Status and prospects for supersymmetry on the lattice

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USQCD All Hands Meeting, JLab, 19 April 2014

Proposal: *Lattice $\mathcal{N} = 4$ supersymmetric Yang–Mills with 2, 3 and 4 colors*
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Context: Why lattice supersymmetry

Supersymmetry is extremely interesting, especially non-perturbatively

- Widely-studied potential roles in BSM physics
  Central to ongoing LHC experimental program
  $\longrightarrow$ current results seriously constrain simplest scenarios
- More generally, symmetries improve analytic control
  $\longrightarrow$ Insight into confinement, dynamical symmetry breaking,
  conformal field theories (conformal window, dilatons), etc.
- Dualities: gauge–gauge (Seiberg) & gauge–gravity (AdS/CFT)
  $\longrightarrow$ potential non-perturbative definition of string theory
- AdS/CFT-inspired modelling of quark–gluon plasma,
  finite-density phase diagram, condensed matter systems, etc.

Context: Why not lattice supersymmetry

There is a problem with supersymmetry on the lattice

Recall: supersymmetry extends Poincaré symmetry
by spinorial generators $\mathcal{Q}_{\alpha}^{i}$ and $\overline{\mathcal{Q}}_{\dot{\alpha}}^{i}$ with $i = 1, \cdots, \mathcal{N}$

The resulting algebra includes $\left\{\mathcal{Q}, \overline{\mathcal{Q}}\right\} \propto \sigma_{\mu} P^{\mu}$

P^{μ} generates infinitesimal translations, which don't exist on the lattice

Consequence for lattice calculations

Quantum effects generate (typically many) susy-breaking operators

Fine-tuning their couplings to restore susy is generally not practical

Two special cases in four dimensions

Minimal ($\mathcal{N} = 1$) supersymmetric Yang–Mills

Theory of SU(N) gauge field and its fermionic superpartner gaugino,
a massless Majorana fermion in the adjoint rep.

Only relevant supersymmetry-breaking operator is gaugino mass
$\Longrightarrow$ chiral symmetry (Ginsparg–Wilson fermions) ensures susy

When there are scalar fields
we must preserve some susy sub-algebra on the lattice

Possible for only one (particularly interesting) 4-dim. system:

Maximal ($\mathcal{N} = 4$) supersymmetric Yang–Mills

Theory of SU(N) gauge field, four gauginos and six adjoint scalars

SciDAC-supported USQCD program pursues both these directions

$\mathcal{N} = 1$ super Yang–Mills status and prospects

Current status (computations on non-USQCD resources)

Domain wall fermions in CPS for SYM with SU(2) gauge group
(SU(2) adjoint rep. in SU(3) fundamental rep. data structures)

—Revisiting gaugino condensation (Giedt et al., Endres 2009)
—Starting to explore low-lying spectrum $\longrightarrow$ disconnected correlators

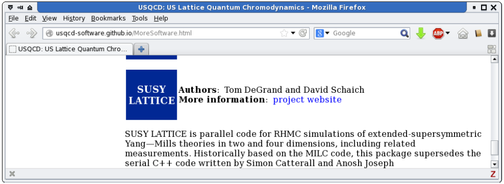
Next steps

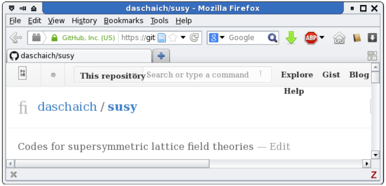
Immediate: CPS $\longrightarrow$ QHMC for general SU(N) with Möbius DWF

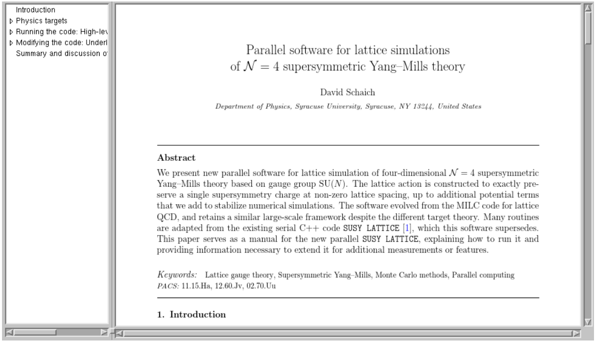
Later: Add N_F fundamental quark+squark fields $\Longrightarrow$ super QCD
—Must fine-tune scalar sector of SQCD
—Possible (but not easy) via reweighting

$\mathcal{N} = 4$ supersymmetric Yang–Mills on the lattice

MILC-based software available through usqcd.org







Parallel software for lattice simulations of $\mathcal{N} = 4$ supersymmetric Yang–Mills theory

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Abstract

We present new parallel software for lattice simulation of four-dimensional $\mathcal{N} = 4$ supersymmetric Yang–Mills theory based on gauge group SU(N). The lattice action is constructed to exactly preserve a single supersymmetry charge at non-zero lattice spacing, up to additional potential terms that we add to stabilize numerical simulations. The software evolved from the MILC code for lattice QCD, and retains a similar large-scale framework despite the different target theory. Many routines are adapted from the existing serial C++ code SUSY LATTICE [1], which this software supersedes. This paper serves as a manual for the new parallel SUSY LATTICE, explaining how to run it and providing information necessary to extend it for additional measurements or features.

Keywords: Lattice gauge theory, Supersymmetric Yang–Mills, Monte Carlo methods, Parallel computing
PACS: 11.15.Ha, 12.60.Jv, 02.70.Uu

1. Introduction

$\mathcal{N} = 4$ supersymmetric Yang–Mills on the lattice

MILC-based software available through usqcd.org

Why four-dimensional $\mathcal{N} = 4$ SYM is special

Four Majorana gauginos $\longrightarrow$ 16 fermionic components
$\Longrightarrow$Gauge&fermion&scalar fields can be discretized on equal footing

$\Longrightarrow$ Lattice can exactly preserve one of 16 supersymmetries

The construction is straightforward, but too messy for this talk

Skip to the consequences:

- Need **five** links in four dimensions $\Longrightarrow$ A_4^* lattice
  (analog of triangular lattice in two dimensions)
- Links are not unitarized $\Longrightarrow$ gauge group U(N) $\simeq$ SU(N)$\otimes$U(1),
  must suppress strong-coupling lattice phase in U(1) sector

Lattice action for $\mathcal{N} = 4$ numerical computations

Schematic lattice action: $(\lambda = g^2 N, \quad a = 1, \cdots, 5)$

$$S = \frac{N}{\lambda}\left[F_{ab}^2 + (\mathcal{D}_a\mathcal{U}_a)^2 + \chi_{ab}\mathcal{D}_a\psi_b + \eta\mathcal{D}_a\psi_a + \epsilon_{abcde}\chi_{de}\mathcal{D}_c\chi_{ab}\right]$$
$$+ \mu^2\left(\mathcal{U}_a^2 - 1\right)^2 + \kappa|\det\mathcal{P} - 1|^2$$

—First line preserves single supersymmetry $\mathcal{Q}$, other 15 broken

$\mathcal{U}_a$ are links, η, ψ_a and χ_{ab} are fermion components

—μ term regulates flat directions, stabilizes continuum limit

—κ term suppresses lattice phase from U(1) sector ($\mathcal{P}$ is plaquette)

—All simulations are pfaffian-phase-quenched (more later)

Both μ and κ deformations break $\mathcal{Q}$ supersymmetry

but are required to carry out numerical computations

Also need to lift fermion zero modes $\longrightarrow$ anti-periodic temporal BCs

Preservation / restoration of supersymmetries

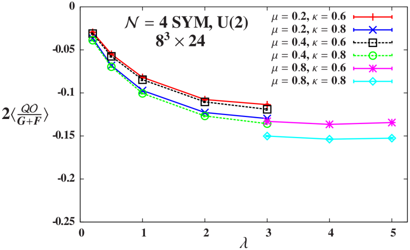
Need observables to monitor supersymmetry

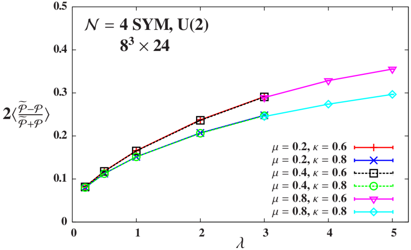
Exactly preserved $\mathcal{Q}$ supersymmetry $\longrightarrow$ Ward identity $\langle \mathcal{QO} \rangle = 0$

$\Longrightarrow$ Ward identity violations measure $\mathcal{Q}$ breaking (**left**)

Restoration of other 15 supersymmetries

follows from restoration of discrete "R" symmetries (**right**)





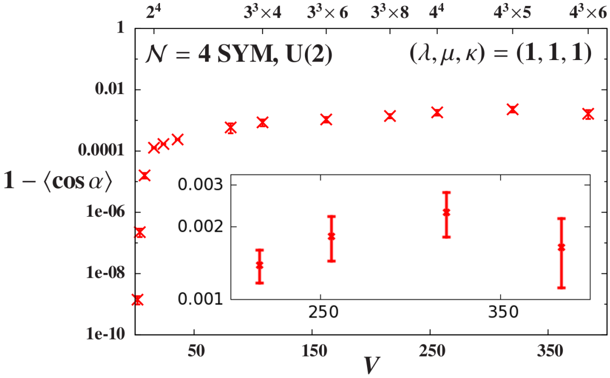
Both plots show $\mathcal{O}(10\%)$ supersymmetry breaking,

little dependence on μ or κ

Majorana fermions $\longrightarrow$ complex pfaffian

No indication of a sign problem

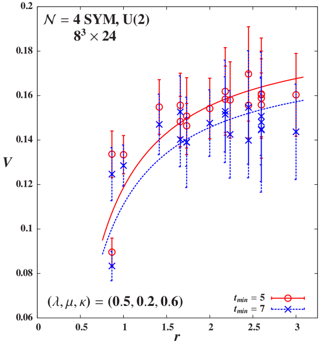
- Pfaffian $P = |P|e^{i\alpha}$ is nearly real and positive, $1 - \langle\cos(\alpha)\rangle \ll 1$
- Fluctuations in pfaffian phase don't grow with the lattice volume
- Insensitive to number of colors $N = 2, 3, 4$

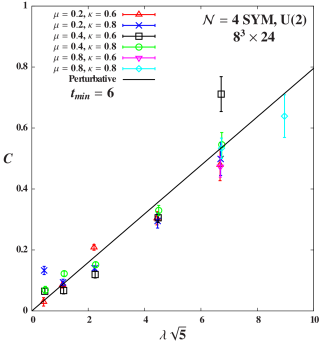


Static potential, comparison with continuum theory

Coulombic at both weak and strong coupling, as expected

Coulomb coefficient agrees with leading-order perturbation theory





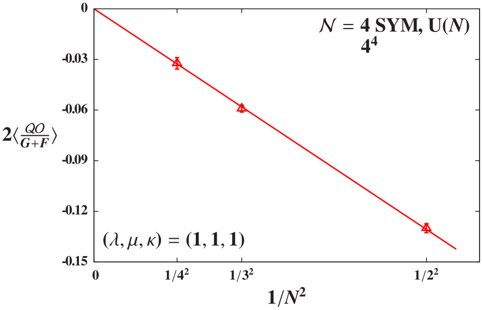
Results fairly noisy $\longrightarrow$ working on smearing for A_4^* lattice

2014 proposal: $\mathcal{N} = 4$ SYM with 2, 3 and 4 colors

So far we have focused on the simplest case U(2),
but continuum theory is anchored in the large-N limit

Code allows U(N) with arbitrary N; we have tested $N = 2$, 3 and 4

First results indicate susy breaking suppressed $\propto 1/N^2$,
computational costs scale $\propto N^5$ (empirically)



Recapitulation

- Strongly-coupled supersymmetric field theories very interesting
  to study through lattice calculations

- Barriers to 4d lattice supersymmetry have been overcome
  for both $\mathcal{N} = 1$ and $\mathcal{N} = 4$ supersymmetric Yang–Mills
  (Not discussed: lower-dim. systems also worth further study)

- SciDAC-supported USQCD program
  studying $\mathcal{N} = 1$ and $\mathcal{N} = 4$ SYM, eventually $\mathcal{N} = 1$ SQCD
  2014 proposal: $\mathcal{N} = 4$ SYM with 2, 3 and 4 colors

- It will be healthy to have more groups studying lattice susy
  $\longrightarrow$ publicly-available code to reduce barriers to entry

SPC Q1: Where is the continuum limit?

Short answer: We must look at large distances, extrapolating $1/L \to 0$

More details

- λ can be fixed: $\mathcal{N} = 4$ SYM has line of conformal fixed points
- Want $\mu \to 0$ as $1/L \to 0$ to restore supersymmetry
  Due to form of flat directions, fixed $\mu^2 V$ should maintain stability
- κ is coupling of $F_{\mu\nu}F^{\mu\nu}$ term in U(1) sector, which decouples
- So far we observe little dependence on μ or κ

We have developed observables to monitor $\mathcal{N} = 4$ susy restoration
in case we have to tune any couplings

Q2: What are plans for smearing?

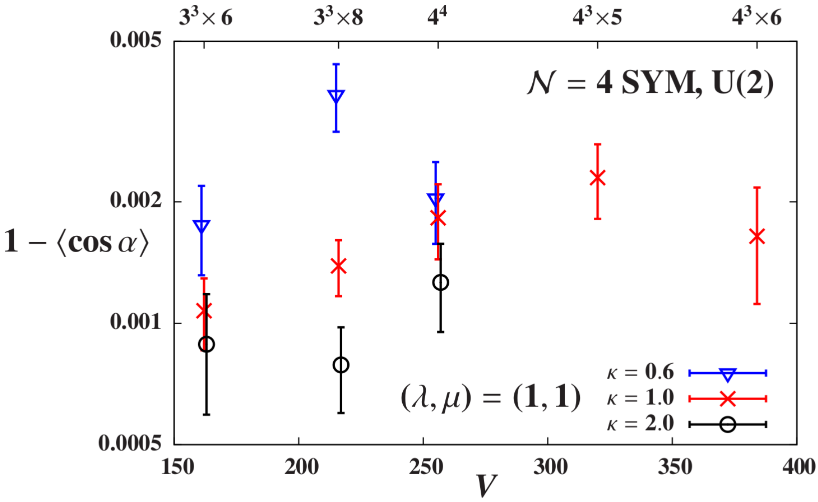
Short answer: Just improving measurements, not smearing the action

More details

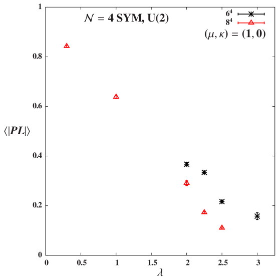
- Non-trivial to smear on A_4^* lattice with five non-unitarized links
- Initial HYP-based tests indicate more careful work needed
- Currently studying stout- and APE-like approaches
- Smearing the $\mathcal{N} = 4$ SYM lattice action appears more challenging
  Any smeared action **must** preserve susy sub-algebra

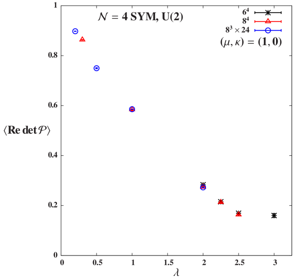
Q3: How does the pfaffian phase depend on λ, μ, κ?

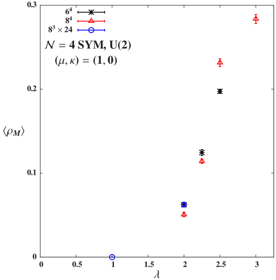
Short answer: Little dependence on κ, fluctuations grow with λ



Backup: Lattice phase due to U(1) sector







1. Polyakov loop collapses $\Longrightarrow$ confining phase
   (**not** present in continuum $\mathcal{N} = 4$ SYM)

2. Plaquette determinant is variable in U(1) sector
   Drops at same coupling λ as Polyakov loop

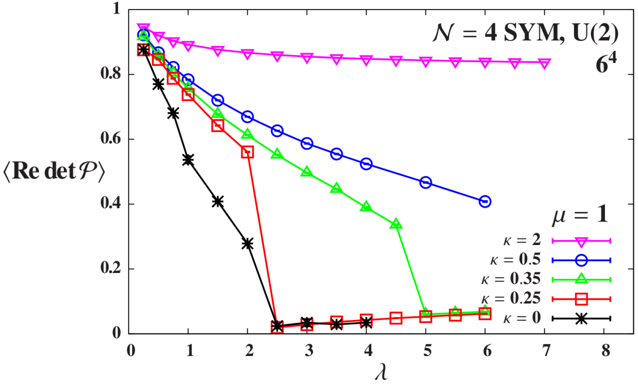
3. ρ_M is density of U(1) monopole world lines (DeGrand & Toussaint)
   Non-zero when Polyakov loop and plaq. determinant collapse

Backup: Suppressing the U(1) sector

$\Delta S = \kappa |\det \mathcal{P} - 1|^2$ suppresses the strongly-coupled lattice phase

Produces $2\kappa F_{\mu\nu} F^{\mu\nu}$ term in U(1) sector

$\Longrightarrow$ QED critical $\beta_c = 0.99 \longrightarrow$ critical $\kappa_c \approx 0.5$

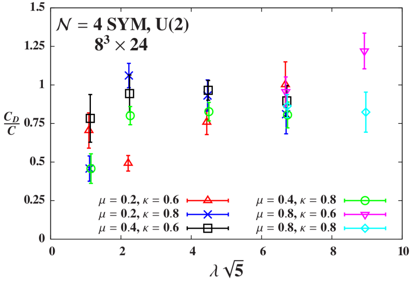


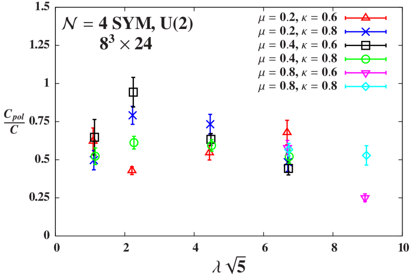
Backup: More tests of the static potential

Left: Projecting Wilson loops from U(N) $\longrightarrow$ SU(N)

$\Longrightarrow$ factor of $\frac{N^2-1}{N^2} = 3/4$

Right: Unitarizing links removes scalars $\Longrightarrow$ factor of $1/2$



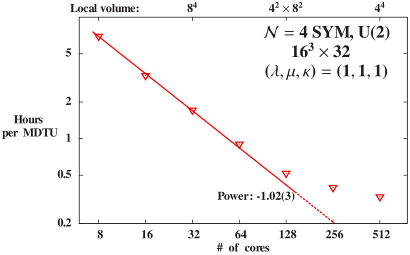


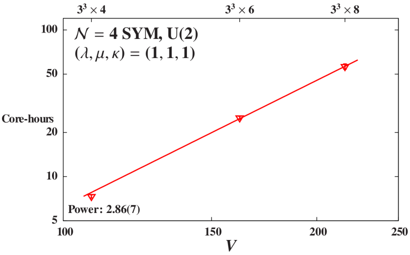
Both expected factors present, although (again) noisily

Backup: $\mathcal{N} = 4$ SYM code performance at Fermilab

Left: Strong scaling for U(2) $16^3 \times 32$ RHMC gauge generation

Right: Weak scaling for $\mathcal{O}(N_\Psi^3)$ pfaffian calculation (fixed local volume)





Both plots on log–log axes with power-law fits