Going Beyond QCD on the Lattice

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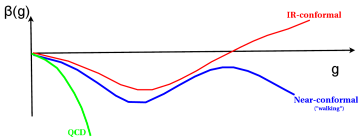
Syracuse High Energy Theory Seminar, 23 April 2013

arXiv:1301.1355, arXiv:1303.7129 and work in progress
with Anqi Cheng, Anna Hasenfratz and Gregory Petropoulos

Motivational overview

- Why study new strong dynamics?
  - Generically important for BSM despite results from LHC so far
  - Room for improved understanding
- Why use lattice gauge theory?
  - (Difficult and expensive approach)
  - Promises non-perturbative insights, systematically improvable
- What have we learned?
  - Must synthesize different methods
  - Most effective methods often differ from familiar lattice QCD techniques
  - Should explore range of couplings, try to reach chiral limit

Near-conformal systems are very different from QCD



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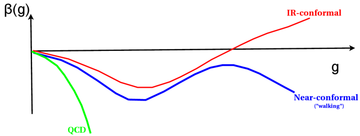

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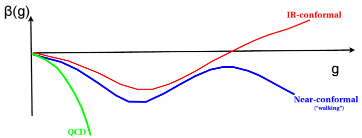

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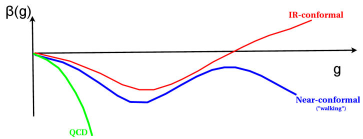


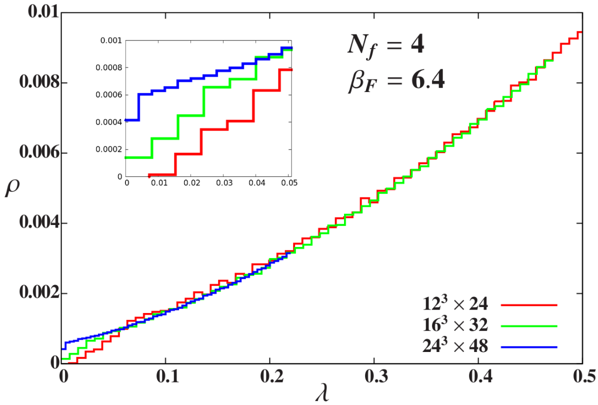
Outline: two improved methods for going beyond QCD

1. Improved method to extract mass anomalous dimension $\gamma_m(\mu)$ from eigenmodes of the massless Dirac operator
   - Proof-of-principle tests from QCD-like SU(3) model with $N_F = 4$
   - IR-conformal dynamics from $N_F = 12$ (arXiv:1301.1355)
2. Improved method to determine RG β function (arXiv:1212.0053) shows $N_F = 12$ IR fixed point more directly
3. **If time permits:** $N_F = 8$ as our most interesting/confusing results
   - $\gamma_m(\mu)$ from Dirac eigenmodes clearly contrasts with $N_F = 4$
   - Current results consistent with both IR conformality and "walking"
   - Further progress requires better understanding novel lattice phase

Not today: Hadron spectrum, finite-size scaling, non-zero temp. (arXiv:1303.7129 and more to appear)

Dirac operator eigenvalues λ and spectral density $\rho(\lambda)$

$\mathcal{L}_F = \overline{\Psi}\left(\not{D} + m_F\right)\Psi$ where $\not{D}$ is the **massless** Dirac operator



$\beta_F \sim 12/g_0^2$ (perturbative)

$\rho(\lambda)$ is histogram of eigenvalues λ

Finite-volume effects visible at small λ

$\langle\overline{\psi}\psi\rangle \propto \rho(\lambda \to 0)$ in ∞-volume chiral limit (Banks & Casher)

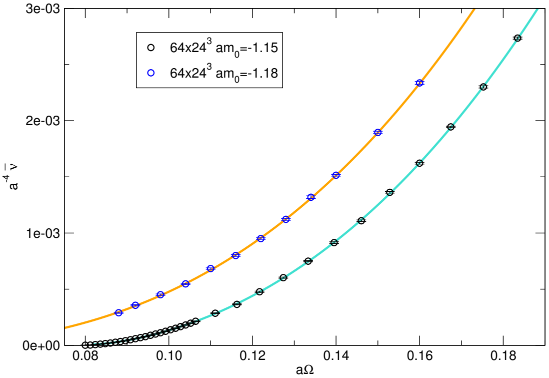
Focus on overlapping regions where different volumes agree instead of studying finite-size scaling for low modes

$\gamma_m^\star$ from eigenvalue mode number $\nu(\lambda)$

L. Del Debbio & R. Zwicky *Phys. Rev.* **D82**:014502 (2010)

In the chiral limit $\rho(\lambda) \sim \lambda^\alpha \quad \Longrightarrow \quad \nu(\lambda) = V \int_{-\lambda}^{\lambda} \rho(\omega) d\omega \sim V\lambda^{1+\alpha}$

Exponent α related to anomalous dimension: $\quad 1 + \gamma_m^\star = \dfrac{4}{1+\alpha}$



A. Patella PRD **86**:025006 (2012)
SU(2), $N_F = 2$ in adjoint
believed IR-conformal

$\gamma_m^\star = 0.371(20)$
for fit range $[0.091, 0.18]$

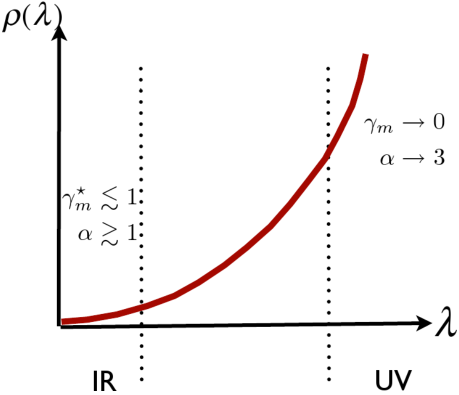
Inspired us to look at $\nu(\lambda)$
and improve technique

Scale-dependent $\gamma_m(\lambda)$ in **IR-conformal** systems

λ defines an energy scale;
fitting $\nu(\lambda) \propto \lambda^{1+\alpha(\lambda)}$ accesses $1 + \gamma_m(\lambda) = \frac{4}{1+\alpha(\lambda)}$ at that scale

For IR-conformal systems:
UV: Asymp. freedom $\Rightarrow \gamma_m(\lambda) \to 0$
corresponding to $\alpha(\lambda) \to 3$

IR: Fixed point $\Longrightarrow \gamma_m(\lambda) \to \gamma_m^\star$
$\gamma_m^\star$ scheme-independent,
expect $\gamma_m^\star \lesssim 1$



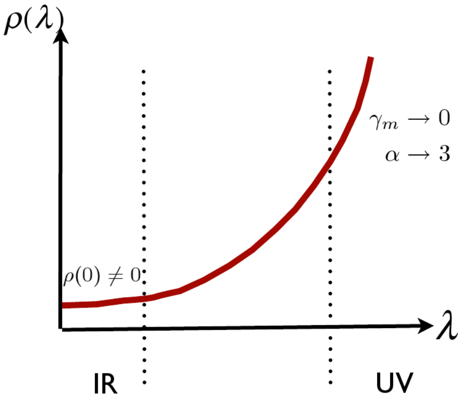
Form of $\rho(\lambda)$ changes from $\rho(\lambda) \propto \lambda^3$ in the UV to $\rho(\lambda) \propto \lambda^{\alpha_\star}$ in the IR

Scale-dependent $\gamma_m(\lambda)$ in **chirally broken** systems

λ defines an energy scale;
fitting $\nu(\lambda) \propto \lambda^{1+\alpha(\lambda)}$ accesses $1 + \gamma_m(\lambda) = \frac{4}{1+\alpha(\lambda)}$ at that scale

For chirally broken systems:
UV: Asymp. freedom $\Rightarrow \gamma_m(\lambda) \to 0$
corresponding to $\alpha(\lambda) \to 3$

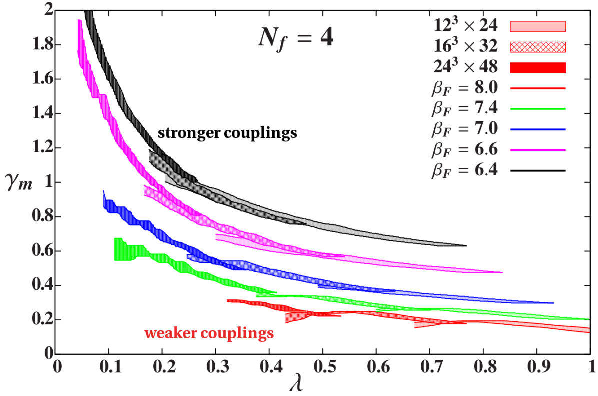
IR: $\langle\overline{\psi}\psi\rangle \propto \rho(0) > 0 \Longrightarrow \alpha(\lambda) \to 0$
would produce "$\gamma_m(\lambda) \to 3$"
but $\rho(\lambda)$ no longer $\sim \lambda^\alpha$



On the lattice we proceed by fitting $\nu(\lambda) \propto \lambda^{1+\alpha}$ in a limited range of λ

Results for QCD-like $N_F = 4$

Fit $\nu(\lambda) \propto \lambda^{1+\alpha}$ in a limited range of λ to find $1 + \gamma_m(\lambda) = \dfrac{4}{1+\alpha(\lambda)}$



1000 eigenvalues on each volume

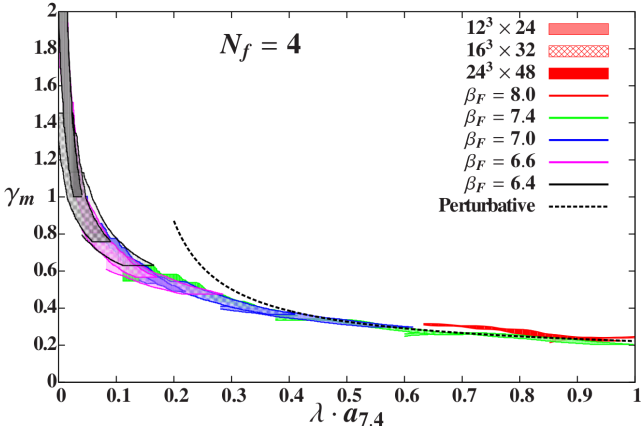
Fit ranges included in error bands

We see chiral symmetry breaking (χSB) on $24^3 \times 48$

Focus on overlapping regions where different volumes agree instead of studying finite-size scaling for low modes

Combine multiple couplings and volumes for $N_F = 4$

- Rescale $\lambda \to \left(\frac{a_{7.4}}{a}\right)^{1+\gamma_m} \lambda$ to plot in terms of single lattice spacing
- Relative lattice spacings from Wilson flow & MCRG matching
- Match to one-loop perturbation theory at $\lambda \cdot a_{7.4} = 0.8$

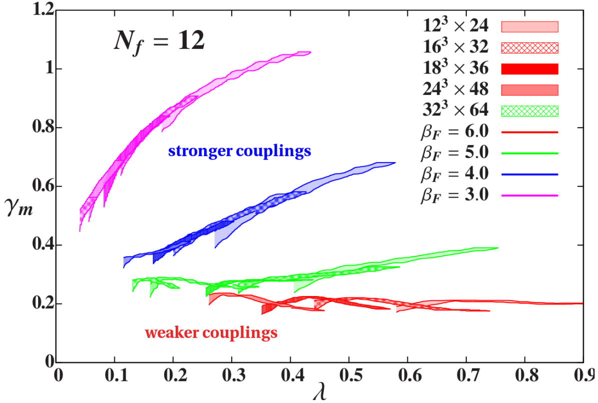


Universal curve from χSB to asymp. freedom

Strong test of method & control over systematics

$N_F = 12$ anomalous dimension indicates IR fixed point

Fit $\nu(\lambda) \propto \lambda^{1+\alpha}$ in a limited range of λ to find $1 + \gamma_m(\lambda) = \dfrac{4}{1 + \alpha(\lambda)}$



1000 eigenvalues on each volume

$m \leq 0.0025$, all have $\rho(0) = 0$

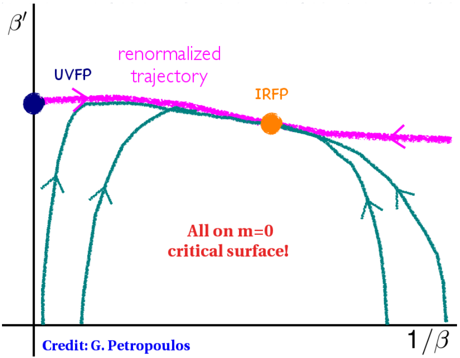
All converge to $\gamma_m^\star = 0.32(3)$ at $\lambda \to 0$ IRFP

- Strong dependence on irrelevant gauge coupling $\beta_F \sim 12/g^2$
- γ_m increasing with λ is a sort of "backward flow" at strong coupling

Wilson renormalization group (RG) approach

Fixed points & flows in ∞-dimensional space of couplings

Renormalized trajectory (RT) connects UV fixed point to IRFP, depends on real-space RG transformation (renormalization scheme)



RG transformation integrates out short-distance (UV) modes

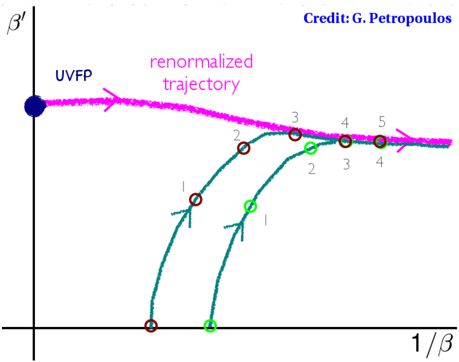
– Flow lines approach FPs/RT in irrelevant directions
– Flow lines go away from FPs in relevant directions

Scaling dimensions determine the speed of the flow

Directly following the flow lines is very difficult

Monte Carlo RG in bare parameter space

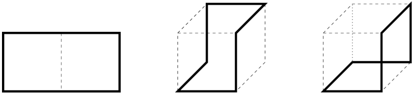
In Monte Carlo calculations we repeatedly block the lattice
Each block transformation probes a different renormalization scheme



Block L^4 lattice N_b times,
$\left(\frac{L}{2}\right)^4$ lattice $N_b - 1$ times

Match blocked actions (all $\langle\mathcal{O}\rangle$)
to find couplings β_L and $\beta_{L/2}$
for which $a(\beta_L) = a(\beta_{L/2})/2$

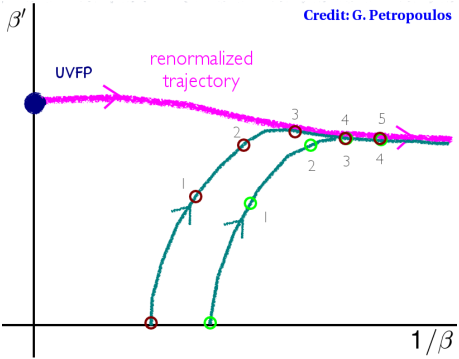
We match small Wilson loops

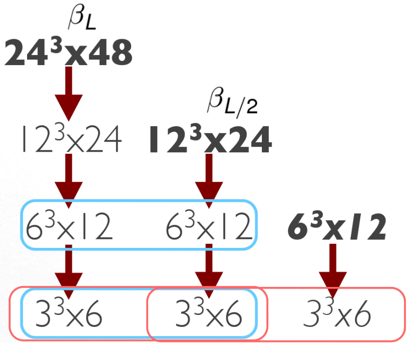


For $N_b \to \infty$, predicts bare step scaling function $s_b \equiv \beta_L - \beta_{L/2}$
(projecting the RG flow onto a single dimension)

The **need** to optimize MCRG

Only a few blocking steps are possible on a finite lattice
$\Longrightarrow$ MCRG can only function if it is optimized
so that lattice systems reach the RT with a single blocking step





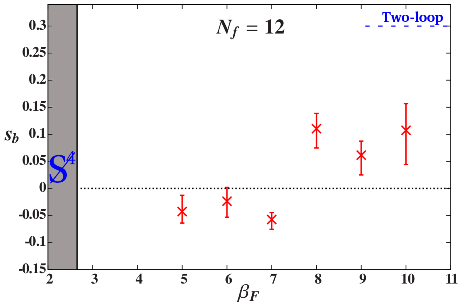
Optimized matching predicts bare step scaling function s_b for $N_b = 3$–4

Two ways to optimize MCRG (arXiv:1212.0053)

1) Traditional optimization

Move the renormalized trajectory to the lattice system
by tuning some parameter in the block (RG) transformation

Step scaling function represents a composite of multiple β functions
probing a different renormalization scheme at each coupling



Uncertainties dominated
by spread in predictions for s_b
from different operators

(Recall on $m = 0$ critical surface)

Two ways to optimize MCRG (arXiv:1212.0053)

1) Traditional optimization

Move the renormalized trajectory to the lattice system
by tuning some parameter in the block (RG) transformation

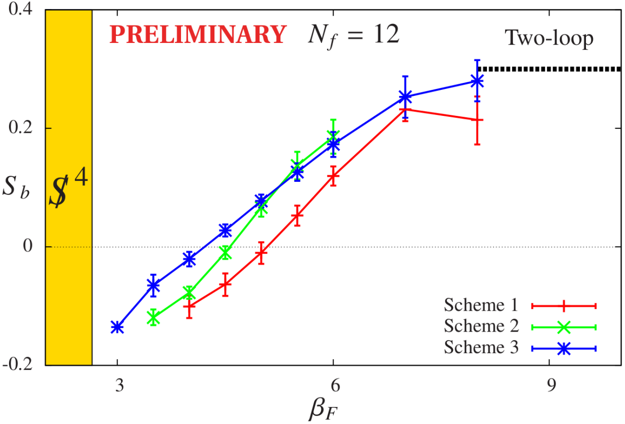
Step scaling function represents a composite of multiple β functions
probing a different renormalization scheme at each coupling

2) Improved optimization

Move the lattice system to the renormalized trajectory!
Fix the renormalization scheme (block transformation)
and tune the amount of “Wilson flow” applied to the lattices

The Wilson flow removes UV fluctuations
without changing the lattice spacing being matched

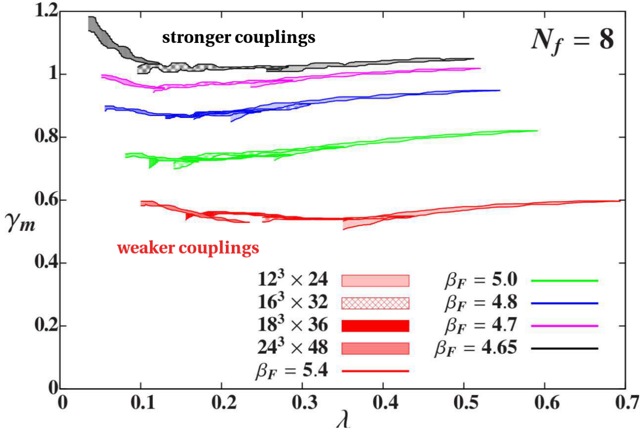
W-MCRG results for $N_F = 12$ (arXiv:1303.7129)



- Clear IR fixed point where $s_b = 0$
- IR fixed point moves for different renormalization schemes
- Slope related to $\gamma_g^\star$, should not change for different schemes (preliminary results give small negative $\gamma_g^\star \sim -0.2$)

$N_F = 8$ behaves very differently

Fit $\nu(\lambda) \propto \lambda^{1+\alpha}$ in a limited range of λ to find $1 + \gamma_m(\lambda) = \dfrac{4}{1+\alpha(\lambda)}$



1000 eigenvalues on each volume

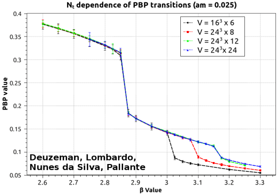
$m = 0$, all have $\rho(0) = 0$

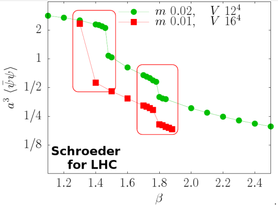
Hit lattice phase for $\beta_F \lesssim 4.65$ (when $\gamma_m \gtrsim 1$)

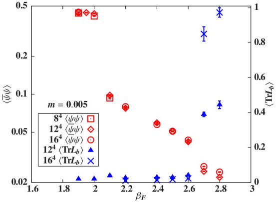
Unlike QCD, γ_m roughly independent of λ at fixed coupling β_F

Novel lattice phase present for both $N_F = 8$ and 12

Several groups find novel **intermediate** phase (for $N_F = 12$)



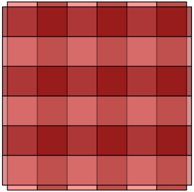




Seems to have no continuum limit $\longrightarrow$ "lattice phase"

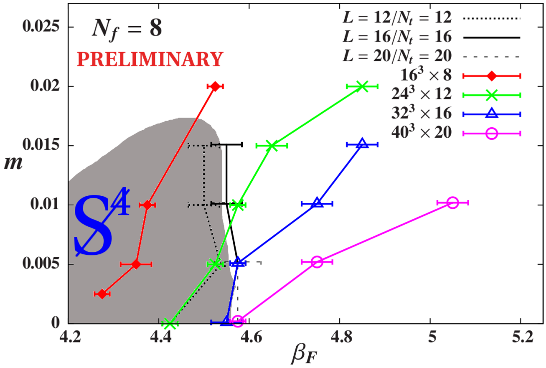
Bordered by phase-separating first-order transitions

Exhibits spontaneous single-site shift symmetry breaking ("$\cancel{S}^4$")



Finite-temperature transitions run into $\not{S}^4$ phase

Prevents clear observation of **spontaneous** chiral symmetry breaking



For large enough mass,
transitions act as in QCD

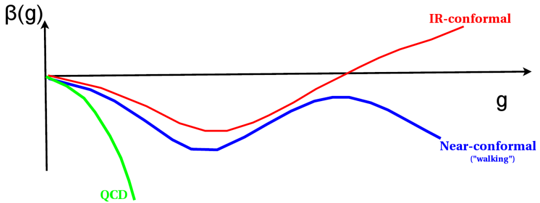
The crucial chiral limit
is still mysterious

Run $24^3 \times 48$ with $m = 0$
on both sides of transition
Run $32^3 \times 64$ with $m = 0$
down to $\beta_F = 4.7$

Changing the action changes the bare β_F at which $\not{S}^4$ phase appears
But the meson spectrum and $\gamma_m \sim 1$ from the Dirac eigenmodes
are the same where the $\not{S}^4$ phase appears for different actions

Recapitulation: New strong dynamics on the lattice

Near-conformal systems are very different from QCD



I hope these examples have helped
to illustrate some of what we've learned:

- Must synthesize different methods
- Most effective methods often differ from familiar lattice QCD techniques
- Should explore range of couplings, try to reach chiral limit

Thank you!

Thank you!

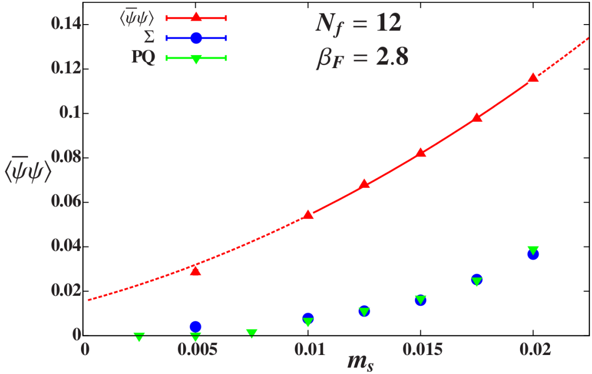
Collaborators

Anqi Cheng, Anna Hasenfratz, Gregory Petropoulos

Funding and computing resources

Backup: $\langle\overline{\psi}\psi\rangle$ in three ways

The chiral condensate directly probes chiral symmetry,
explicitly broken by non-zero fermion mass on lattice



"Direct" $\langle\overline{\psi}\psi\rangle$ uses $m_{valence} = m_{sea}$

Σ measured from $m_v = 0$ eigenmodes

Partially quenched with $m_v \to 0$

Mildest example of sensitivity to method
(same target quantity from same lattice gauge configurations)

Backup: Fermion mass dependence of $\langle\overline{\psi}\psi\rangle$

$\langle\overline{\psi}\psi\rangle$ depends on both valence mass m_v and sea mass m_s
For massless Dirac operator, $\rho(\lambda)$ depends only on m_s

$$\langle\overline{\psi}\psi\rangle_{m_v;\ m_s} = m_v \int \frac{\rho(\lambda, m_s)}{\lambda^2 + m_v^2} d\lambda + m_v^5 \int \frac{\rho(\lambda, m_s)}{\left(\lambda^2 + m_v^2\right) \lambda^4} d\lambda + \gamma_1 m_v \Lambda^2 + \gamma_2 m_v + \mathcal{O}\left(1/\Lambda\right),$$

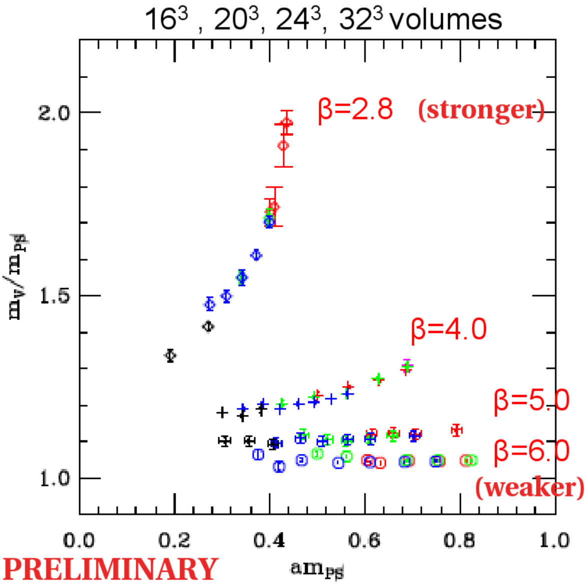
where $\Lambda = 1/a$ is the UV cutoff (Leutwyler & Smilga)

Quadratic UV divergence complicates chiral extrapolation

Can address with partially-quenched ($m_v \neq m_s$) measurements,
to extrapolate $m_v \to 0$ with fixed m_s
Can also remove m_v dependence via $\Sigma_{m_s} = \pi\rho(0, m_s) = \langle\overline{\psi}\psi\rangle_{m_v=0;\ m_s}$

It is a good check that these two approaches agree!

Backup: dependence on coupling



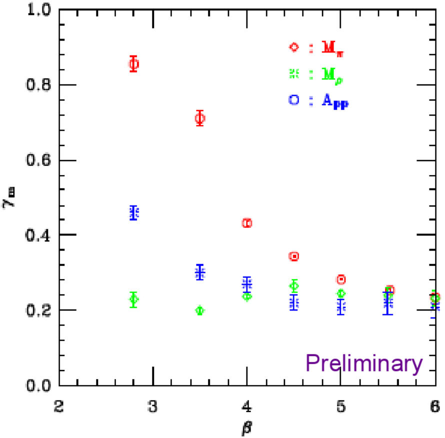
M_V / M_P as function of relevant parameter (fermion mass $\rightsquigarrow aM_P$)

Strong dependence on $\beta_F \sim 12/g^2$

Should all converge in $a \to 0$ continuum limit

Backup: $\gamma_m^\star$ from finite-size scaling

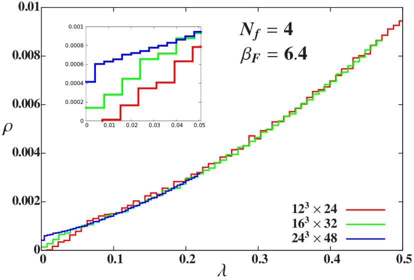
Conformality $\Rightarrow$ hadron masses depend on scaling variable $Lm^{1/(1+\gamma_m^\star)}$
Predict $\gamma_m^\star$ from collapse of data with different m and L onto a single scaling curve

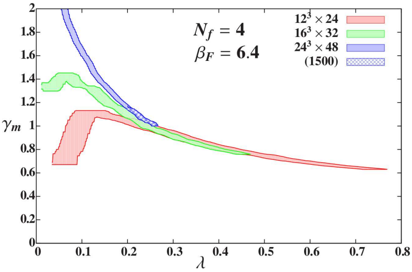


A_{pp} is pseudoscalar amplitude, seems more stable than F_P

Clear dependence on nearly-marginal gauge coupling β_F

Backup: Finite-volume effects in $\gamma_m(\lambda)$ from $\nu(\lambda)$





- As discussed above, $\langle\overline{\psi}\psi\rangle \propto \rho(\lambda \to 0) > 0 \Longrightarrow \gamma_m \nearrow 3$,
  but scaling $\rho(\lambda) \propto \lambda^\alpha$ breaks down in this situation
- Finite-volume effects can produce a "gap" with $\rho(0) = 0$
  This is a different breakdown of the scaling, leading to $\gamma_m \searrow 0$
- Both of these effects are unphysical;
  we remove the finite-volume transients from most γ_m plots

Backup: A bit about the Wilson flow

Evolution of gauge links $U(x,\mu)$ in a "flow time" t:

$$\frac{d}{dt}V_t(x,\mu) = -g_0^2\left[\frac{\delta}{\delta V_t(x,\mu)}S_W(V_t)\right]V_t(x,\mu),$$

where $V_{t=0}(x,\mu) = U(x,\mu)$ and S_W is the Wilson gauge action

$$S_W(U) = \beta\sum_{\{P\}}\mathrm{Re}\mathrm{Tr}\left[1 - P(U)\right]$$

$$P_{x,\mu\nu}(U) = U_{x,\mu}U_{x+\widehat{\mu},\nu}U^\dagger_{x+\widehat{\nu},\mu}U^\dagger_{x,\nu}$$

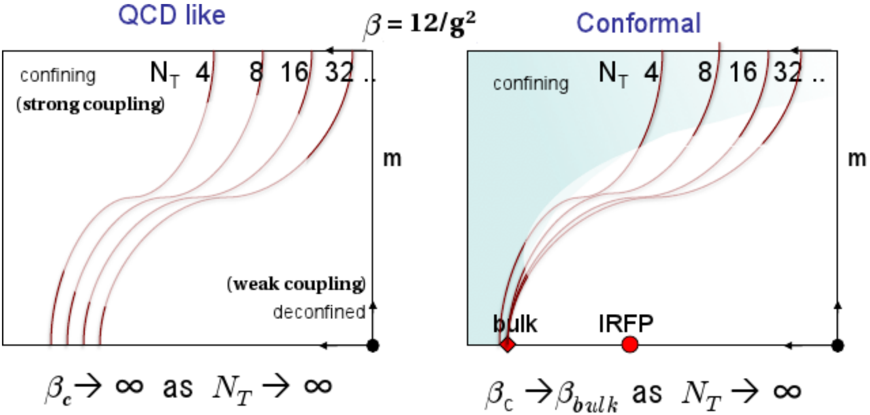
Solution:

$$V_t(x,\mu) = \exp\left[-tg_0^2\frac{\delta}{\delta U(x,\mu)}S_W(U)\right]U(x,\mu)$$

$\Longrightarrow$ numerical integration of **infinitesimal stout smearing steps**

Backup: Qualitative expectations for phase diagram

Fermion mass vs. gauge coupling; critical surface is $m = 0$ chiral limit



Hope for clear distinction between QCD-like and conformal cases from scaling $\Delta\beta$ of finite-temperature transitions as N_T increases

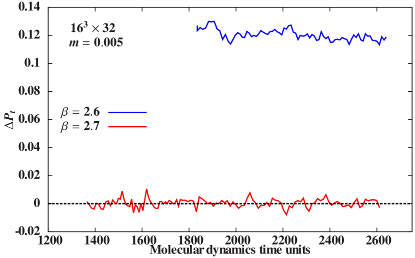
Backup: Order parameters for $\cancel{S}^4$ phase

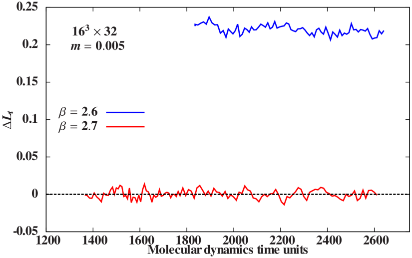
Staggered lattice actions possess exact single-site shift symmetry which is spontaneously broken in the intermediate phase

Order parameters (any or all μ)

$$\Delta P_\mu = \left\langle \mathrm{ReTr}\,\Box_n - \mathrm{ReTr}\,\Box_{n+\mu} \right\rangle_{n_\mu \text{ even}}$$

$$\Delta L_\mu = \left\langle \alpha_{\mu,n} \overline{\chi}_n U_{\mu,n} \chi_{n+\mu} - \alpha_{\mu,n+\mu} \overline{\chi}_{n+\mu} U_{\mu,n+\mu} \chi_{n+2\mu} \right\rangle_{n_\mu \text{ even}}$$

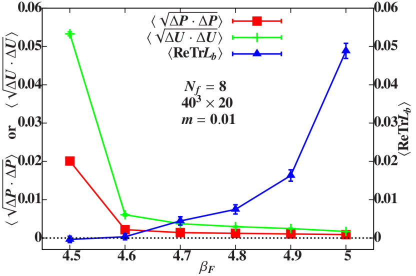


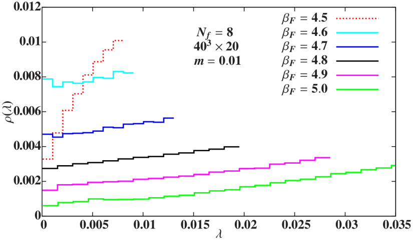


$\cancel{S}^4$ has never been seen before, but is clear in our data

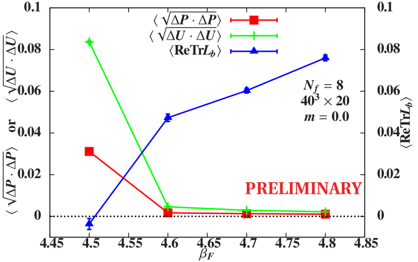
Backup: Sample $N_F = 8$ transition signals

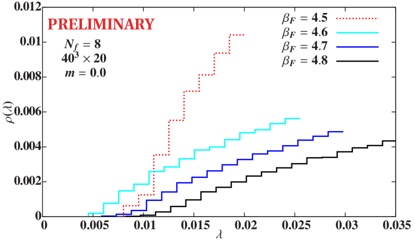
m = 0.01





m = 0





For $m = 0.01$, clear χSB with $\rho(0) > 0$ between transitions