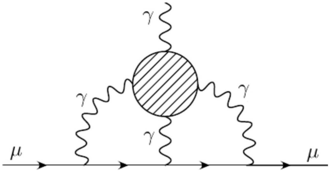
†Mainly from unknown Higgs mass, which is unknown no longer

What are these hadronic contributions?

Vacuum polarization (VP)



Light-by-light scattering (LbL)



Blobs represent all possible intermediate hadronic states,
not perturbatively calculable

Two possible ways to calculate hadronic vacuum polarization:

- Insert total $\sigma\,(e^+e^- \to$ hadrons) into dispersion relation
- Direct evaluation from first principles in lattice QCD

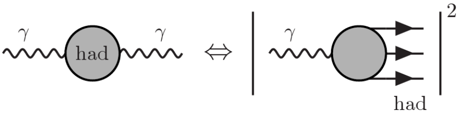
Hadronic LbL calculations are harder (no dispersion relation)...
(table used "Glasgow consensus" based on several different models)

How does the dispersive VP calculation work?

Since the vacuum polarization $\Pi(q^2)$ is an analytic function,

$$\Pi(q^2) = \frac{1}{\pi}\int_0^\infty ds \frac{\mathrm{Im}\,[\Pi(s)]}{s - q^2}$$

Recall optical theorem: $\mathrm{Im}\,[\Pi(s)] = \dfrac{s}{4\pi\alpha}\sigma_{tot}\left(e^+e^- \rightarrow \text{ hadrons}\right)$



Inserting resulting $\Pi(q^2)$ into vertex function $\Gamma(q^2 = 0)$ gives

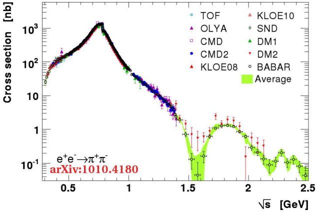
$$a^{\text{(LO HadVP)}} = \alpha^2 \int_{4m_\pi^2}^\infty ds\, K\left(s/m^2\right)\ \sigma_{tot}(s)$$

$K\left(s/m^2\right)$ is a "known function": too messy & opaque to write down
K strongly weighted to low energies $\mathcal{O}(m)$
$\Longrightarrow$ hadronic effects (especially ρ pole) more important for a_μ than a_e

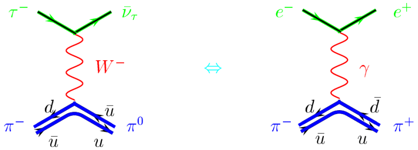
What's the catch? (Why is this "basically useless"?)

We have two ways to determine $\sigma_{tot}\,(e^+e^- \to \text{ hadrons})$:

e^+e^- annihilation & radiative return



$\text{Br}\left(\tau^- \to \nu_\tau \pi^0 \pi^-\right)$



Of course, they don't agree:

$e^+e^- \Rightarrow 3.6\sigma$ discrepancy $\qquad \tau \Rightarrow 2.4\sigma \qquad$ combined $\Rightarrow 3.0\sigma$

- τ data require isospin correction involving hadronic uncertainties
- e^+e^- data involve radiative corrections
- Different e^+e^- data sets disagree by 1.9σ
- e^+e^- data sets disagree with τ by up to 2.3σ (reduced from $\sim 3\sigma$)

A first-principles alternative sure would be nice!

How can we calculate the VP on the lattice?

The lattice vacuum polarization relation looks similar,
but we get it by analytic continuation to euclidean momenta $Q^2 > 0$

$$a^{\text{(LO HadVP)}} = 4\alpha^2 \int_0^\infty dQ^2\, f\left(Q^2/m^2\right)\, \left[\Pi(Q^2) - \Pi(0)\right]$$

$f\left(Q^2/m^2\right)$ is another "known function", diverges as $Q^2 \to 0$
$\Longrightarrow$ integral dominated by $Q^2 \approx m^2$

$\Pi(Q^2)$ calculated directly from lattice currents $J_\mu(x) \sim \overline{\psi}(x)\gamma_\mu\psi(x)$

$$\left(\widehat{Q}^2\delta_{\mu\nu} - \widehat{Q}_\mu\widehat{Q}_\nu\right)\Pi(\widehat{Q}^2) = \sum_x e^{i\widehat{Q}\cdot(x-y)}\,\langle J_\mu(x)J_\nu(y)\rangle$$

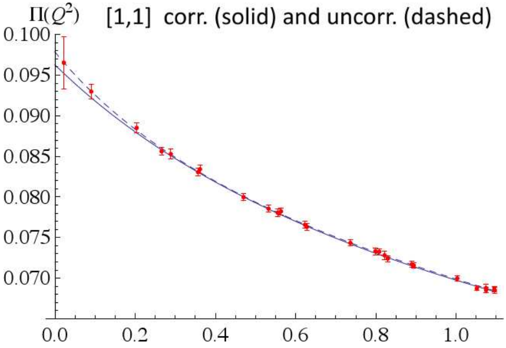
$\left(\widehat{Q}_\mu = 2\pi n_\mu/L_\mu\right)$

First complication: requires (non-trivial) conserved lattice currents
to avoid longitudinal lattice artifacts $\widehat{Q}_\mu\widehat{Q}_\nu\Pi^L(\widehat{Q}^2)$

"First complication"… ?

Next complication: integrand dominated by $Q^2 \approx m^2$
$\widehat{Q} \gtrsim 2\pi/L \approx$ 400–600 MeV for typical $L \approx$ 2–3 fm

- Lattice momenta too large, require $\widehat{Q}^2 \to 0$ extrapolation
  sensitive to model/parameterization, fit range, …
- Lowest lattice momenta have largest statistical uncertainties

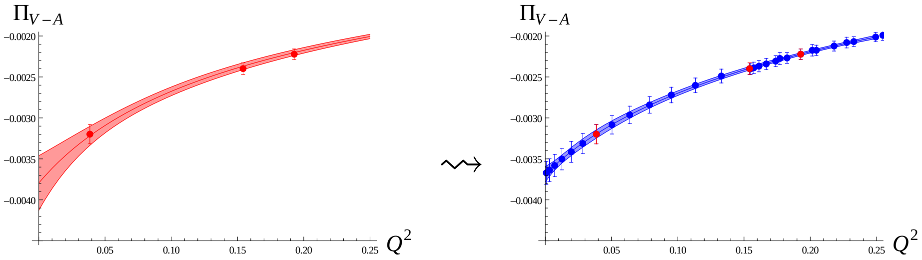


(arXiv:1205.3695)

What can we do about $Q^2 \to 0$ extrapolation?

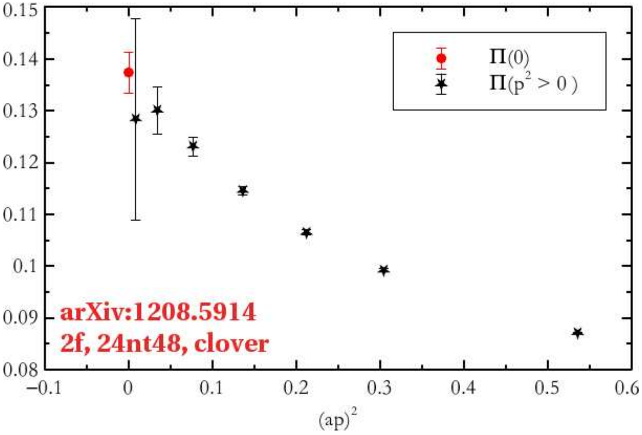
“(Partially) Twisted Boundary Conditions”

- Couple fermions to external abelian field
  (equivalent to adding phase to fermion fields at lattice boundaries)
- Allows access to arbitrary Q^2, not just lattice modes $2\pi n/L$
- Increases computational cost, but much cheaper than larger L
- Already being done by several lattice groups calculating $\Pi(Q^2)$



What can we do about $Q^2 \to 0$ extrapolation?

- Recent proposal aims to extract $\Pi(Q^2 = 0)$ with no extrapolation
- Taylor expand $\Pi(Q^2 = 0)$ w.r.t. spatial momenta
- Error scales with statistics (unlike uncertainty from extrapolation)



What else do we need to extrapolate?

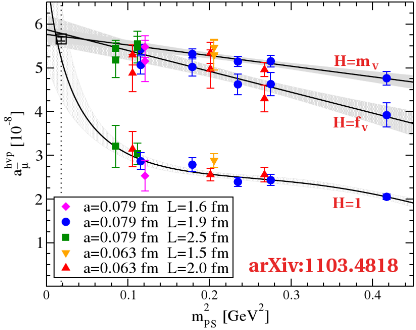
Lattice calculations (still) carried out at non-physical quark masses
$a^{(\text{LO HadVP})}$ used to be very sensitive to chiral extrapolation

Last year, new trick reduced sensitivity: (2011 Ken Wilson Lattice Award)

Reformulate "known function"

$$f\left(\frac{Q^2}{m_\mu^2}\right) \longrightarrow f\left(\frac{Q^2}{H_{lat}^2}\cdot\frac{H_{phys}^2}{m_\mu^2}\right)$$

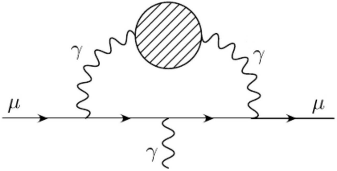
H_{lat} is hadronic scale (m_ρ or f_ρ)
that absorbs chiral dependence
Trivially cancels at physical point



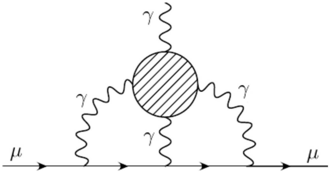
Smallest uncertainty quoted by any lattice calculation,
still $\sim 5\times$ larger than that claimed by e^+e^- and $\tau \rightarrow$ hadrons (box)

Wasn't this supposed to be the easy part?

Vacuum polarization (VP)



Light-by-light scattering (LbL)



We've been considering the leading-order (in α) VP contribution...

Hadronic LbL contribution smaller, but much more challenging

No dispersion relation, models mainly consider light meson exchange in combined large-N and chiral extrapolations

On the lattice we can calculate four-point correlator $\Pi^{\mu\nu\rho\sigma}(Q, P_1, P_2)$...
32 (of >100) Lorentz structures contribute to $(g-2)$,
need to integrate over P_1 and P_2 (cost $\propto V^2$), extrapolate $Q^2 \to 0$

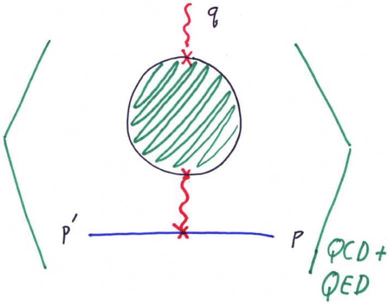
Isn't there an easier way to do light-by-light?

An **alternative** lattice approach to hadronic light-by-light scattering:

Lattice QCD+QED

Include photons along with gluons in lattice calculation

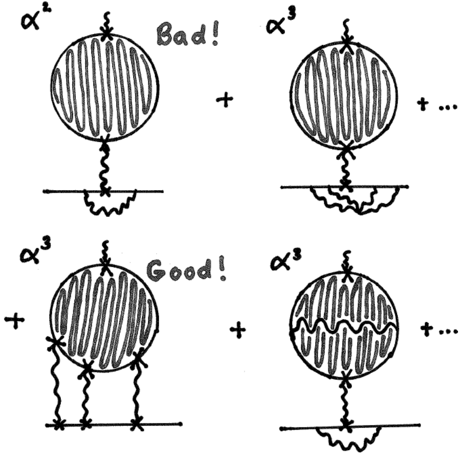
Only need to correlate hadronic loop and muon line
(one internal photon attached by hand for next step...)



Isn't there an easier way to do light-by-light?

Problem

Lattice QCD+QED generates additional unwanted terms,
including one at higher order than $\mathcal{O}(\alpha^3)$ light-by-light contribution



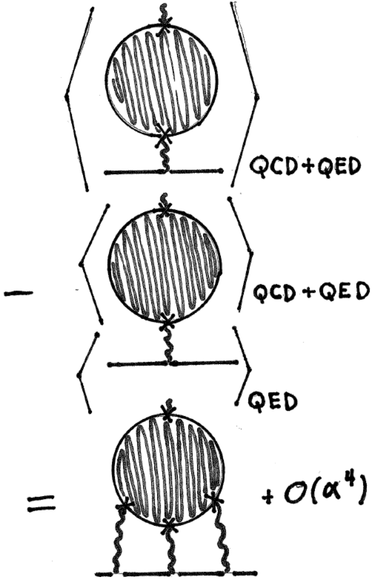
Isn't there an easier way to do light-by-light?

Trick

$\langle\cdot\rangle$ means average over gauge fields, both photons and gluons

Same gauge fields in correlator (top) and separate averages (middle)

$\Longrightarrow$ All unwanted terms cancel, leaving light-by-light $+\mathcal{O}(\alpha^4)$



Does this lattice QCD+QED approach actually work?

$a_\mu(\mathrm{HLbL})$ in 2+1f lattice QCD+QED (PRELIMINARY)

(Tom Blum, Lattice 2012)

Signal may be emerging in the model ballpark:

- $F_2(0.18\ \mathrm{GeV}^2) = (0.142 \pm 0.067) \times \left(\frac{\alpha}{\pi}\right)^3$ Q~425 MeV
- $F_2(0.11\ \mathrm{GeV}^2) = (0.038 \pm 0.095) \times \left(\frac{\alpha}{\pi}\right)^3$ Q~330 MeV
- $a_\mu(\mathrm{HLbL/model}) = (0.084 \pm 0.020) \times \left(\frac{\alpha}{\pi}\right)^3$

(domain wall fermions)

Lattice size 24^3, $m_\pi = 329$ MeV, $m_\mu \approx 190$ MeV

100 configurations
216 measurements/cfg.

model value/error is "Glasgow Consensus" (arXiv:0901.0306 [hep-ph])

Not yet: Need to reduce statistical errors,
estimate systematics, extrapolate $Q^2 \to 0$, ...

Do we really need to worry about hadronic LbL?

$a_\mu^{\text{(HadLbL)}} \times 10^{11} = 105(26)$ smaller than $\Delta a_\mu \times 10^{11} = 287(80)$

Upcoming experiments require improved prediction!

- Fermilab E989 will repeat Brookhaven E821, reduce $\delta a_\mu^{\text{(exp)}}$ by $4\times$
  Data taking to begin in 2015
- J-PARC E34 approved earlier this year
  Comparable precision from completely different method
  Data taking to begin in 2016

Also over next 3–5 years, more e^+e^- and $\tau \to$ hadrons data
will decrease associated $\delta a_\mu^{\text{(LO HadVP)}}$ by $\sim 2\times$ (Blum, Lattice 2012)
(...and hopefully agree!)

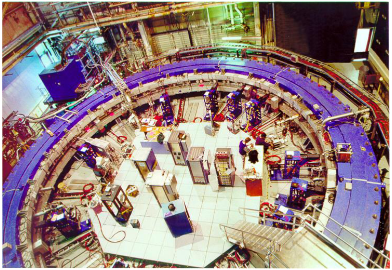
$\Longrightarrow$ Hadronic light-by-light will start to dominate SM uncertainty

How will Fermilab E989 work?

Same as E821, but better

Storage ring shipped NY $\rightarrow$ Chicago
Same "magic" momentum, B field, etc.

Approved and mostly built,
data taking to begin 2015



Improvements

Statistics 20$\times$ more protons per year than **total** at E821 (μ^+ only)

Systematics 10$\times$ longer decay channel to reduce pion background
Finer segmentation to reduce pileup in calorimeters

Statistics:	$\delta a_\mu \times 10^{11} = 54 \longrightarrow 12$
Systematics:	$\delta a_\mu \times 10^{11} = 33 \longrightarrow 12$
Total:	$\delta a_\mu \times 10^{11} = 63 \longrightarrow 16$

What does theory need to remain comparable?

Experiment:	$\delta a_\mu \times 10^{11} = 63 \longrightarrow 16$
Standard model:	$\delta a_\mu \times 10^{11} = 49$

Standard model uncertainty dominated by hadronic contributions

LO vacuum polarization	$\delta a_\mu \times 10^{11} = 42$
Light-by-light	$\delta a_\mu \times 10^{11} = 26$

Need both contributions reduced to $\delta a_\mu \times 10^{11} \sim 10$

LO vacuum polarization	$\searrow\!\sim\!4\times$	(to $\sim$0.2%)	(easier?)
Light-by-light	$\searrow\!\sim\!2\times$	(to $\sim$10%)	(optimistic?)

- e^+e^- and $\tau \to$ hadrons probably limited to $\delta a_\mu \times 10^{11} \sim 20$–30
- Light-by-light models already questionable at $\delta a_\mu \times 10^{11} \sim 26$–40

Lattice QCD systematically improvable

$\longrightarrow$ **will become indispensable!**

What are the prospects for lattice calculations?

Hadronic vacuum polarization on the lattice

Currently have ~5–10% uncertainties, may have ~1–2% in 3–5 years
To reach required sub-percent precision, we will need to:

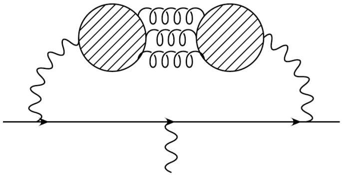
- Work at physical quark masses (requires large volumes), including $m_u \neq m_d$ and different electric charges
- Improve control over Q^2 dependence and $Q^2 \to 0$ limit (seems to be much recent progress)
- Determine charm-quark contribution (comparable to total light-by-light?)
- Worry about (quark-line-)**disconnected diagrams. . .**

Hadronic light-by-light on the lattice

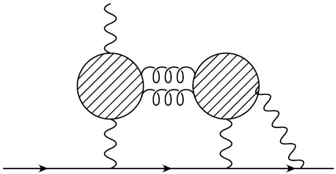
Currently only have proof-of-principle explorations
Need to improve methods **in addition to** dealing with issues above
Goal: combined lattice+models $\longrightarrow$ 10% in ~5 years

“Disconnected diagrams”?

Vacuum polarization (VP)



Light-by-light scattering (LbL)



Extremely expensive to evaluate on lattice

cost $\propto$ lattice volume (usually estimated stochastically)

Disconnected vacuum polarization

Cancels in flavor-SU(3) limit, Zweig suppressed

May be as large as 1–2% $\longrightarrow$ needed to attain required precision

Disconnected light-by-light

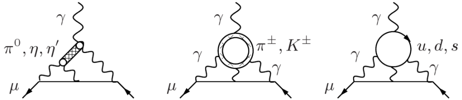
May be comparable to connected piece…

So what's the plan?

- Experiments should have $\sim 4\times$ improved results in ~ 5 years
- Comparable SM (lattice!) predictions may require the next decade

Are there possible intermediate steps?

- $\sim$1–2% precision for the vacuum polarization contribution would be sensitive to current e^+e^- vs. τ disagreement
- $\pi \to \gamma^*\gamma^{(*)}$ easier than full LbL, would help check models (upcoming experiments PrimEx@Jlab and KLOE@Frascati)
- Similar: quark condensate magnetic susceptibility $\langle \overline{q}\sigma^{\mu\nu} q\rangle_{\vec{B}}$, $\langle AVV\rangle$, $\langle VVVV\rangle$ for fixed fiducial momenta



Typical LbL model: light meson exchange in large-N+chiral expansion

Lots of interesting & important lattice projects to explore!

Backup: “Will you talk about the $h \to \gamma\gamma$ decay rate?”

No

A possible connection between new physics for Δa_μ and an enhanced $h \to \gamma\gamma$ decay rate was recently proposed

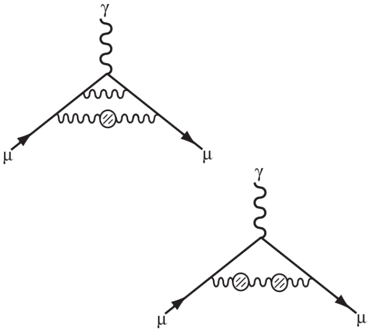
Cf. arXiv:1207.1313 and arXiv:1208.2973

Unless I don’t get volunteers for future meetings…

Backup: What about higher-order VP contribution?

Finally some good news:

NLO just requires inserting $\Pi(Q^2)$ into 17 simple QED diagrams



Lattice result already agrees with e^+e^- and $\tau \to$ hadrons

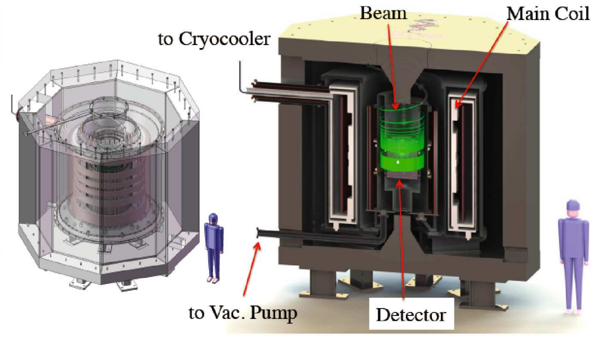
with comparable uncertainty

Precision already comparable to future experiments' goal,

further improvement will come for free from better $\Pi(Q^2)$

Backup: How will J-PARC E34 work?

- Muons accelerated from muonium $\longrightarrow$ no background from pions
- No electric field for focusing $\longrightarrow$ don't need "magic" momentum
- Approved, data taking scheduled to begin in 2016



Compared to E989

momentum $\searrow$ 10×	$B \nearrow$ 2×	diameter $\searrow$ 20×
cyclotron $T \searrow$ 20×	precession $T \searrow$ 2×	$\#\mu^+ \nearrow$ 10×

Backup: What about $(g-2)_\tau$?

Interesting since contribution to a_ℓ of new physics around scale Λ is generically expected to be $a_\ell^{(new)} \propto m_\ell^2/\Lambda^2$

However,

- τ lifetime too short for storage rings, hadronic decays are messy (95% confidence level estimated from $\sigma_{tot}(e^+e^- \to e^+e^-\tau^+\tau^-)$ at LEP)
- $m_\tau \gg m_\mu \implies$ hadronic effects even more important

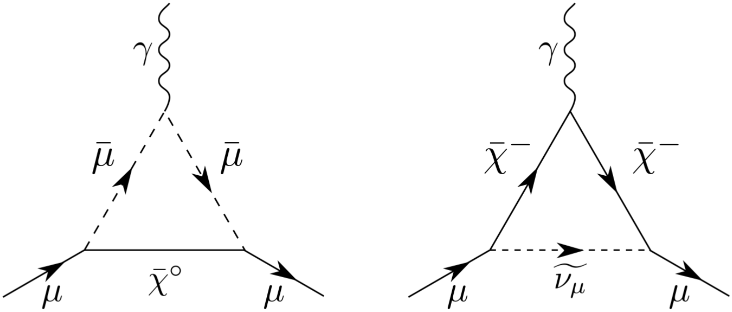
Experiment:	$-5\,200\,000 < a_\tau \times 10^8 < 1\,300\,000$
Standard model:	$a_\tau \times 10^8 = 117\,721(5)$

For the future – prospects for Super B Factories (arXiv:0807.2366)

KEK and INFN could measure $(g-2)_\tau$ from $\tau^+\tau^-$ spin correlations
Expect $\delta a_\tau \times 10^8 \sim 500$ from 75/ab
Comparable sensitivity from τ polarization analysis using polarized e^+e^- beams

Backup: What about SUSY?

Representative contributions:



"Generic" supersymmetric prediction: (PDG)

$$a_\mu^{(SUSY)} \times 10^{11} \simeq \mathrm{sign}(\mu) 130 \cdot \left(\frac{100\ \mathrm{GeV}}{M_{SUSY}}\right) \tan\beta$$

If supersymmetry (or other new physics) discovered (@13 TeV LHC?) then $(g-2)_\mu$ could lift degeneracy in parameters