



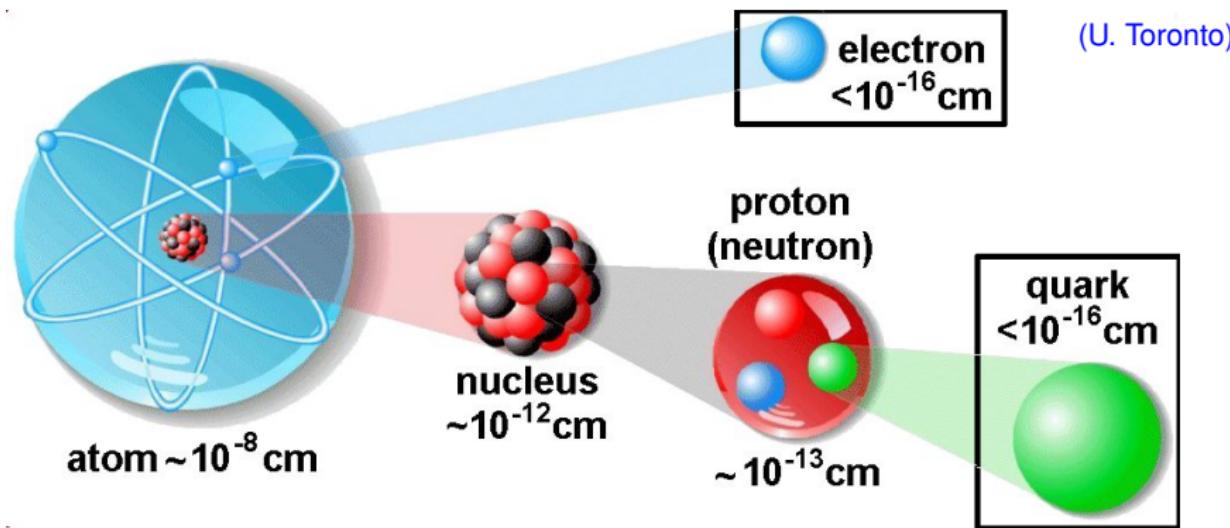
# Measuring the S Parameter on the Lattice

David Schaich

(Dissertation: *Strong Dynamics and Lattice Gauge Theory*)

12 May 2011

# Setting the scene

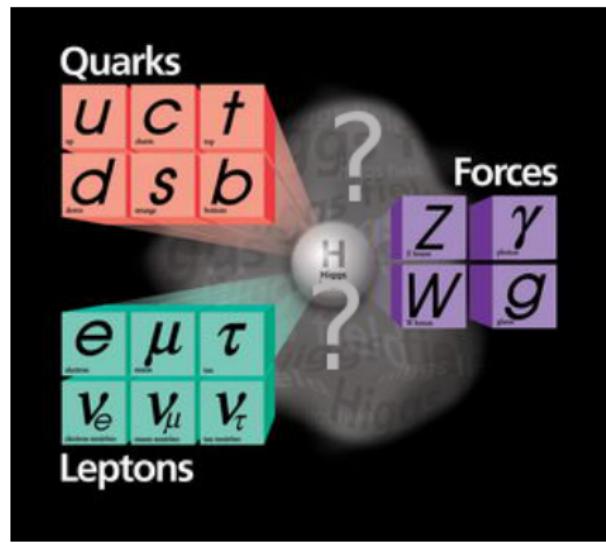


What explains the masses of *elementary* (non-composite) particles?

# Elementary particle masses

Symmetries of nature appear to require *massless* elementary particles

The **Higgs mechanism** hides these symmetries

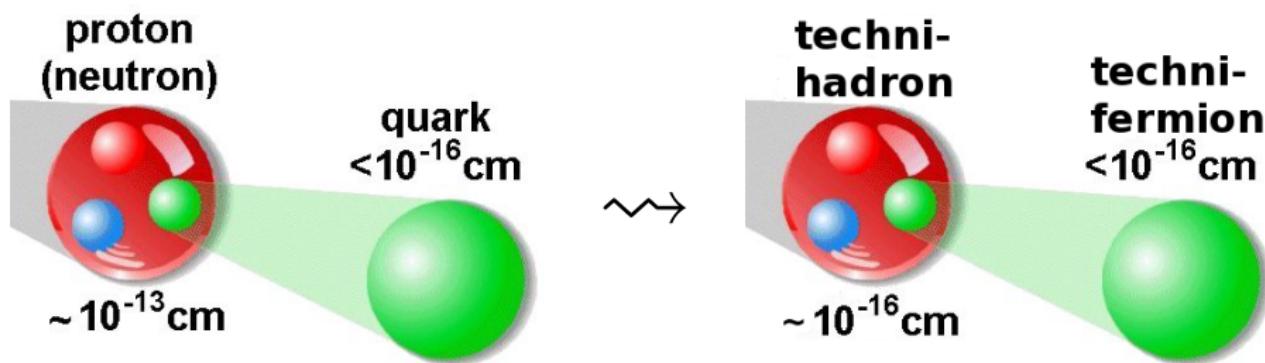


Englert and Brout, 1964;  
Higgs, 1964; Guralnik,  
Hagen and Kibble, 1964

The physics behind the Higgs mechanism remains unknown!

Proposal: The Higgs mechanism results  
from the dynamics of a new **strong** interaction

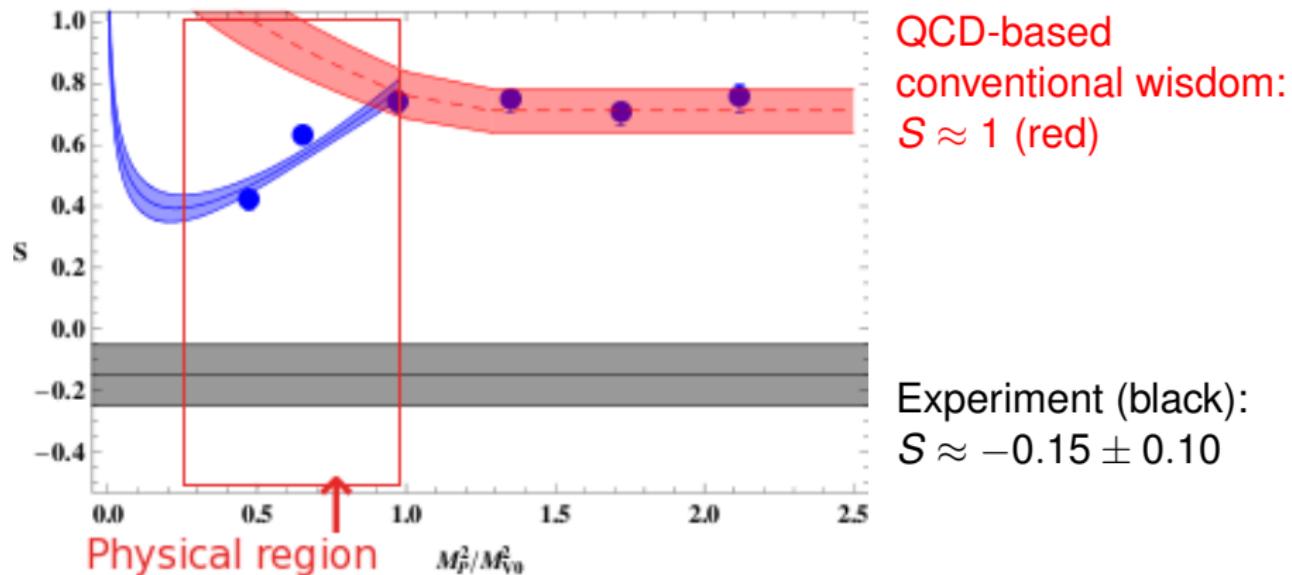
Superficially similar to quantum chromodynamics (QCD),  
the fundamental theory of the strong nuclear force



Despite superficial similarity, the dynamics can be very different...

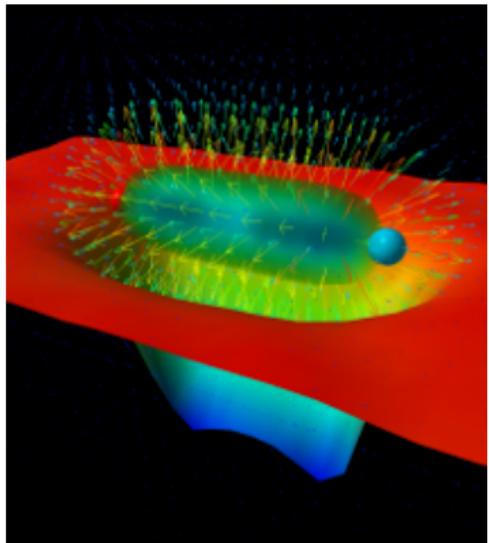
# Different dynamics (a preview of my main results)

“S” parameterizes information about the Higgs mechanism (more later)



Dynamics of a realistic technicolor theory **must** be different than QCD  
... so why is the conventional wisdom based on QCD?

# It's a **strong** interaction!

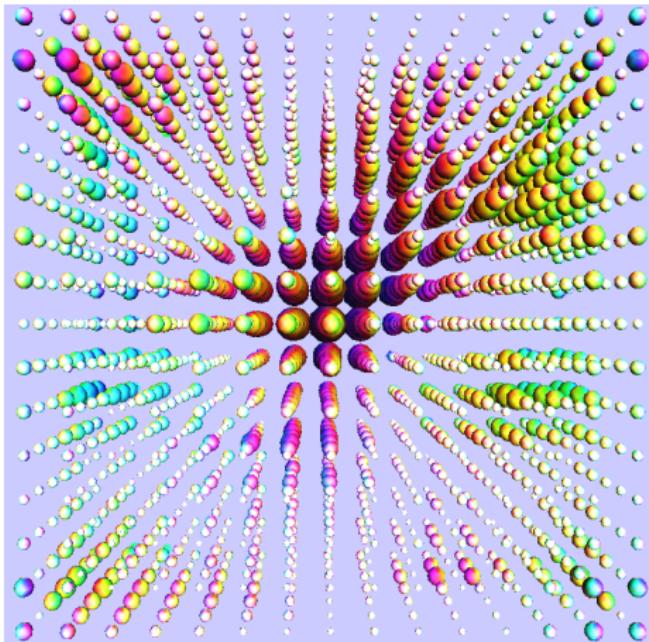


(Derek Leinweber)

## Implications

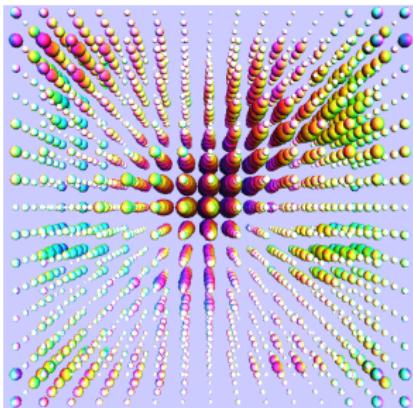
- Internal structure of composite particles very complicated
- Standard techniques (perturbation theory) inapplicable

To perform non-perturbative calculations,  
define the theory on a space-time lattice  
(Kenneth Wilson, 1974)



(Claudio Rebbi)

# Lattice quantum field theory



Numerically evaluate observables  $\mathcal{O}$  from the defining functional integral

$$\langle \mathcal{O} \rangle = \frac{\int dU \ \mathcal{O}(U) \ e^{-S(U)}}{\int dU \ e^{-S(U)}}$$

$U$ : four-dimensional field configurations

$S$ : action giving probability distribution  $e^{-S}$

## Generic workflow of lattice projects

- ① Generate and save a stochastic sample of field configurations
- ② Carry out measurements on saved configurations (my work)

Both steps require inverting large ( $\sim 100M \times 100M$ ) sparse matrices

$$\sum_y [D(U)]_{x,y} \psi_y = \eta_x \qquad \qquad \sim 90\% \text{ of cost}$$

~400M core-hours on clusters and supercomputers

Livermore Nat'l Lab, NSF Teragrid, USQCD (DoE), BU SCF



# Recap

- Use lattice quantum field theory
- Study strongly-interacting physics beyond QCD
- Apply to the Higgs mechanism

## Next

Parameterize information about the Higgs mechanism

$S$  parameter provides most stringent constraint

Experiment:  $S \approx -0.15 \pm 0.10$

QCD:  $S = 0.32 \pm 0.03$

# The $S$ parameter

(Michael Peskin and Tatsu Takeuchi, 1991)

$$S = 4\pi N_D \lim_{Q^2 \rightarrow 0} \frac{d}{dQ^2} \Pi_{V-A}(Q^2) - \Delta S_{SM}$$

- ① The vacuum polarization function  $\Pi_{V-A}(Q^2)$  is schematically

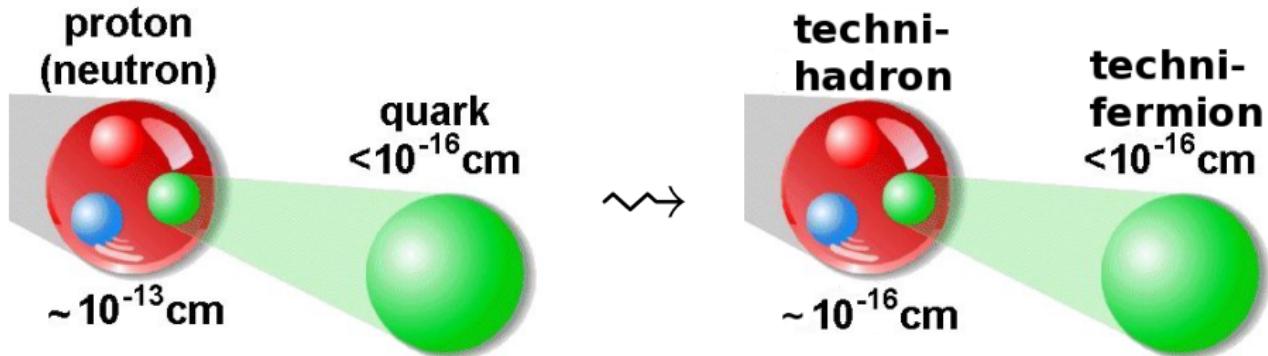


On the lattice, it is the difference of vector and axial-vector two-point correlation functions

$$\Pi_{V-A}^{\mu\nu}(Q) = Z \sum_x e^{iQ \cdot (x + \hat{\mu}/2)} \text{Tr} \left[ \langle V^{\mu a}(x) V^{\nu b}(0) \rangle - \langle A^{\mu a}(x) A^{\nu b}(0) \rangle \right]$$

- ②  $N_D \geq 1$  is the number of pairs of contributing techni-fermions
- ③  $\Delta S_{SM}$  is subtracted so that  $S = 0$  in the standard model  
(the simplest model of the Higgs mechanism)

## The model (last stop before data)



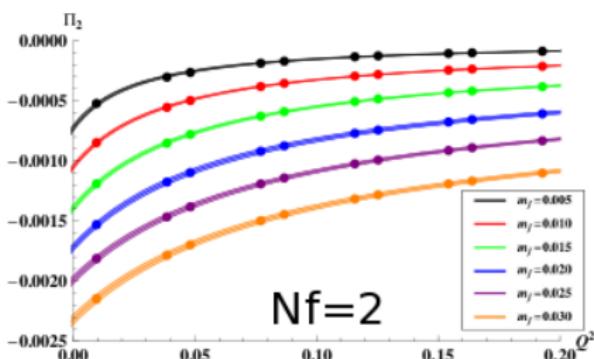
Take advantage of experience with lattice QCD

- QCD is an SU(3) gauge theory with  $N_f = 2$  light fermions
- Study an SU(3) gauge theory with  $N_f = 6$  light fermions
- Compare  $N_f = 2$  and  $N_f = 6$  calculations to see what changes
  - 1 Match lattice spacing (energy scale) to compare directly
  - 2 Perform calculations with a range of light fermion masses  $m_f$

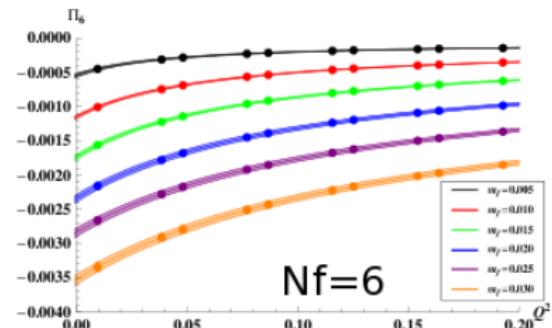
# Polarization function data and fits

$$\Pi_{V-A}^{\mu\nu}(Q) = Z \sum_x e^{iQ \cdot (x + \hat{\mu}/2)} \text{Tr} \left[ \langle V^{\mu a}(x) V^{\nu b}(0) \rangle - \langle A^{\mu a}(x) A^{\nu b}(0) \rangle \right]$$

(Z is a renormalization constant I compute non-perturbatively)



Nf=2



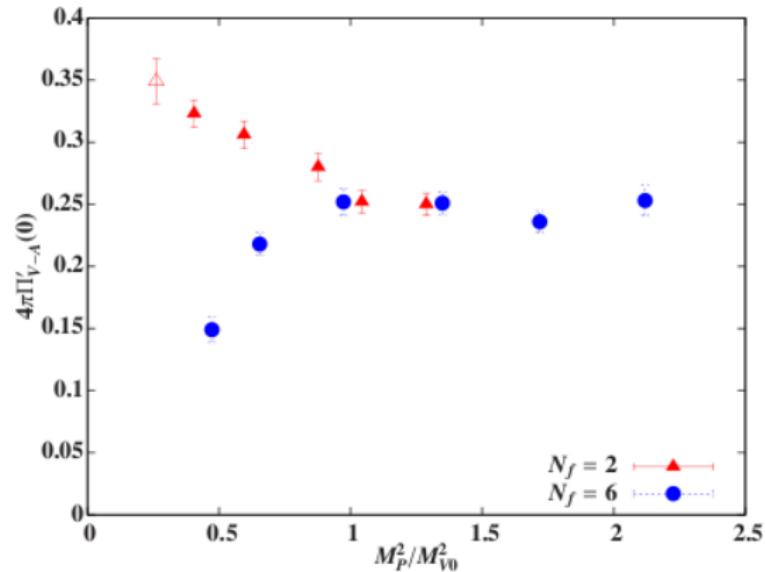
Nf=6

Very smooth data  $\Rightarrow$  fit with functional form  $\frac{a_0 + a_1 Q^2}{1 + b_1 Q^2 + b_2 Q^4}$

extract slope at  $Q^2 = 0$

Results stable and  $\chi^2/dof \ll 1$  as  $Q^2$  fit range varies

# Fit results for $\Pi'_{V-A}(0)$ , $N_f = 2$ and $N_f = 6$



Vertical axis:  $4\pi\Pi'_{V-A}(0)$

where

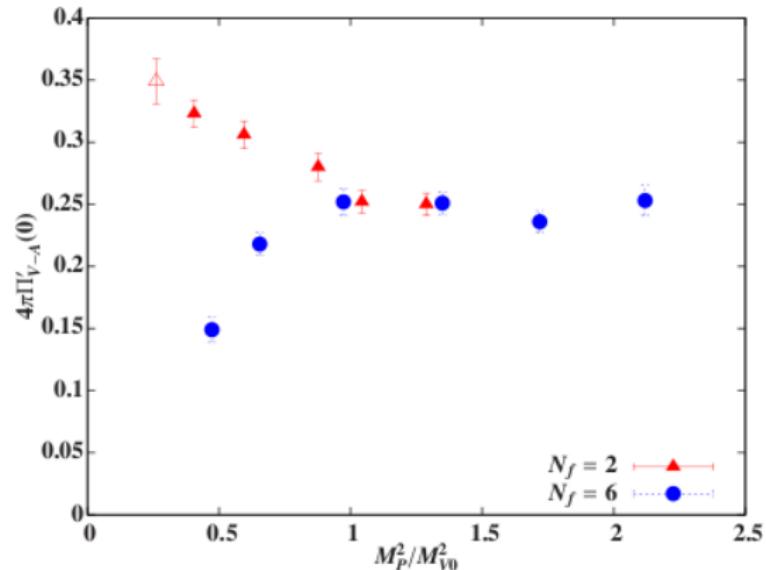
$$\Pi'(0) = \lim_{Q^2 \rightarrow 0} \frac{d}{dQ^2} \Pi(Q^2)$$

$$S = 4\pi N_D \Pi'_{V-A}(0) - \Delta S_{SM}$$

Horizontal axis:  $M_P^2/M_{V0}^2$  gives a more physical comparison than  $m_f$

- $M_P$  is the pseudoscalar ground state mass at each  $m_f$
- $M_{V0}$  is the vector ground state mass in the limit  $m_f \rightarrow 0$   
(matched between  $N_f = 2$  and  $N_f = 6$ )

# Fit results for $\Pi'_{V-A}(0)$ , $N_f = 2$ and $N_f = 6$



Vertical axis:  $4\pi\Pi'_{V-A}(0)$

where

$$\Pi'(0) = \lim_{Q^2 \rightarrow 0} \frac{d}{dQ^2} \Pi(Q^2)$$

$$S = 4\pi N_D \Pi'_{V-A}(0) - \Delta S_{SM}$$

For large  $M_P^2 > M_{V0}^2$ , techni-fermions are very heavy  
and have little effect on the dynamics  
Expect results to agree in this heavy-fermion region

# The last steps from slope to $S$

$$S = 4\pi N_D \lim_{Q^2 \rightarrow 0} \frac{d}{dQ^2} \Pi_{V-A}(Q^2) - \Delta S_{SM}$$

- ①  $N_D$  pairs of techni-fermions contributing to  $S$

Most straightforward to have  $N_D = N_f/2$

(only  $N_D \geq 1$  is required by the Higgs mechanism)

- ②  $\Delta S_{SM} = \frac{1}{4} \int_{4M_P^2}^{\infty} \frac{ds}{s} \left[ 1 - \left( 1 - \frac{M_{V0}^2}{s} \right)^3 \Theta(s - M_{V0}^2) \right]$

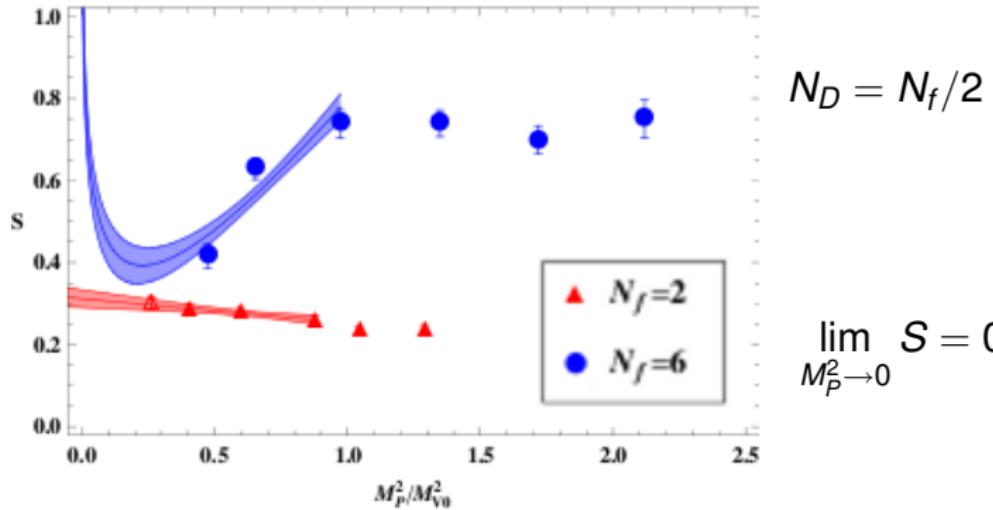
Diverges logarithmically in the limit  $M_P^2 \rightarrow 0$

(divergence exactly cancels if  $N_f = 2$ )

Small ( $\lesssim 10\%$ ) reduction in this work ( $M_P^2 > 0$  on the lattice)

# $S$ parameter, $N_f = 2$ and $N_f = 6$

$$S = 4\pi(N_f/2)\Pi'_{V-A}(0) - \Delta S_{\text{SM}}$$

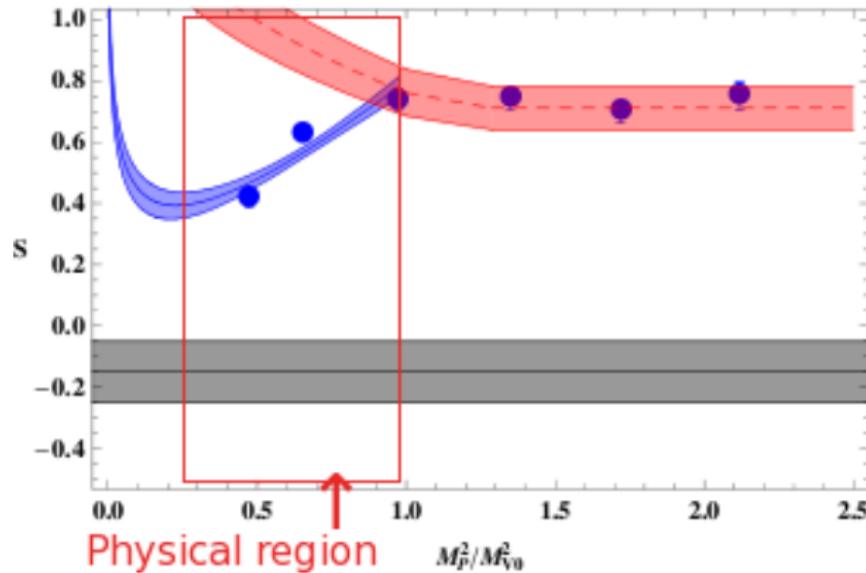


$$\lim_{M_P^2 \rightarrow 0} S = 0.31(2) \text{ for } N_f = 2$$

Linear fit to light points  $M_P^2 < M_{V0}^2$  guides the eye,  
accounts for logarithmic divergence as  $M_P^2 \rightarrow 0$

$$S = A + BM_P^2 + \frac{1}{12\pi} \left[ \frac{N_f^2}{4} - 1 \right] \log \left( \frac{M_{V0}^2}{M_P^2} \right)$$

## Main result: deviation from conventional wisdom



Conventional wisdom:  
 $S \approx 1$  (red)

Experiment (black):  
 $S \approx -0.15 \pm 0.10$

This is the most straightforward case with maximal  $N_D = N_f/2 = 3$

# Systematic effects

- **Finite volume**

At most percent-level effect for  $N_f = 2$  (except lightest point)  
Being directly checked for  $N_f = 6$

- **Nonzero lattice spacing  $a$  between sites**

Effects reduced to  $\mathcal{O}(a^2)$  by our lattice action

- **Chiral symmetry breaking**

Explicitly broken by many lattice actions, in addition to  $M_P^2 > 0$

Effects reduced by using (expensive) chiral lattice action

$M_P^2 \rightarrow 0$  limit is subtle (recall cancellation from  $\Delta S_{SM}$ )

- **Autocorrelations and topological effects**

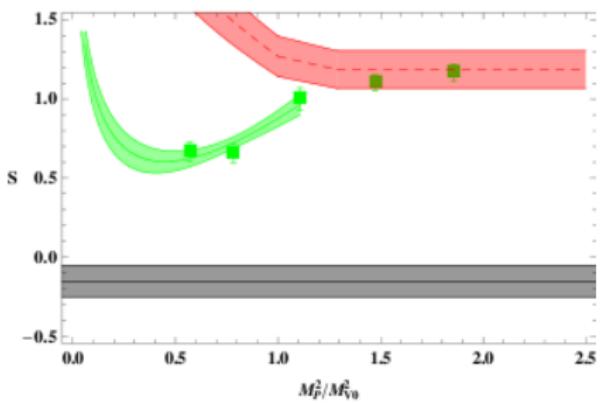
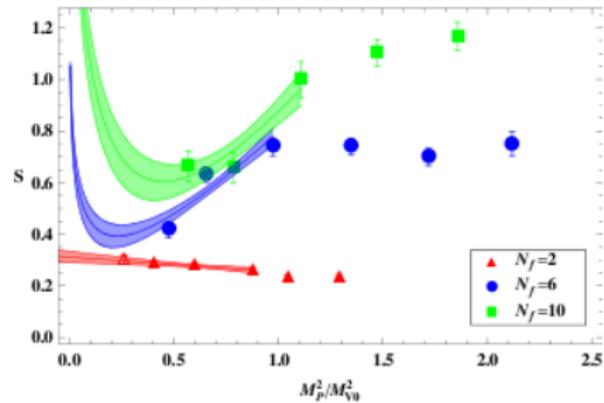
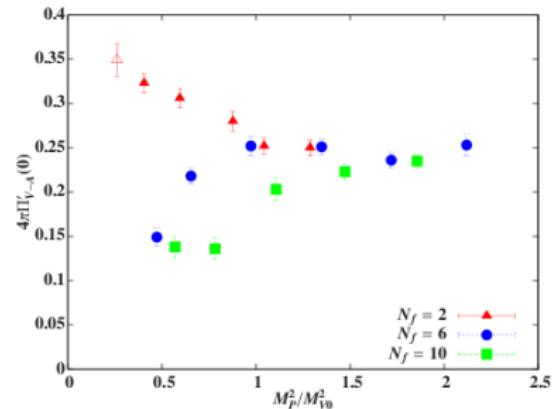
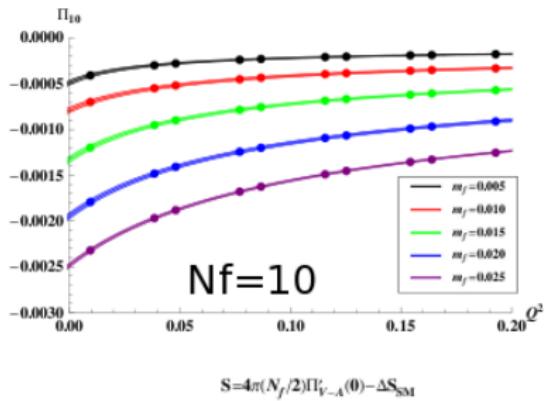
(!)

Exploratory calculations with short (but expensive) simulations

Combined statistical and systematic errors of 10–20% ⇒ result robust

(PRL in press)

# Preliminary results for $N_f = 10$ (ongoing work)

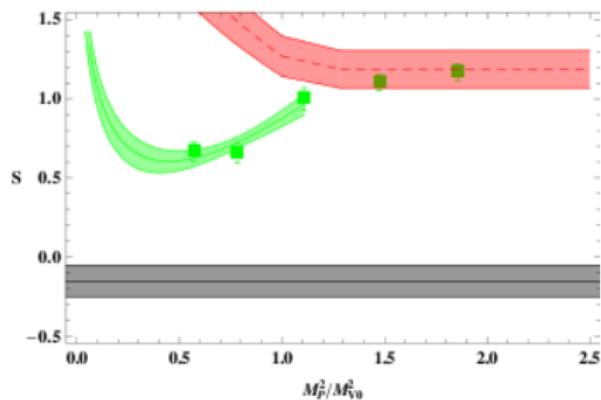
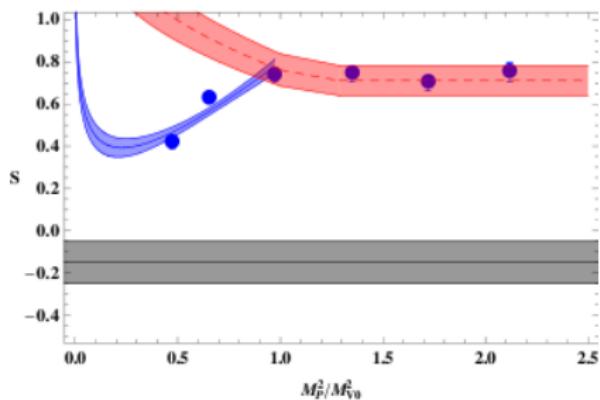


# Conclusion

- Higgs mechanism required by elementary particle masses
- If the Higgs mechanism involves strong interactions,  
the lattice can provide non-perturbative information
- The  $S$  parameter provides the most stringent constraint

For SU(3) gauge theory with  $N_f = 6$

I find  $S$  closer to experiment than the conventional wisdom



# Thank you!

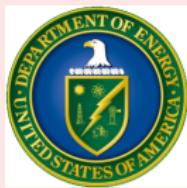
# Acknowledgements

## At BU

Ron Babich, Rich Brower, Michael Cheng, Mike Clark, Saul Cohen, James Osborn, Claudio Rebbi

Tom Appelquist, Mike Buchoff, George Fleming, Fu-Jiun Jiang, Joe Kiskis, Meifeng Lin, Ethan Neil, Pavlos Vranas

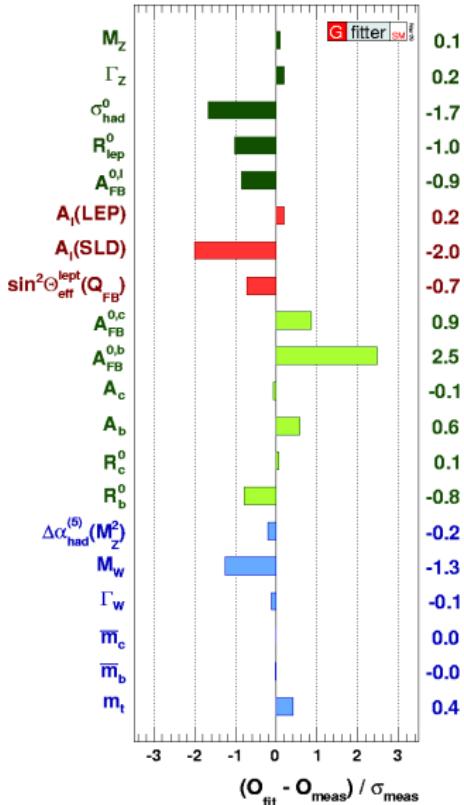
## Funding and computing resources



# The end

- 1 Introduction
- 2 Lattice quantum field theory
- 3  $S$  parameter on the lattice
- 4 Backup: EWSB
- 5 Backup: ETC, walking and the conformal window
- 6 Backup: Lattice topics
- 7 Backup: More on the  $S$  parameter
- 8 Backup: Lattice Strong Dynamics Collaboration results
  - Backup: Scale setting
  - Backup: NLO $\chi$ PT
  - Backup: Condensate enhancement
  - Backup: Parity doubling
  - Backup: Conserved currents
- 9 Backup: Asides and extensions

# Experimental confirmation of electroweak theory



(Gfitter Group)

## Gauge invariance example: electromagnetism

Electric and magnetic fields in terms of potentials  $\Phi$  and  $\mathbf{A}$

$$\mathbf{E} = -\frac{\partial \mathbf{A}}{\partial t} - \nabla \Phi \quad \mathbf{B} = \nabla \times \mathbf{A}$$

$\mathbf{E}$  and  $\mathbf{B}$  are invariant under the gauge transformation

$$\Phi \rightarrow \Phi - \frac{\partial \Lambda}{\partial t} \quad \mathbf{A} \rightarrow \mathbf{A} + \nabla \Lambda$$

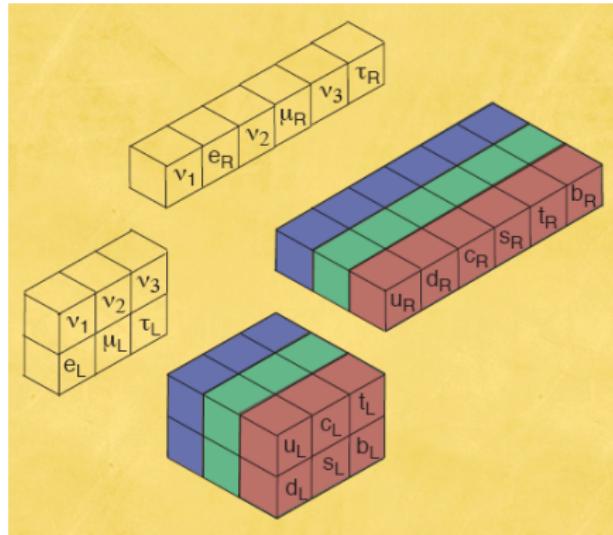
In four-vector notation,  $A_\mu = (\Phi, \mathbf{A}) \rightarrow A_\mu + \partial_\mu \Lambda$

Photon mass term in lagrangian is

$$\tfrac{1}{2} m_\gamma^2 A_\mu A^\mu = \tfrac{1}{2} m_\gamma^2 (\mathbf{A} \cdot \mathbf{A} - \Phi^2)$$

Forbidden by gauge invariance!

# Massless fermions from chiral gauge theory



(Chris Quigg)

Fermion mass term in lagrangian is  $m\bar{\psi}\psi = m(\bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L)$

$$\bar{\psi}_L\psi_R \sim (\bar{\psi}_\uparrow \quad \bar{\psi}_\downarrow)_L \cdot (\psi)_R$$

Forbidden by gauge invariance!

## Fermion masses in standard model

Need to make a gauge-invariant object involving

$$\bar{\psi}_L \psi_R \sim (\bar{\psi}_\uparrow \quad \bar{\psi}_\downarrow)_L \cdot (\psi)_R$$

Standard model solution: stick in a Higgs  $\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$

$$\lambda_\psi (\bar{\psi}_\uparrow \quad \bar{\psi}_\downarrow)_L \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} (\psi)_R$$

With vacuum  $\langle \Phi \rangle = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}$ , identify  $m_\psi = \lambda_\psi v / \sqrt{2}$ .

All fermion masses and mixing are arbitrary free parameters!

## Gauge boson masses in standard model

$$\Phi = \begin{pmatrix} \phi_1 + i\phi_2 \\ v/\sqrt{2} + h + i\phi_3 \end{pmatrix}$$

$$\mathcal{L}_\Phi = (\mathcal{D}^\mu \Phi)^\dagger (\mathcal{D}_\mu \Phi) + \mu^2 \Phi^\dagger \Phi - \lambda (\Phi^\dagger \Phi)^2 \Rightarrow v = \sqrt{-\mu^2/\lambda}$$

$$\mathcal{D}_\mu = (\partial_\mu + \frac{i}{2}g_1 B_\mu) \mathbb{I} + \frac{i}{2}g_2 W_\mu^a \sigma^a$$

$W^\pm$  and  $Z$  masses pop out of  $(\mathcal{D}^\mu \Phi)^\dagger (\mathcal{D}_\mu \Phi)$ . Relevant terms:

$$\begin{aligned} & \frac{v^2}{8} (0 \quad 1) \begin{pmatrix} -g_2 W_\mu^3 - g_1 B_\mu & g_2 (W_\mu^1 - iW_\mu^2) \\ g_2 (W_\mu^1 + iW_\mu^2) & g_2 W_\mu^3 - g_1 B_\mu \end{pmatrix}^2 \begin{pmatrix} 0 \\ 1 \end{pmatrix} \\ & \equiv \frac{g_2^2 v^2}{8} (0 \quad 1) \begin{pmatrix} \dots & \sqrt{2} W_\mu^+ \\ \sqrt{2} W_\mu^- & (g_1^2 + g_2^2)^{1/2} Z_\mu / g_2 \end{pmatrix}^2 \begin{pmatrix} 0 \\ 1 \end{pmatrix} \\ & \equiv M_W^2 W^{+\mu} W_\mu^- + \frac{1}{2} M_Z^2 Z^\mu Z_\mu + \dots \end{aligned}$$

$$M_W = \frac{1}{2} g_2 v = (M_Z/g_2) \sqrt{g_1^2 + g_2^2} \equiv M_Z \cos \theta_W$$

# Gauge boson masses in technicolor

Now we have pions with

$$\mathcal{L}_\chi = F_P^2 \text{Tr} \left[ (\mathcal{D}^\mu \Sigma)^\dagger (\mathcal{D}_\mu \Sigma) \right] / 4$$

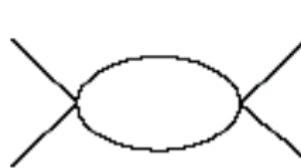
$$\Sigma = \exp(2i\sigma^a \pi^a / F_P) \sim q_L \bar{q}_R$$

$$\mathcal{D}_\mu = \mathbb{I} \partial_\mu - \frac{i}{2} g_2 \mathcal{W}_\mu^a \sigma^a \quad \mathcal{W}_\mu^a = (W_\mu^1, W_\mu^2, W_\mu^3 - g_1 B_\mu / g_2)$$

$W^\pm$  and  $Z$  masses pop out of  $F_P^2 \text{Tr} |\mathcal{D}_\mu \Sigma|^2 / 4$ . Relevant terms:

$$\begin{aligned} (\partial_\mu \pi^a)^2 - F_P g_2 (\partial^\mu \pi^a) \mathcal{W}_\mu^a / 2 + F_P^2 g_2^2 (\mathcal{W}_\mu^a)^2 / 16 &= [F_P g_2 \mathcal{W}_\mu^a / 4 - \partial_\mu \pi^a]^2 \\ &= F_P^2 g_2^2 [(W_\mu^1)^2 + (W_\mu^2)^2] / 8 + F_P^2 (g_2^2 + g_1^2) Z_\mu^2 / 8 \\ &\equiv M_W^2 W^{+\mu} W_\mu^- + \tfrac{1}{2} M_Z^2 Z^\mu Z_\mu \quad \longrightarrow F_P = v \end{aligned}$$

## Triviality of fundamental Higgs


$$\Rightarrow \beta = \frac{3\lambda^2}{2\pi^2} > 0$$

$$\lambda(\mu) \simeq \frac{1}{[1/\lambda(\Lambda)] + (3/2\pi^2) \log(\Lambda/\mu)} < \frac{2\pi^2}{3 \log(\Lambda/\mu)}$$
$$\Lambda \simeq M_H \exp \left( \frac{4\pi^2 v^2}{3M_H^2} \right)$$

$$M_H = 115 \text{ GeV} \longrightarrow \Lambda \sim 10^{28} \text{ GeV}$$

$$M_H = 700 \text{ GeV} \longrightarrow \Lambda \sim 1000 \text{ GeV}$$

# (Extended) technicolor in a picture

Massless SU(2)  
Gauge fields

$A_1 \ A_2 \ A_3$

**A new strong force**

Techni-quarks  
Techni-gluons

Massless particle  
fermion fields

$\Psi_e, \Psi_\mu, \dots$



**Spontaneous chiral symmetry breaking by the strong dynamics**

$W^+ \ Z^0 \ W^-$



$$m_\pi = 0$$

$$\langle \bar{T}T \rangle \neq 0$$



$\bar{T}T \bar{\Psi}_e \Psi_e, \dots$

$$m_{W,Z} \sim F_\pi^{tc}$$

TC

$$m_e \sim \langle \bar{T}T \rangle, \dots$$

ETC

P. Vranas, LLNL

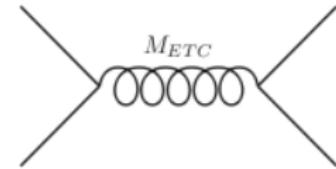
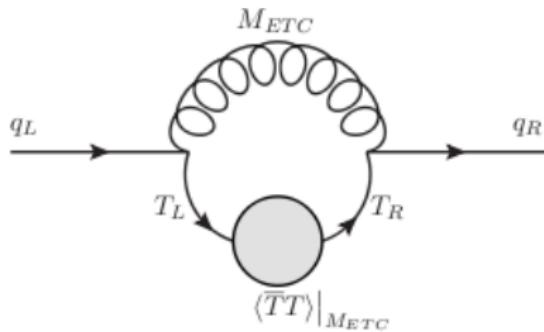
# Fermion masses in extended technicolor

Integrating out ETC gauge bosons produces four-fermion operators that provide both SM fermion masses and FCNCs

Masses:  $\frac{(\bar{T}T)(\bar{q}q)}{M_{ETC}^2}$

FCNCs:  $\frac{(\bar{q}q)(\bar{q}q)}{M_{ETC}^2}$

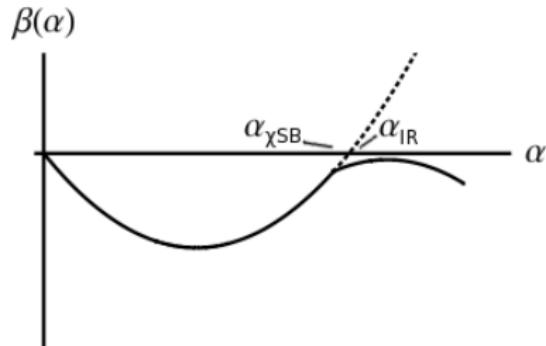
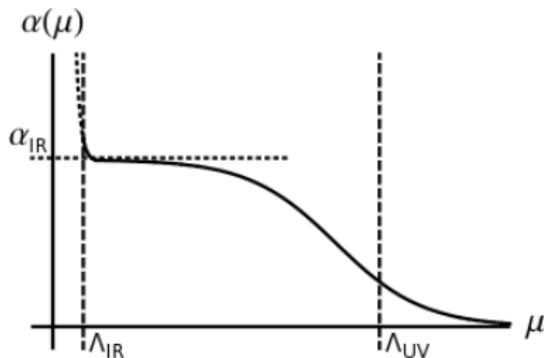
FCNCs required by CKM mixing, limit obtainable SM fermion masses.



# “Walking” Technicolor

$$\langle \bar{T}T \rangle|_{M_{ETC}} = \langle \bar{T}T \rangle|_{\Lambda_{TC}} \exp \left( \int_{\Lambda_{TC}}^{M_{ETC}} \frac{d\mu}{\mu} \gamma(\mu) \right) \approx \langle \bar{T}T \rangle|_{\Lambda_{TC}} \left( \frac{M_{ETC}}{\Lambda_{TC}} \right)^\gamma$$

- $\gamma(\mu) \sim 1$  for  $\Lambda_{TC} \lesssim \mu \lesssim M_{ETC}$  enhances fermion masses
- Implies large, slowly-running (“walking”) coupling, small  $\beta$  function



## Perturbative Yang–Mills $\beta$ function

For  $SU(N_c)$  Yang–Mills theory with  $N_f$  fermions in representation  $r$

$$\beta(g) = \mu \frac{\partial g}{\partial \mu} = \beta_0 g^3 + \beta_1 g^5 + \beta_2 g^7 + \dots$$

$$\beta_0 = -\frac{1}{(4\pi)^2} \left( \frac{11}{3} N_c - \frac{4}{3} N_f C(r) \right)$$

$$\beta_1 = -\frac{1}{(4\pi)^4} \left[ \frac{34}{3} N_c^2 - \left( \frac{13}{3} N_c - \frac{1}{N_c} \right) N_f C(r) \right]$$

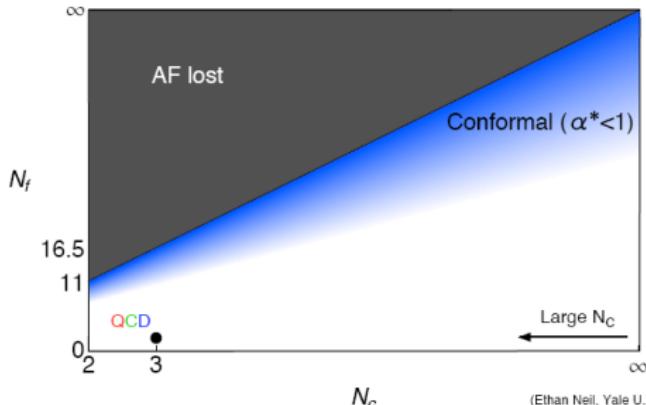
Higher-order  $\beta_i$  depend on choice of renormalization scheme

$$C(N) = \frac{1}{2} \quad C(\text{Adj}) = N_c \quad C_2(N) = \frac{d(\text{Adj})}{d(N)} C(N) = \frac{N_c^2 - 1}{2N_c}$$

# Conformal window

- Strongly-coupled gauge theories can look **very** different than QCD
- With many fermions, theory has perturbative IR fixed point;  
it is in an IR-conformal phase with no spontaneous  $\chi$ SB
- The **conformal window** ranges from loss of asymptotic freedom to some (unknown) critical  $N_f^c < N_f^{AF}$
- With  $N_f \lesssim N_f^c$ , may be approximately conformal (walking!) for some range of scales

Visualization of conformal window  
for  $SU(N_c)$  fermions in  
fundamental rep:



(Ethan Neil, Yale U.)

# Anomalous dimension

From “rainbow approximation” to Schwinger–Dyson equation

$$\left[ \frac{\vec{p}}{\text{---}} \bullet \text{---} \right]^{-1} = \text{---} \bullet \text{---} \quad \begin{array}{c} \text{k-p} \\ \curvearrowleft \\ \text{---} \end{array}$$

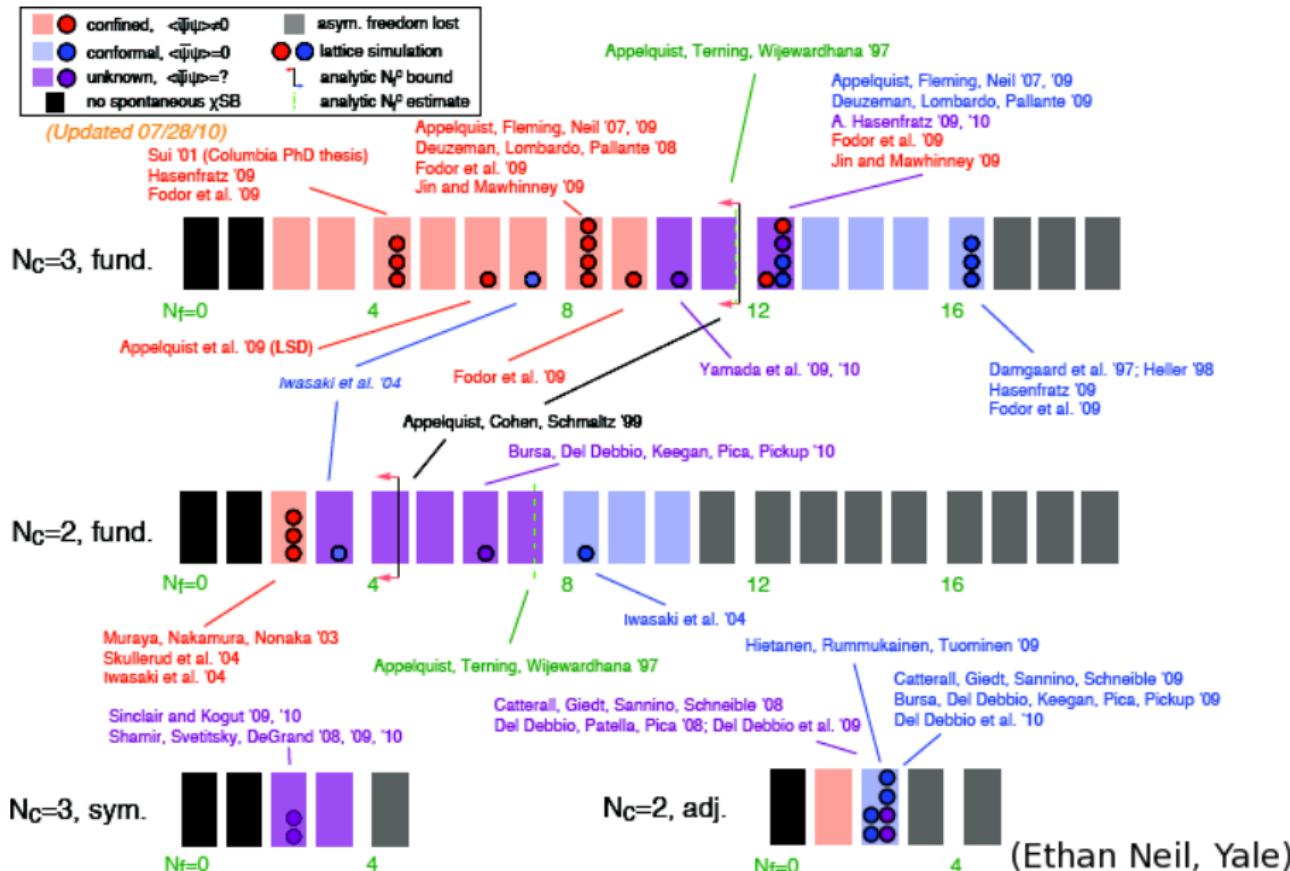
$$\gamma(\mu) = 1 - \sqrt{1 - 3C_2(r)\alpha(\mu)/\pi} \leq 1$$

Assume spontaneous chiral symmetry breaking when

$$\alpha(\mu) \geq \frac{\pi}{3C_2(r)} \equiv \alpha_{\chi SB}$$

When  $\alpha(\mu) = \alpha_{\chi SB}$ , this gives  $\gamma(\mu) = 1$

# Searching for conformal windows



# Iwasaki gauge action

$$U_{x,\mu} = \exp [i a g A_\mu(x + \hat{\mu}/2)] \quad (\text{directed from } x + \hat{\mu} \text{ to } x)$$

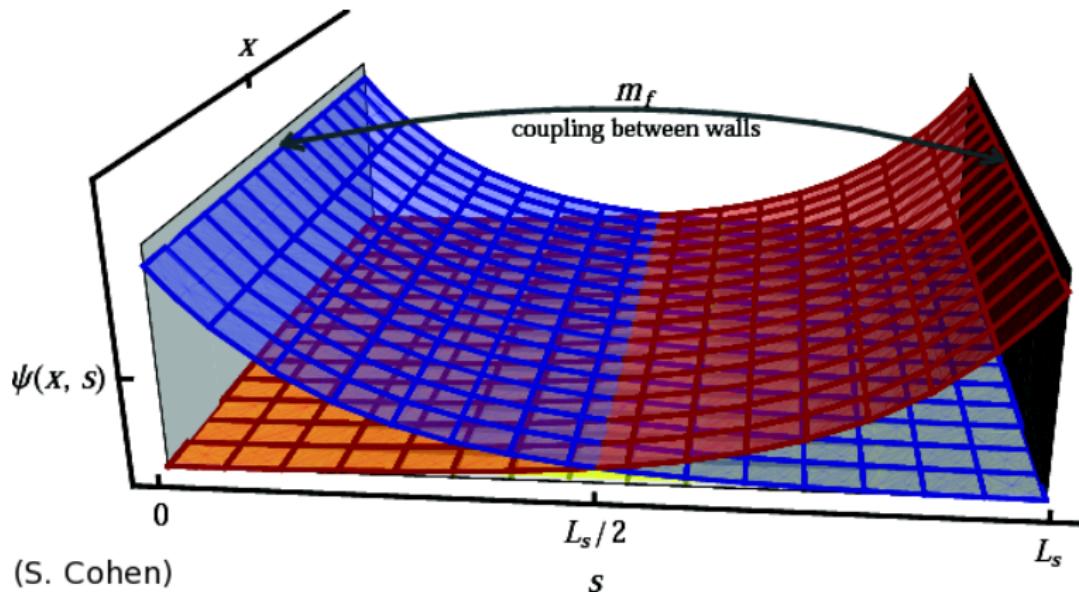
$$P_{x,\mu\nu} = \text{Tr} \left[ U_{x,\mu} U_{x+\hat{\mu},\nu} U_{x+\hat{\nu},\mu}^\dagger U_{x,\nu}^\dagger \right]$$

$$R_{x,\mu\nu} = \text{Tr} \left[ U_{x,\mu} U_{x+\hat{\mu},\mu} U_{x+\hat{\mu}+\hat{\nu},\nu} U_{x+\hat{\mu}+\hat{\nu},\mu}^\dagger U_{x+\hat{\nu},\mu}^\dagger U_{x,\nu}^\dagger \right]$$

$$S_G = \frac{2N_c}{g^2} \sum_x \left\{ 3.648 \sum_{\mu,\nu; \mu < \nu} \left( 1 - \frac{1}{N_c} P_{x,\mu\nu} \right) - 0.331 \sum_{\mu,\nu} \left( 1 - \frac{1}{N_c} R_{x,\mu\nu} \right) \right\}$$

$$\rightarrow \int \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \mathcal{O}(a^2) \quad \text{as } a \rightarrow 0$$

# Domain wall fermions



- Add fifth dimension of length  $L_s$
- Exact chiral symmetry at finite lattice spacing in the limit  $L_s \rightarrow \infty$
- At finite  $L_s$ , “residual mass”  $m_{\text{res}} > 0$ ;  $m = m_f + m_{\text{res}}$

## Domain wall Dirac operator

$$D_{x,y}^W(M_5) = (4 - M_5)\delta_{x,y} - \frac{1}{2} \left[ (1 + \gamma^\mu) U_{x,\mu}^\dagger \delta_{x,y+\mu} \right. \\ \left. + (1 - \gamma^\mu) U_{x,\mu} \delta_{x+\mu,y} \right]$$

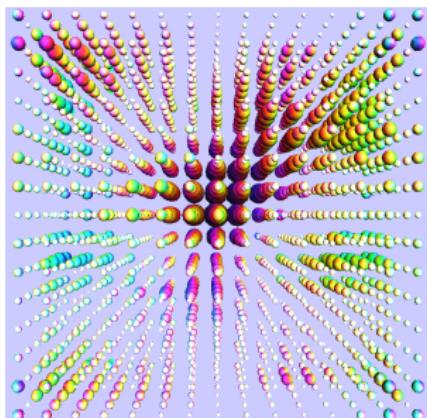
$$D_{s,s'}(m) = \left[ D^W(M_5) + 1 \right] \delta_{s,s'} + P_L [(1+m)\delta_{s,L_s-1}\delta_{s',0} - \delta_{s+1,s'}] \\ + P_R [(1+m)\delta_{s,0}\delta_{s',L_s-1} - \delta_{s,s'+1}]$$

$$D(m) = \begin{pmatrix} D^W + 1 & -P_L & 0 & \cdots & mP_R \\ -P_R & D^W + 1 & -P_L & \cdots & 0 \\ 0 & -P_R & D^W + 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ mP_L & 0 & 0 & \cdots & D^W + 1 \end{pmatrix}$$

$$P_L = \frac{1}{2}(1 - \gamma_5), P_R = \frac{1}{2}(1 + \gamma_5); \quad M_5 < 2 \text{ is height of domain wall}$$

# Hybrid Monte Carlo algorithm

- ① Generate random “momenta” with Gaussian distribution  $e^{-p^2/2}$
- ② Molecular dynamics evolution through fictitious MD “time” to produce new four-dimensional field configuration
- ③ Use MD discretization errors in Metropolis accept/reject step



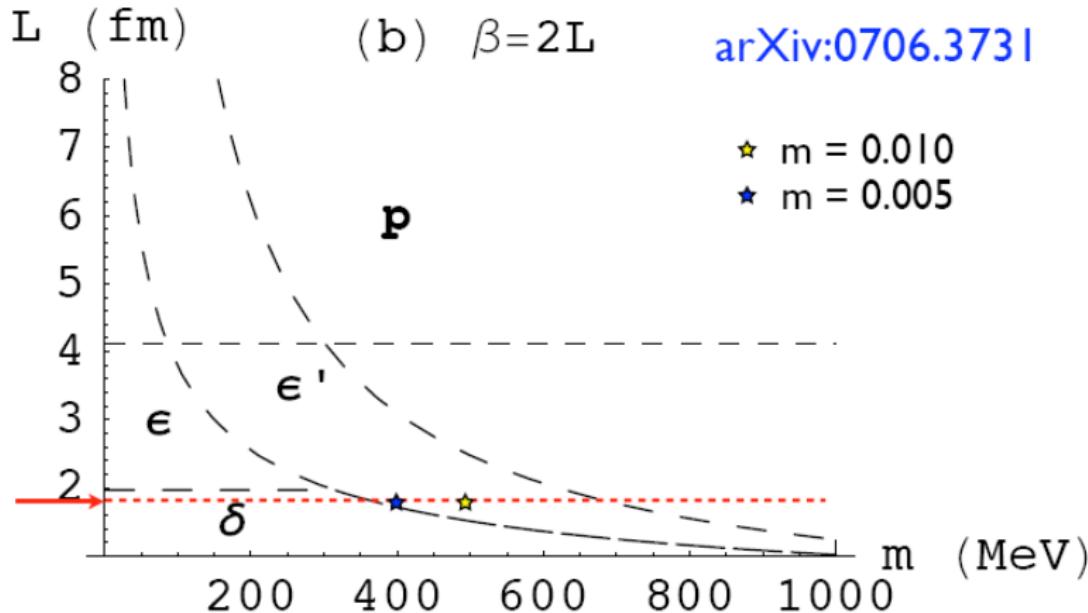
Numerically evaluate observables from the defining functional integral

$$\langle \mathcal{O} \rangle = \frac{\int \mathcal{D}U \ \mathcal{O}(U) \ e^{-S(U)}}{\int \mathcal{D}U \ e^{-S(U)}}$$

$U$ : four-dimensional field configurations  
 $S$ : action giving probability distribution  $e^{-S}$

Finite-volume effects depend on “regime”

# Phase Diagram



Boundaries can move as function of flavor

(Mike Buchoff)

## Polarization functions for the $S$ parameter

$$\gamma \text{---} \bullet \text{---} \gamma = ig_2 g_1 \cos \theta_w \sin \theta_w \Pi_{ee} \delta^{\mu\nu} + \dots$$

$$Z \sim \text{wavy line} \quad \text{black circle} \quad \text{wavy line} \gamma = ig_2 g_1 (\Pi_{3e} - \sin^2 \theta_w \Pi_{ee}) \delta^{\mu\nu} + \dots$$

$$Z \text{---} \text{---} \text{---} Z = \frac{i g_2 g_1}{\cos \theta_w \sin \theta_w} (\Pi_{33} - 2 \sin^2 \theta_w \Pi_{3e} + \sin^4 \theta_w \Pi_{ee}) \delta^{\mu\nu} + \dots$$

$$W \sim \text{wavy line} \sim W = ig_2^2 \Pi_{11} \delta^{\mu\nu} + \dots$$

$$\Pi_{VV} = 2\Pi_{3e}$$

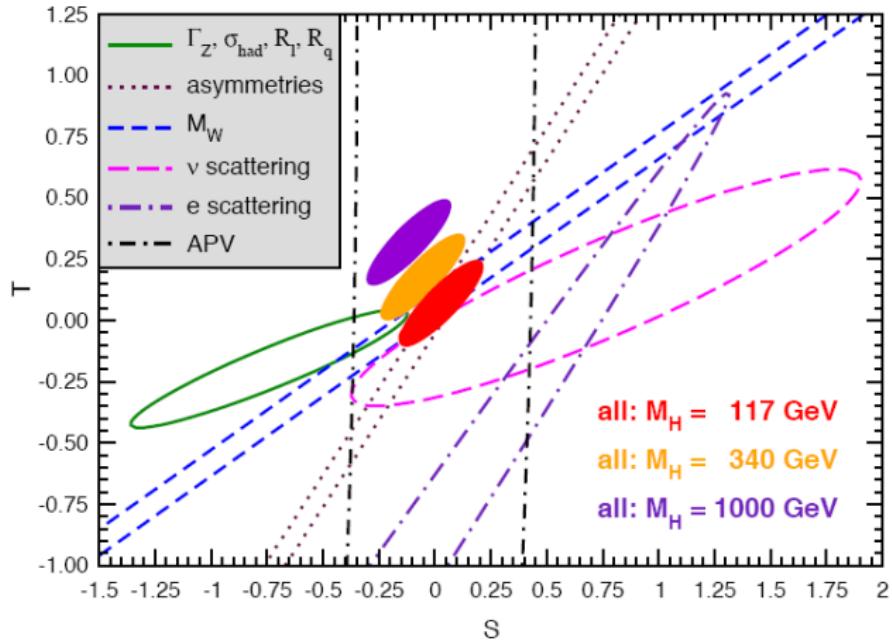
$$\Pi_{AA} = 4\Pi_{33} - 2\Pi_{3e}$$

$$S = 4\pi \lim_{Q^2 \rightarrow 0} \frac{d}{dQ^2} \left[ \Pi_{VV}(Q^2) - \Pi_{AA}(Q^2) \right] - \Delta S_{SM}$$

# Experimentally, $S \lesssim 0$

Extract  $S$  from global fit to experimental data

- ▶  $Z$  decay partial widths and asymmetries
- ▶  $M_W, M_Z$
- ▶ Neutrino scattering cross sections
- ▶ Atomic parity violation



(PDG)

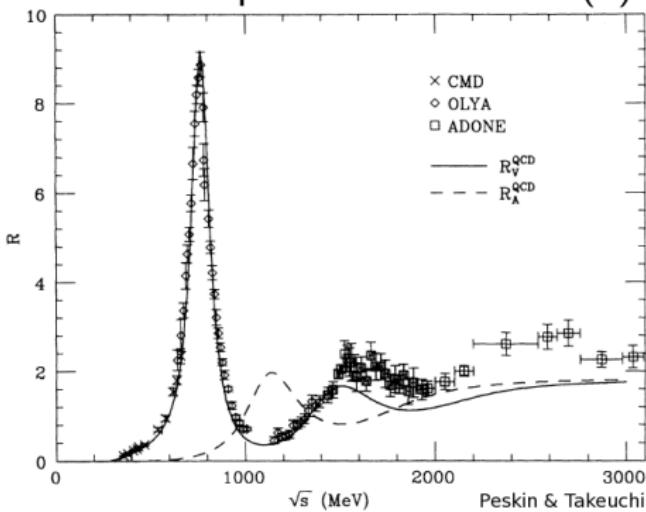
# $S$ from scaling up QCD

Analyticity and optical theorem relate polarization functions  $\Pi(Q^2)$   
to spectral functions  $R(s)$

$$R(s) = \frac{\sigma(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)}$$

$$\Pi(Q^2) = \Pi(0) + \frac{Q^2}{12\pi^2} \int_0^\infty \frac{ds R(s)}{s + Q^2}$$

$$S = 4\pi N_D \Pi'_{V-A}(0) - \Delta S_{SM}$$



$$S = \frac{1}{3\pi} \int_0^\infty \frac{ds}{s} \left\{ N_D (R_V - R_A) - \frac{1}{4} \left[ 1 - \left( 1 - \frac{M_H^2}{s} \right)^3 \Theta(s - M_H^2) \right] \right\}$$

Replacing the QCD scale with the electroweak scale,  $S = 0.32 \pm 0.03$

# $S$ parameter on the lattice

$$\gamma \sim \text{wavy line} \quad \bullet \quad \gamma = i g_2 g_1 \cos \theta_v \sin \theta_o \Pi_{\nu e} \delta^{\mu\nu} + \dots$$

$$Z \sim \text{wavy line} \quad \bullet \quad \gamma = i g_2 g_1 (\Pi_{3e} - \sin^2 \theta_v \Pi_{ee}) \delta^{\mu\nu} + \dots$$

$$Z \sim \text{wavy line} \quad \bullet \quad Z = \frac{i g_2 g_1}{\cos \theta_v \sin \theta_o} (\Pi_{33} - 2 \sin^2 \theta_v \Pi_{3e} + \sin^4 \theta_v \Pi_{ee}) \delta^{\mu\nu} + \dots$$

$$W \sim \text{wavy line} \quad \bullet \quad W = i g_2^2 \Pi_{11} \delta^{\mu\nu} + \dots$$

$$S = 4\pi N_D \Pi'_{V-A}(0) - \Delta S_{SM}$$

$$\Pi_{VV} = 2\Pi_{3Q}$$

$$\Pi_{AA} = 4\Pi_{33} - 2\Pi_{3Q}$$

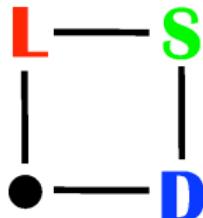
On the lattice, correlators involve a single pair of fermions

$$\Pi_{V-A}^{\mu\nu}(Q) = Z \sum_x e^{iQ \cdot (x + \hat{Q}/2)} \text{Tr} \left[ \langle \mathcal{V}^{\mu a}(x) V^{\nu b}(0) \rangle - \langle \mathcal{A}^{\mu a}(x) A^{\nu b}(0) \rangle \right]$$

$$\Pi^{\mu\nu}(Q) = \left( \delta^{\mu\nu} - \frac{\hat{Q}^\mu \hat{Q}^\nu}{\hat{Q}^2} \right) \Pi(Q^2) - \frac{\hat{Q}^\mu \hat{Q}^\nu}{\hat{Q}^2} \Pi^L(Q^2) \quad \hat{Q} = 2 \sin(Q/2)$$

- **Conserved currents**  $\mathcal{V}$  and  $\mathcal{A}$  ensure that lattice artifacts cancel
- $\langle \mathcal{V}^{\mu a}(x) \mathcal{V}^{\nu a}(0) \rangle$  and  $\langle \mathcal{A}^{\mu a}(x) \mathcal{A}^{\nu a}(0) \rangle$  require  $\mathcal{O}(L_s)$  inversions
- Renormalization constant  $Z$  computed non-perturbatively  
 $Z = 0.85$  (2f);    0.73 (6f);    0.71 (10f)

# Lattice Strong Dynamics Collaboration



Argonne James Osborn

Berkeley Sergey Syritsyn

Boston Ron Babich, Richard Brower, Saul Cohen,

Claudio Rebbi, DS

Brookhaven Oliver Witzel

Fermilab Ethan Neil

Harvard Mike Clark

Livermore Mike Buchoff, Michael Cheng, Pavlos Vranas, Joe Wasem

UC Davis Joseph Kiskis

Yale Thomas Appelquist, George Fleming,

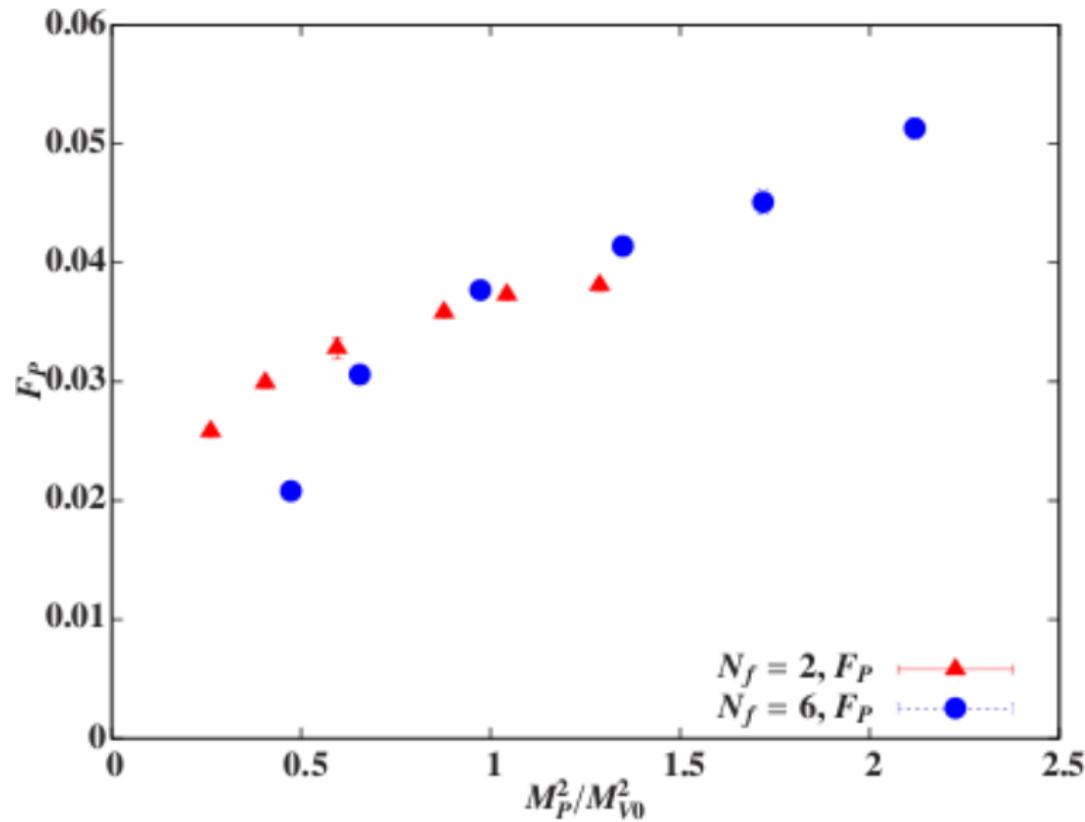
Meifeng Lin, Gennady Voronov

Performing non-perturbative studies of strongly interacting theories  
likely to produce observable signatures at the Large Hadron Collider

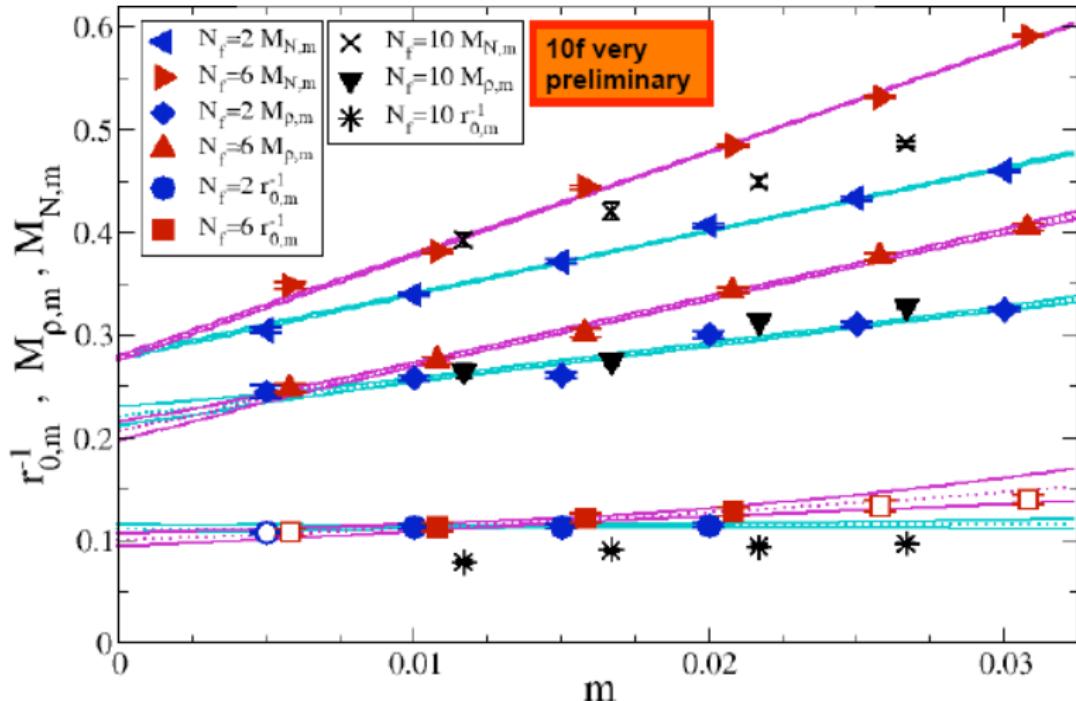
# LSD Philosophy and Simulation Details

- Start from what we know (QCD) and use it as a baseline  
→  $SU(3)$  gauge theory with  $N_f = 2, 6, 10$
- Work on large lattices so finite-volume effects are small  
→  $32^3 \times 64$  with  $0.005 \leq m_f \leq 0.030$  gives  $M_P L \gtrsim 4$
- Directly compare the different theories
  - Tune parameters to match chiral symmetry breaking scale
  - Plot results versus  $M_P^2$  rather than  $m = m_f + m_{res}$
- Exploratory calculations
  - $\mathcal{O}(100)$  independent measurements per point
- Studying spontaneous chiral symmetry breaking
  - Domain wall fermions with  $L_s = 16$
  - $m_{res} \approx 3 \times 10^{-5}$  (2f);  $8 \times 10^{-4}$  (6f);  $2 \times 10^{-3}$  (10f)

# Goldstone decay constant

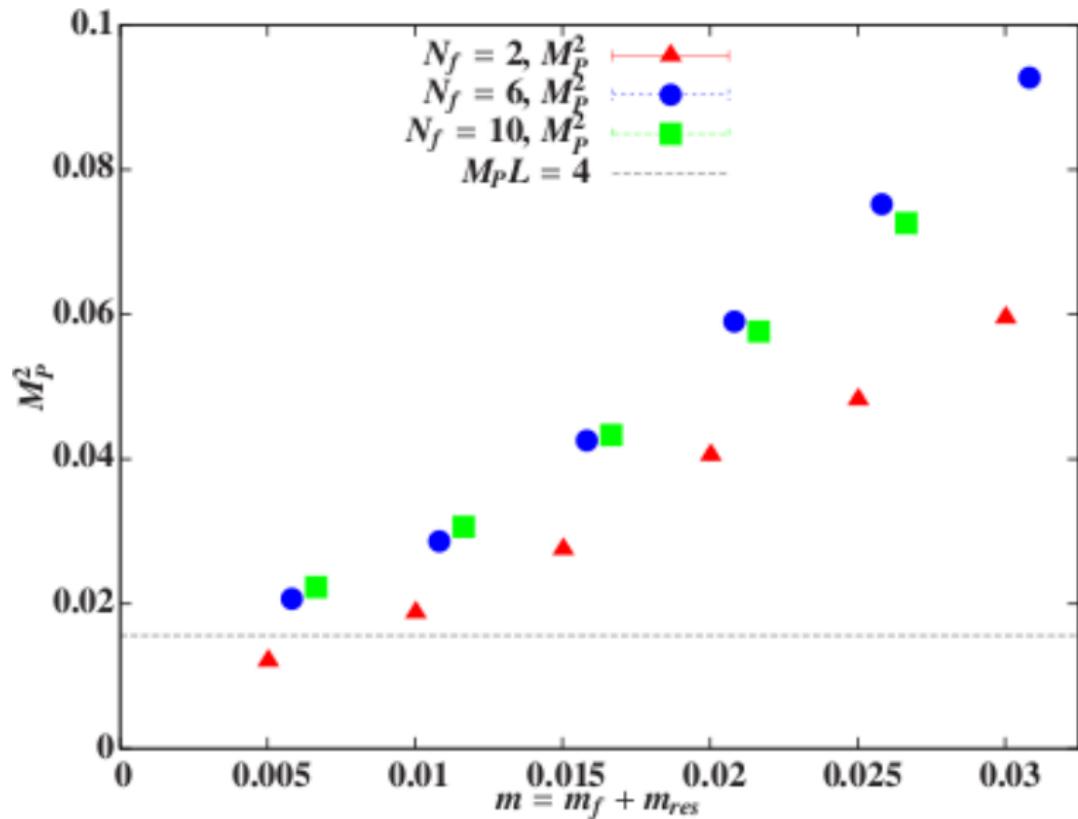


# Sommer scale, vector and nucleon masses



$N_f = 2$  and  $N_f = 6$  all match at 10% level

# Pseudo Nambu–Goldstone boson mass



## NLO $\chi$ PT for general $N_f$

Chiral perturbation theory ( $\chi$ PT) gives an effective field theoretic description of theories with chiral symmetry breaking, in terms of effective low-energy constants.

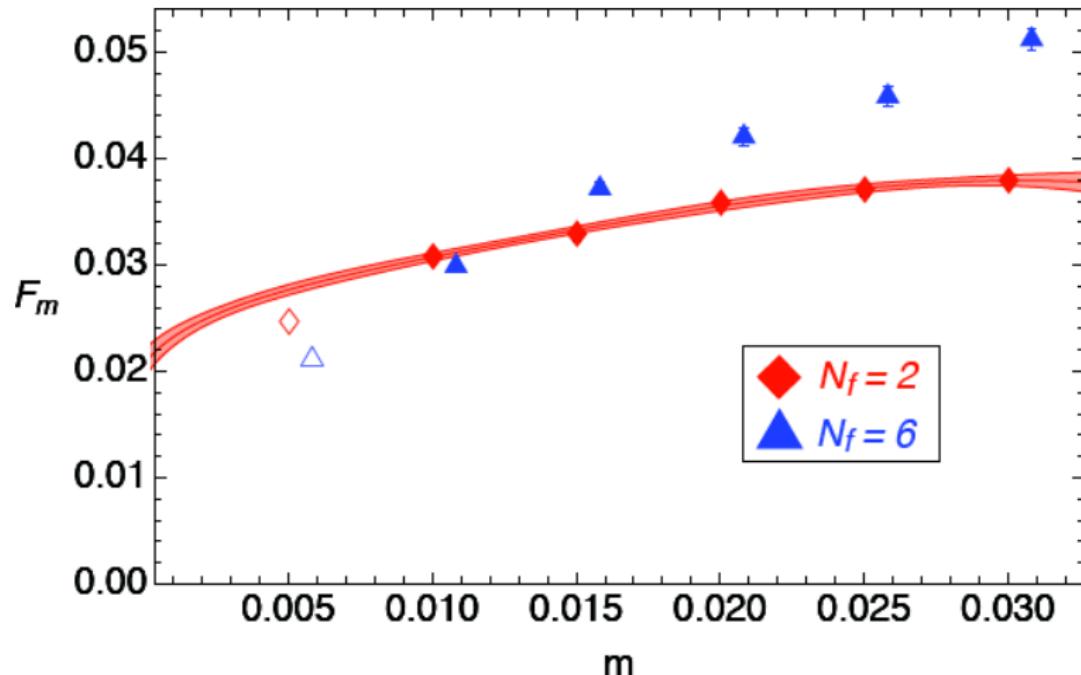
$$\frac{M_P^2}{2m} = B \left\{ 1 + \frac{2mB}{(4\pi F)^2} \left[ \alpha_m + \frac{1}{N_f} \log \left( \frac{2mB}{(4\pi F)^2} \right) \right] \right\}$$

$$F_P = F \left\{ 1 + \frac{2mB}{(4\pi F)^2} \left[ \alpha_F - \frac{N_f}{2} \log \left( \frac{2mB}{(4\pi F)^2} \right) \right] \right\}$$

$$\langle \bar{\psi} \psi \rangle = F^2 B \left\{ 1 + \frac{2mB}{(4\pi F)^2} \left[ \alpha_C - \frac{N_f^2 - 1}{N_f} \log \left( \frac{2mB}{(4\pi F)^2} \right) \right] \right\}$$

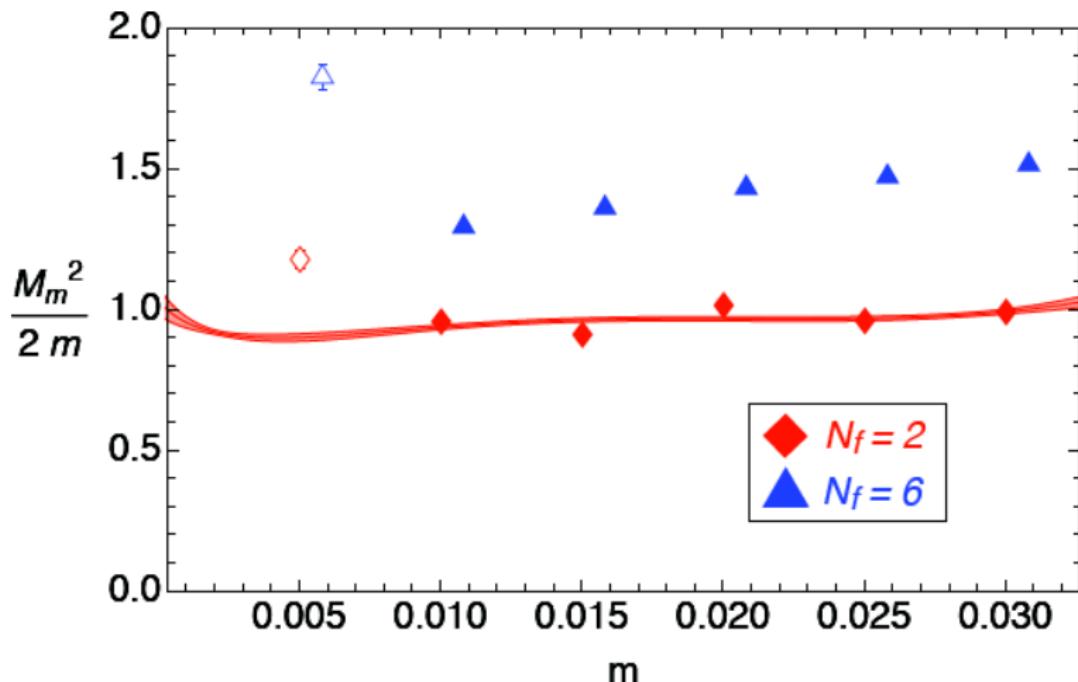
- $\alpha_C$  includes “contact term”  $m\Lambda^2 \sim ma^{-2}$
- NNLO  $M_P^2$  coefficients enhanced by  $N_f^2$  (Johan Bijnens and Jie Lu, 2009)

# Goldstone decay constant with NLO $\chi$ PT fit



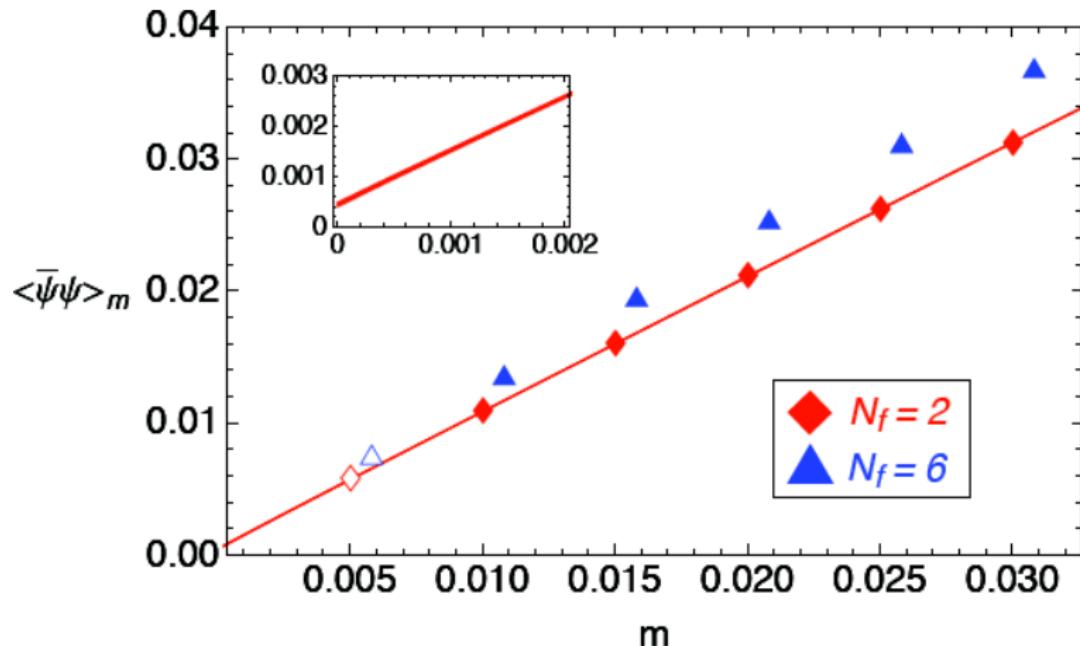
Joint NNLO $\chi$ PT fit to  $N_f = 2$   $F_P$ ,  $M_P^2$ ,  $\langle \bar{\psi}\psi \rangle$

# Pseudo Goldstone boson mass with NLO $\chi$ PT fit



- Slope of  $M_P^2$  with  $m$  significantly larger for  $N_f = 6$
- Plot against  $M_P^2$ , to provide more physical comparison

# Chiral condensate with NLO $\chi$ PT fit



Joint NNLO $\chi$ PT fit to  $N_f = 2$   $F_P$ ,  $M_P^2$ ,  $\langle\bar{\psi}\psi\rangle$   
Linear term clearly dominant

# Chiral condensate enhancement: introduction

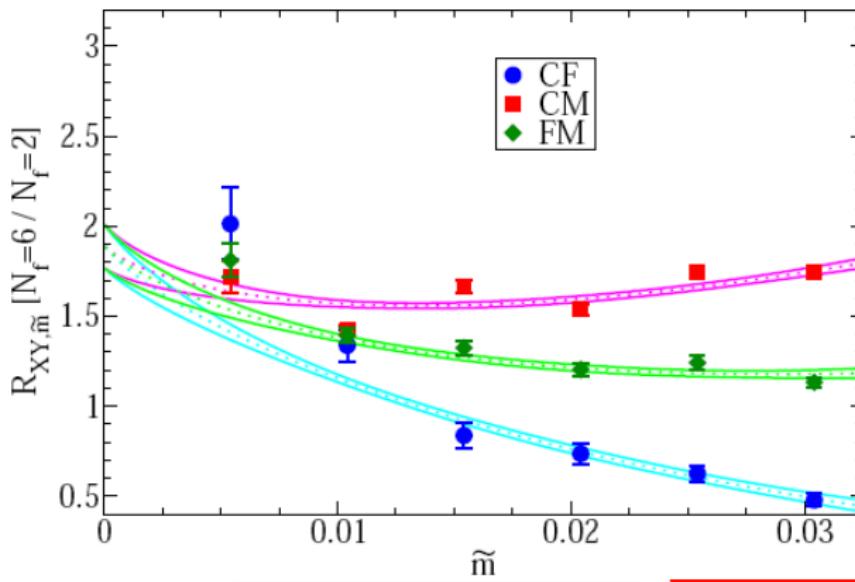
- Search for enhancement through  $\langle \bar{\psi}\psi \rangle / F^3$
- Not RG invariant: keep cutoff fixed in physical units
- Focus on the ratio  $R$  of  $\langle \bar{\psi}\psi \rangle / F^3$  between  $N_f = 6$  and  $N_f = 2$

$$R = \frac{(\langle \bar{\psi}\psi \rangle / F^3)_{6f}}{(\langle \bar{\psi}\psi \rangle / F^3)_{2f}} = \frac{\exp \left( \int_{M_\rho}^{5M_\rho} \frac{\gamma(\mu)}{\mu} \Big|_{6f} d\mu \right)}{\exp \left( \int_{M_\rho}^{5M_\rho} \frac{\gamma(\mu)}{\mu} \Big|_{2f} d\mu \right)}$$

$\overline{MS}$  perturbation theory & perturbative conversion to lattice scheme  
predicts  $R = 1.27(7)$

# Enhancement of $\langle \bar{\psi}\psi \rangle / F^3$ , $N_f = 2$ to $N_f = 6$

Find significant enhancement compared with perturbative  $R = 1.27(7)$

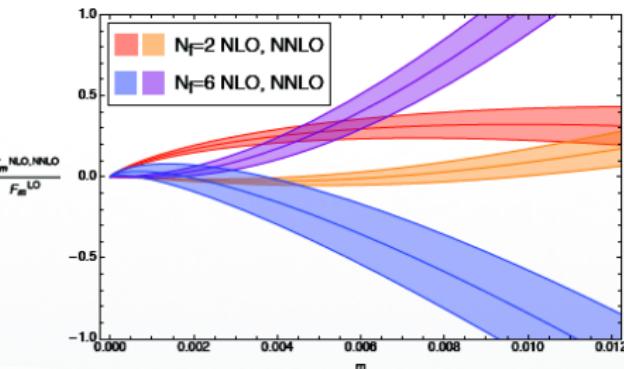
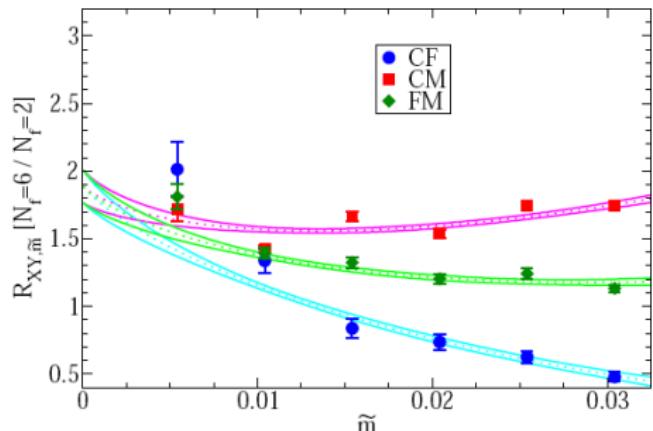


$$FM = M_\pi^2 / 2mF_\pi$$

$$CF = \langle \bar{\Psi}\Psi \rangle / F_\pi^3$$

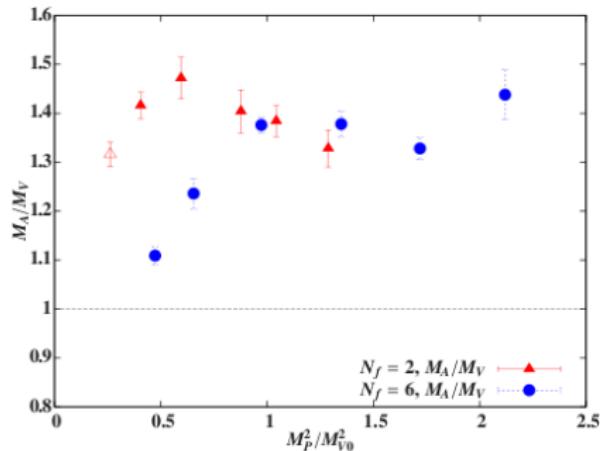
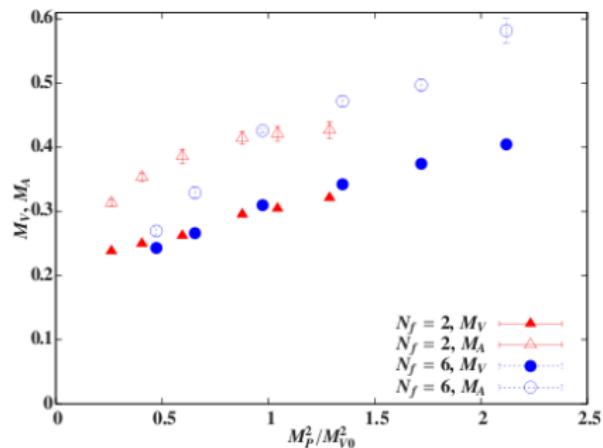
$$CM = \frac{\langle \bar{\Psi}\Psi \rangle}{[\frac{\sqrt{2m}\bar{\Psi}\Psi}{m_\pi}]^3}$$

# NLO $\chi$ PT fits, $N_f = 2$ and $N_f = 6$



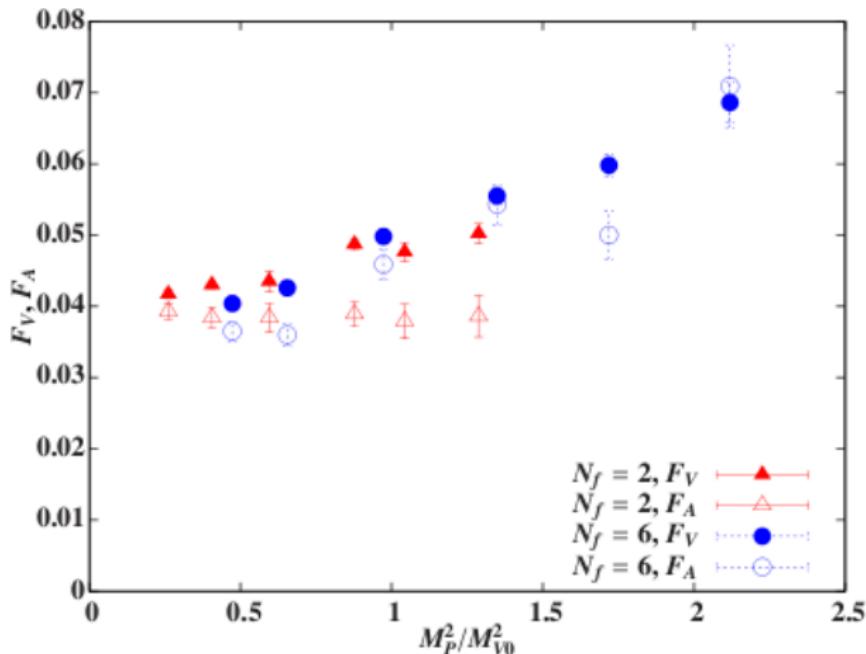
- NLO  $\chi$ PT fits work for  $N_f = 2$  but not  $N_f = 6$  (lighter  $m_f$  required)
- GMOR  $\Rightarrow \frac{\langle \bar{\psi}\psi \rangle}{F_\pi^3} = \frac{M_\pi^3}{\sqrt{(2m)^3 \langle \bar{\psi}\psi \rangle}} = \frac{M_\pi^2}{2mF_\pi} \equiv \mathcal{R}$  as  $m \rightarrow 0$
- Fit ratios to  $\mathcal{R} [1 + \tilde{m}(\alpha_{XY10} + \alpha_{11} \log \tilde{m})]$  where  $\tilde{m} \equiv \sqrt{m_2 m_6}$

# Vector and axial spectrum



Signs of  $N_f = 6$  parity-doubling as  $M_P^2$  decreases  
consistent with reduced  $S$  parameter

# Vector and axial decay constants



Parity-doubling involves  $F_V \approx F_A$  in addition to  $M_V \approx M_A$

# Conserved and local domain wall currents

Conserved currents:

$$\mathcal{V}^{\mu a}(x) = \sum_{s=0}^{L_s-1} j^{\mu a}(x, s) \quad \mathcal{A}^{\mu a}(x) = \sum_{s=0}^{L_s-1} \text{sign}\left(s - \frac{L_s - 1}{2}\right) j^{\mu a}(x, s)$$

$$j^{\mu a}(x, s) = \bar{\Psi}(x + \hat{\mu}, s) \frac{1 + \gamma^\mu}{2} \tau^a U_{x,\mu}^\dagger \Psi(x, s) \\ - \bar{\Psi}(x, s) \frac{1 - \gamma^\mu}{2} \tau^a U_{x,\mu} \Psi(x + \hat{\mu}, s)$$

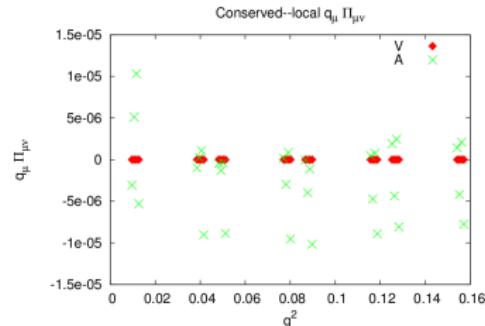
Local currents:

$$V^\mu(x) = \bar{q}(x) \gamma^\mu \tau^a q(x) \quad A^\mu(x) = \bar{q}(x) \gamma^\mu \gamma^5 \tau^a q(x)$$

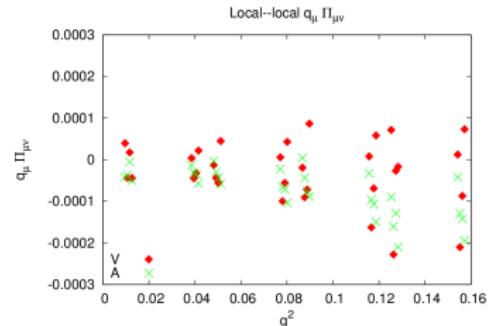
$$q(x) = P_L \Psi(x, 0) + P_R \Psi(x, L_s - 1)$$

# Ward identities and violations

$$\widehat{Q}_\mu \left[ \sum_x e^{iQ \cdot (x + \widehat{\mu}/2)} \langle V_\mu^a(x) V_\nu^a(0) \rangle \right] = 0$$

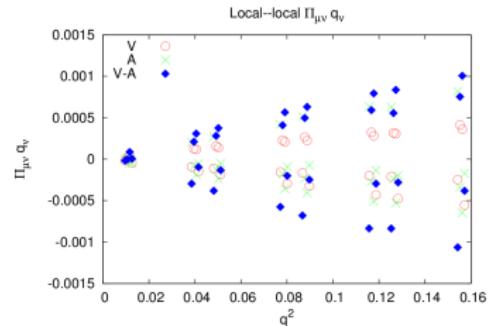
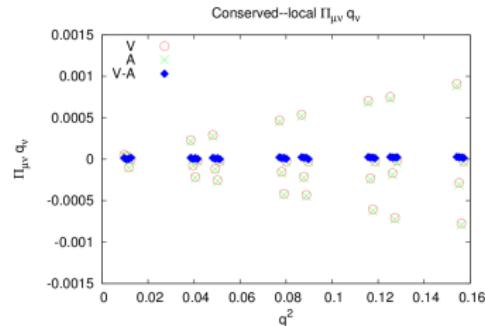


$$\widehat{Q}_\mu \left[ \sum_x e^{iQ \cdot x} \langle V_\mu^a(x) V_\nu^a(0) \rangle \right] \neq 0$$

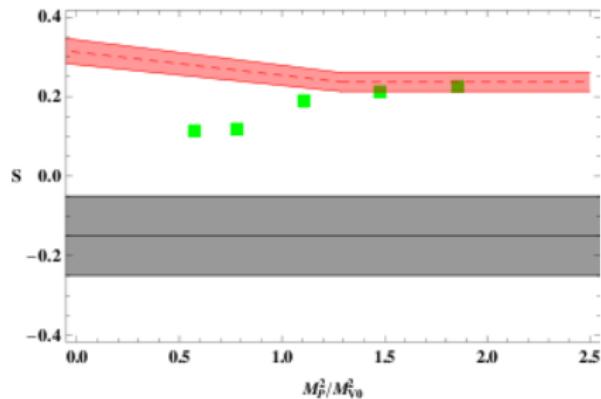
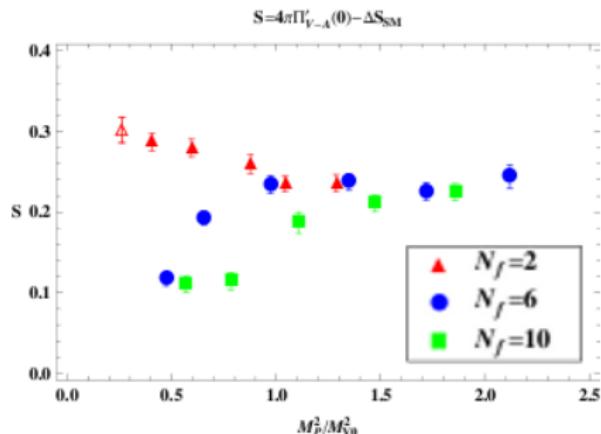
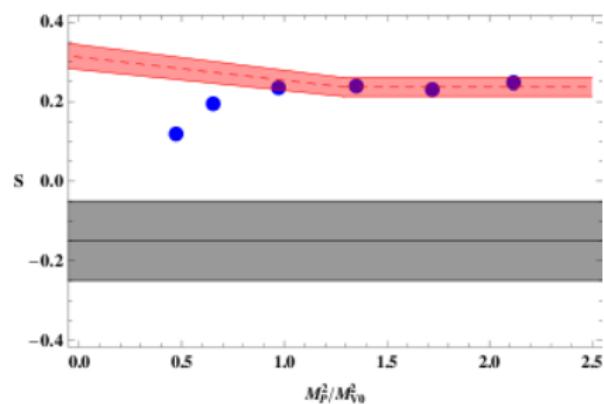


$$\left[ \sum_x e^{iQ \cdot (x + \widehat{\mu}/2)} (\langle V_\mu^a V_\nu^a \rangle - \langle A_\mu^a A_\nu^a \rangle) \right] \widehat{Q}_\nu \approx 0$$

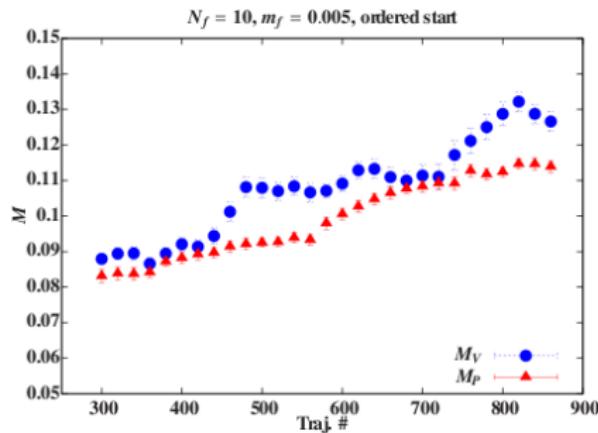
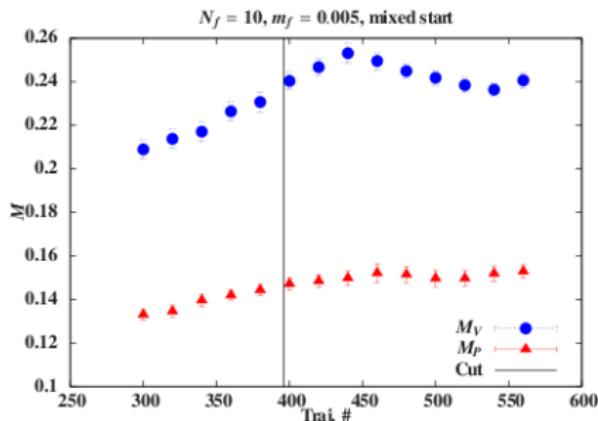
$$\left[ \sum_x e^{iQ \cdot x} (\langle V_\mu^a V_\nu^a \rangle - \langle A_\mu^a A_\nu^a \rangle) \right] \widehat{Q}_\nu \neq 0$$



# Single-doublet $S$ parameter

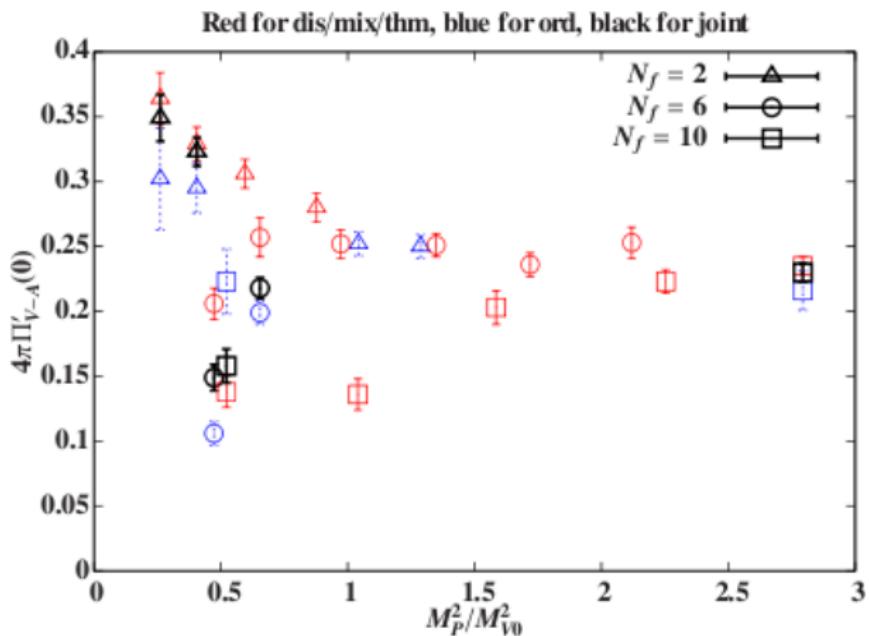


# Illustration of thermalization



- $M_P$  and  $M_V$  calculated using 100 configurations
- Full thermalization analysis considers plaquette,  $\langle \bar{\psi} \psi \rangle$  and correlators themselves
- Also compare runs starting from random vs. constant field

# Results from runs with different starting configuration



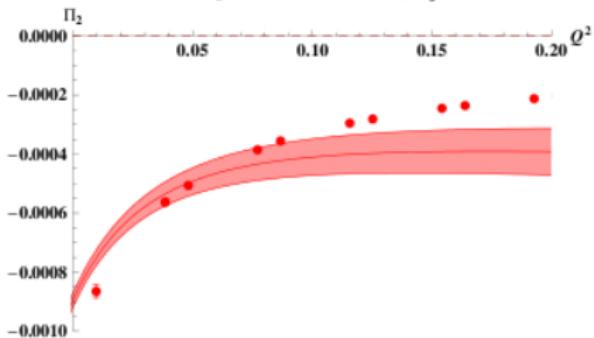
# Single-pole approximations to $\Pi_{V-A}$

$$R_V(s) = 12\pi^2 F_V^2 \delta(s - M_V^2)$$

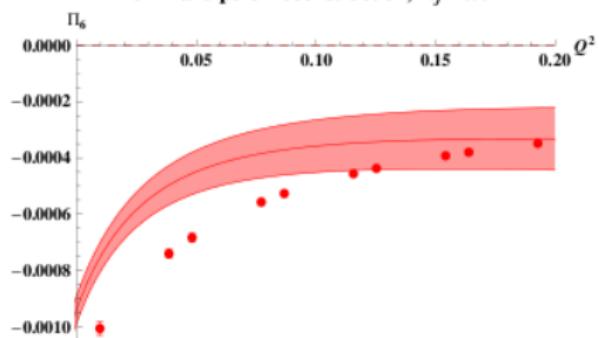
$$R_A(s) = 12\pi^2 F_A^2 \delta(s - M_A^2)$$

$$\Pi_{V-A}(Q^2) = -F_P^2 + \frac{Q^2 F_V^2}{M_V^2 + Q^2} - \frac{Q^2 F_A^2}{M_A^2 + Q^2}$$

2f  $\Pi$  and pole-reconstruction,  $m_f=0.01$



6f  $\Pi$  and pole-reconstruction,  $m_f=0.01$



## $S$ in $\chi$ PT, for $N_f = 2$

$$S = \frac{1}{12\pi} \left( \bar{\ell}_5 + \log \left[ \frac{M_P^2 v^2}{M_H^2 F_P^2} \right] - \frac{1}{6} \right)$$

$\bar{\ell}_5$  is extracted from

(Gasser and Leutwyler)

$$\Pi_{V-A}(Q^2) = -F_P^2 + Q^2 \left[ \frac{1}{24\pi^2} \left( \bar{\ell}_5 - \frac{1}{3} \right) + \frac{2}{3} (1+x) \bar{J}(x) \right]$$

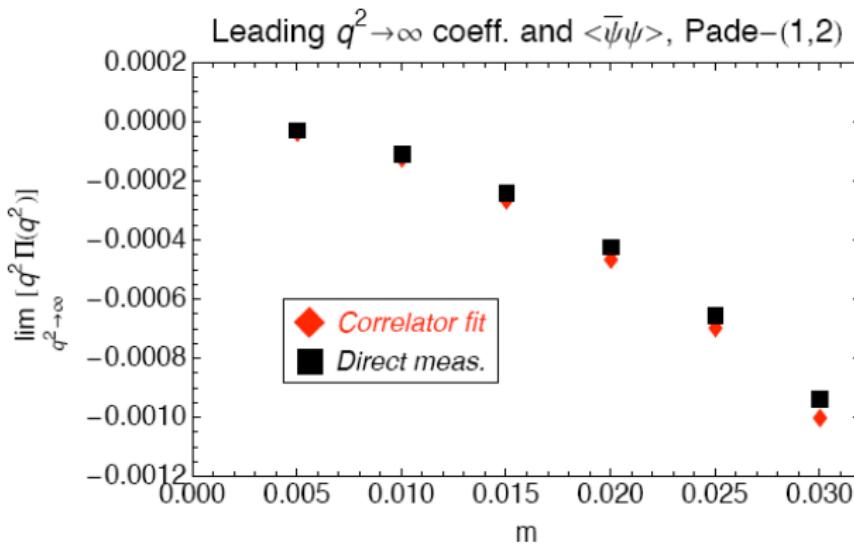
$$\bar{J}(x) = \frac{1}{16\pi^2} \left( \sqrt{1+x} \log \left[ \frac{\sqrt{1+x} - 1}{\sqrt{1+x} + 1} \right] + 2 \right), \quad x \equiv 4M_P^2/Q^2$$

- Our  $N_f \geq 6$  simulations have  $M_P$  too large to apply  $\chi$ PT
- General- $N_f$  corrections for  $\bar{\ell}_5$  not yet known
- Must take only two techni-fermions to the massless limit,  
any others remain massive

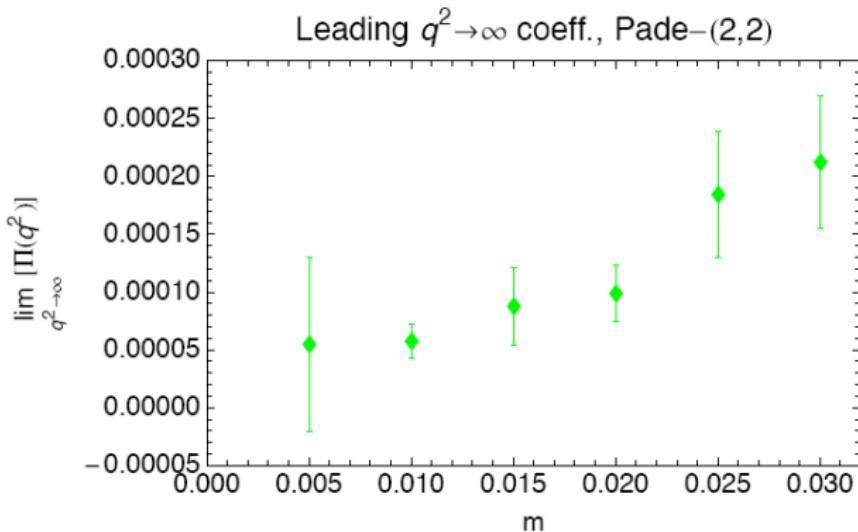
# Comparing Padé and OPE, $N_f = 2$

As  $Q^2 \rightarrow \infty$ ,

$$\Pi_{V-A}(Q^2) \rightarrow \frac{N_c}{8\pi^2} m^2 + \frac{m \langle \bar{\psi}\psi \rangle}{Q^2} + \mathcal{O}(\alpha) + \mathcal{O}(Q^{-4})$$



# Corrections to the first Weinberg sum rule, $N_f = 2$



$Q^4$  term in numerator of (2, 2) Padé is small

$$\frac{a_0 + a_1 Q^2 + a_2 Q^4}{1 + b_1 Q^2 + b_2 Q^4} = \left[ -F_0^2 + \frac{Q^2 F_1^2}{M_1^2 + Q^2} - \frac{Q^2 F_2^2}{M_2^2 + Q^2} \right]$$

## Ongoing refinements and future directions

- Additional runs at  $m_f = 0.0075$  to confirm trend
- Ordered-start runs for  $N_f = 10$  (may help understand topology)
- Direct calculation of finite-volume effects
- “Twisted” boundary conditions to probe smaller  $Q^2$
- $N_f = 8$  simulations with cheaper (staggered) lattice action
- Operator product expansion for  $\Pi_{V-A}^{\mu\nu}(Q)$