Strong Dynamics and Lattice Gauge Theory
— — — Going Beyond QCD — — —

David Schaich (Syracuse)

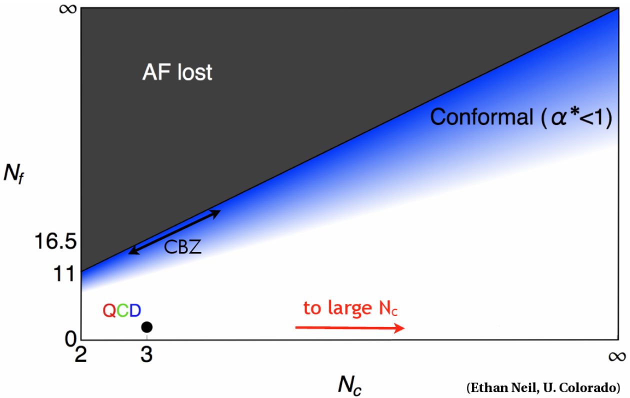
Purdue High Energy Theory Seminar, 7 April 2015

arXiv:1401.0195, arXiv:1404.0984, arXiv:1410.5886 & more to come
with Anqi Cheng, Anna Hasenfratz, Greg Petropoulos and Aarti Veernala

Context: Going Beyond QCD

Focus on non-supersymmetric SU(3) gauge theories
with N_F massless fundamental fermions

Much is still mysterious despite a few points of reference:

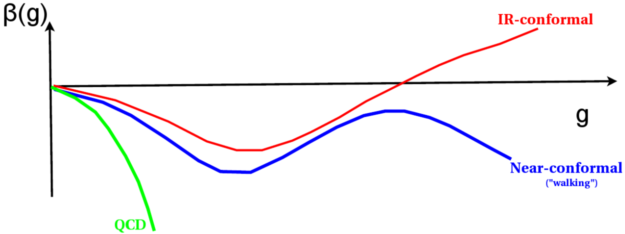


Context: Going Beyond QCD

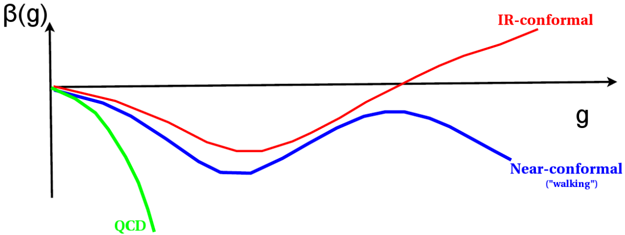
Focus on non-supersymmetric SU(3) gauge theories with N_F massless fundamental fermions

Much is still mysterious despite a few points of reference

Expect dramatically different dynamics as N_F increases:



Why strong dynamics beyond QCD



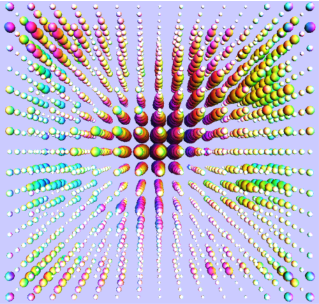
1. Theoretical questions:
   What is the range of possible phenomena in strongly coupled systems?
   What are the most effective methods to study such systems?

2. Phenomenological applications:
   What models of new strong dynamics are ruled out after LHC Run 1?
   What models remain viable and what do they predict for Run 2?

Why lattice gauge theory

Lattice discretization provides non-perturbative,
gauge-invariant regularization of vectorlike gauge theories

Amenable to numerical analysis
$\longrightarrow$ complementary approach to study strongly coupled field theories



Evaluate observables from functional integral
via importance sampling Monte Carlo

$$\langle \mathcal{O} \rangle = \frac{\int \mathcal{D}U \;\; \mathcal{O}(U) \;\; e^{-S[U]}}{\int \mathcal{D}U \;\; e^{-S[U]}}$$

U: four-dimensional field configurations
S: action giving probability distribution e^{-S}

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Proven success for QCD; more challenges in near-conformal contexts:

- Non-zero lattice spacing and finite volume violate conformality
- Slowly running gauge coupling complicates traditional lattice QCD analyses
- Harder to check against experiment and other techniques

—Must compare multiple investigations to form a consistent picture
—Standard lattice QCD techniques likely no longer optimal

Outline: Lattice methods to go beyond QCD

Focus on $N_F = 12$ as an IR-conformal case study

1. **Running coupling** indicates IR fixed point (zero in β function)

2. **Dirac operator eigenmode scaling**
   predicts mass anomalous dimension $\gamma_m^\star = 0.235(27)$

3. **Finite-size scaling** predicts $\gamma_m^\star = 0.235(15)$ and $\gamma_g^\star \approx -0.5$

Not today: Thermal transitions
also behave as expected for an IR-conformal system

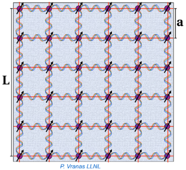
For each method we had to develop novel improvements
that can also be applied to lattice studies of other systems

Corresponding studies of $N_F = 8$ produce more "interesting" results
No conclusive demonstration of spontaneous χSB or IR conformality

Non-perturbative β function (arXiv:1404.0984 & arXiv:1410.5886)

1. Define a scale-dependent running coupling $g_c^2(L; a)$ for lattice spacing "a" and lattice volume L^4

   Lattice spacing related to bare input coupling $\beta_F \equiv 12/g_0^2$ at UV cutoff a^{-1}



2. The discrete β function integrates over scale change $L \longrightarrow sL$

$$\beta_s(g_c^2; L) = \frac{g_c^2(sL; a) - g_c^2(L; a)}{\log(s^2)}$$

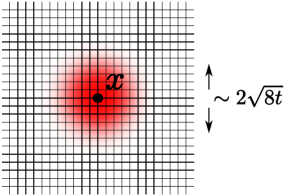
3. Extrapolate $(a/L)^2 \to 0$ at fixed g_c^2 to obtain continuum $\beta_s(g_c^2)$

The gradient flow running coupling

1. Define a scale-dependent running coupling $g_c^2(L; a)$

The Yang–Mills gradient flow integrates an infinitesimal smoothing operation

Local observables measured after "flow time" t depend on original fields within $r \simeq \sqrt{8t}$

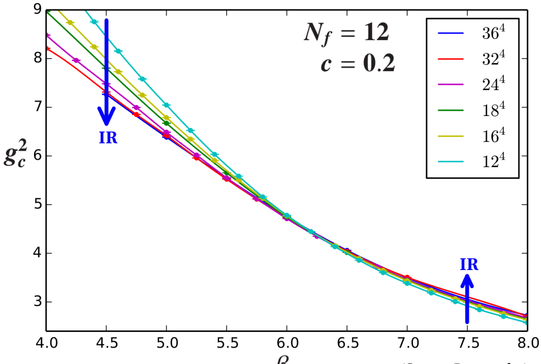


Perturbatively $g^2_{\overline{\mathrm{MS}}}(\mu) \propto t^2 E(t)$ with $\mu = 1/\sqrt{8t}$
where $E = -\frac{1}{2}\mathrm{Tr}\left[G_{\mu\nu}G^{\mu\nu}\right]$ is the energy density

Define running coupling $g_c^2(L; a)$ by fixing $c = L/\sqrt{8t}$

Running coupling data for $N_F = 12$

2. Evolution of g_c^2 upon scale change $L \longrightarrow sL$ produces discrete β_s



(Gregory Petropoulos)

Three possible s:

$s = 2$: $L = 12 \to 24$
$16 \to 32$; $18 \to 36$

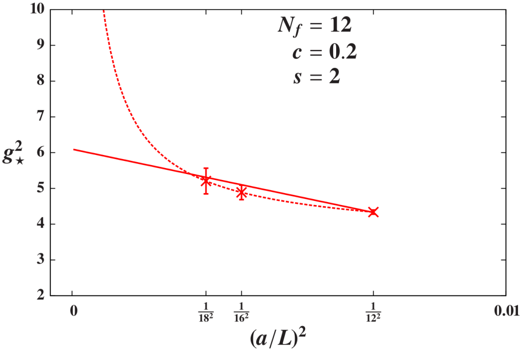
$s = 3/2$: $L = 12 \to 18$
$16 \to 24$; $24 \to 36$

$s = 4/3$: $L = 12 \to 16$
$18 \to 24$; $24 \to 32$

$\beta_s(g_c^2; L)$ for each s and L vanishes around $4 \lesssim g_c^2 \lesssim 5$
Does the IR fixed point remain in the $(a/L)^2 \to 0$ continuum limit?

Continuum extrapolation of finite-volume fixed points

3. Extrapolate $(a/L)^2 \to 0$ to obtain continuum result
   Consider $g_\star^2(L)$ for which $\beta_s(g_\star^2; L) = 0$



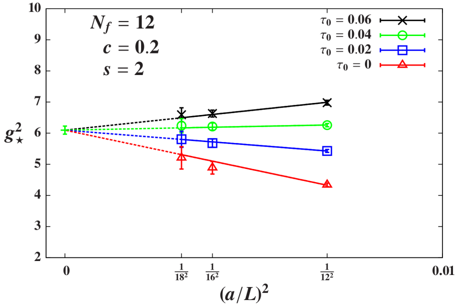
How to distinguish between two possible interpretations?
1) Lattice cutoff effects lead to simple linear extrapolation
2) Conformal fixed point is not a property of the continuum theory

New development: Improved continuum extrapolation

How to distinguish between two possible interpretations?

Recall that the gradient flow involves a smoothing operation
that removes UV lattice artifacts

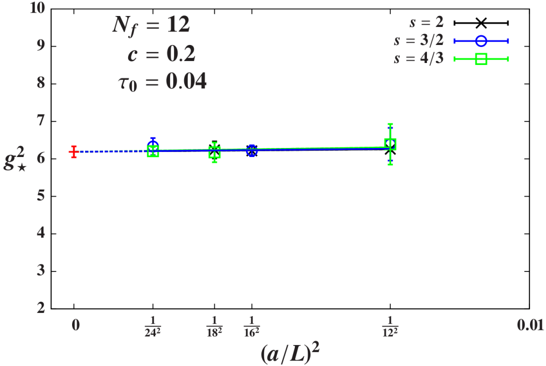
$\Longrightarrow$ Reduce UV fluctuations with pre-smoothing before beginning flow
$\Longrightarrow$ $\widetilde{g}_c^2 \propto t^2 E(t + \tau_0 a^2)$ with $\tau_0 a^2 \ll t$ vanishing in continuum



We find a conformal IR fixed point for $N_F = 12$

Further tests of $N_F = 12$ fixed point

With fixed improvement $\tau_0 = 0.04$ we find the same fixed point for all three discrete β_s with scale change $s = 2,\ 3/2$ and $4/3$



Continuum $g_\star^2$ is renormalization scheme dependent

$g_\star^2 = 6.18(20)$ for $c = 0.2$ scheme $\qquad$ ($g_\star^2 = 5.9$ for four-loop $\overline{\text{MS}}$)

$g_\star^2 = 6.84(32)$ for $c = 0.25$ scheme

$g_\star^2 = 7.13(48)$ for $c = 0.3$ scheme

Checkpoint

Given a conformal IR fixed point for 12-flavor SU(3) gauge theory
what is the corresponding spectrum of anomalous dimensions?

1. **Running coupling** indicates IR fixed point (zero in β function)
   New development: Improved continuum extrapolation
   (arXiv:1404.0984 & arXiv:1410.5886)

2. **Dirac operator eigenmode scaling**
   predicts mass anomalous dimension $\gamma_m^\star = 0.235(27)$

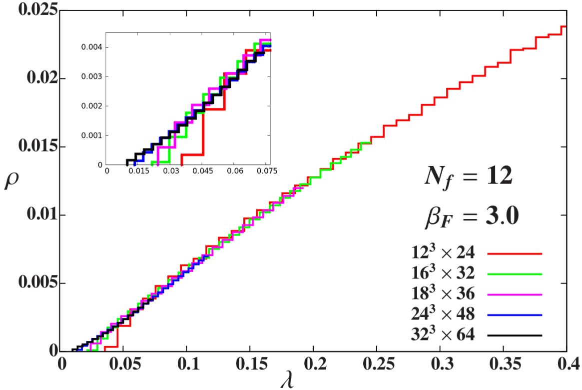
3. **Finite-size scaling** predicts $\gamma_m^\star = 0.235(15)$ and $\gamma_g^\star \approx -0.5$

Dirac operator eigenvalues (arXiv:1301.1355 & arXiv:1311.1287)

$\mathcal{L} \supset \overline{\Psi}\left(\not{D} + m\right)\Psi$ where $\not{D}$ is the **massless** Dirac operator

m has scaling dimension $y_m = 1 + \gamma_m^\star \Longrightarrow \dim\left[\overline{\Psi}\Psi\right] = 3 - \gamma_m^\star$

Spectral density $\rho(\lambda)$ is histogram of eigenvalues λ



$\langle\overline{\psi}\psi\rangle \propto \rho(\lambda \to 0)$
in ∞-volume chiral limit
(Banks & Casher)

Finite-volume effects
visible at small λ

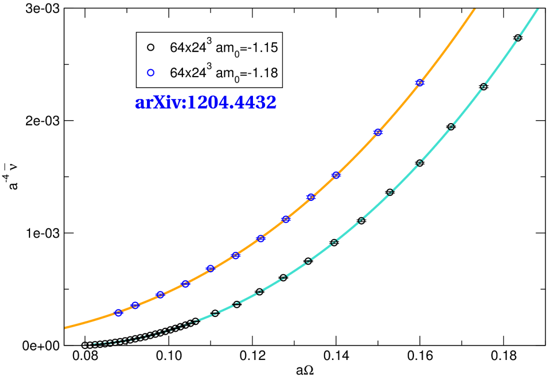
Integral is **mode number**

$$\nu(\lambda) = 2V \int_0^\lambda \rho(\omega)d\omega$$

$\gamma_m^\star$ from eigenvalue mode number $\nu(\lambda)$

At a conformal fixed point $\rho(\lambda) \propto \lambda^\alpha \Longrightarrow \nu(\lambda) \propto V \int \rho(\omega) d\omega \propto V\lambda^{1+\alpha}$

Renormalization group relates $1 + \gamma_m^\star = \dfrac{4}{1+\alpha}$ (Del Debbio & Zwicky)



First test: (Patella)
SU(2) gauge theory,
$N_F = 2$ adjoint fermions

$\gamma_m^\star = 0.371(20)$
for fit range $[0.091, 0.18]$

System is not actually at a conformal fixed point...

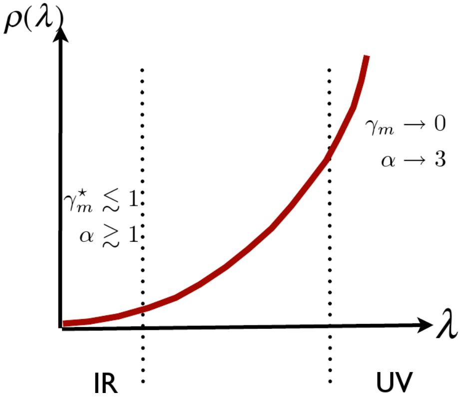
New development: Scale-dependent $\gamma_{\text{eff}}(\lambda)$

λ defines an energy scale $\Longrightarrow$ fitting $\nu(\lambda)$ predicts
effective anomalous dimension $\gamma_{\text{eff}}(\lambda)$ at that scale

For IR-conformal systems

UV: Asymp. freedom $\Rightarrow \gamma_{\text{eff}}(\lambda) \to 0$
corresponding to $\alpha(\lambda) \to 3$

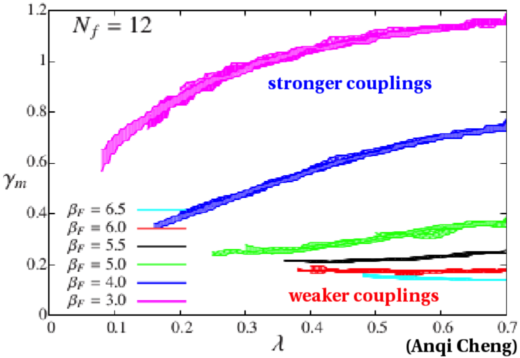
IR: Fixed point $\Longrightarrow \gamma_{\text{eff}}(\lambda) \to \gamma_m^\star$
$\gamma_m^\star$ scheme-independent
typically expect $\gamma_m^\star \lesssim 1$



Ideally monitor evolution from perturbative UV to strongly coupled IR

$\gamma_{\text{eff}}(\lambda)$ from eigenmodes for $N_F = 12$

Fit $\nu(\lambda) \propto \lambda^{1+\alpha}$ in a limited range of λ to find $1 + \gamma_{\text{eff}}(\lambda) = \dfrac{4}{1+\alpha(\lambda)}$



$\nu(\lambda)$ computed stochastically

Fit ranges included in error bands

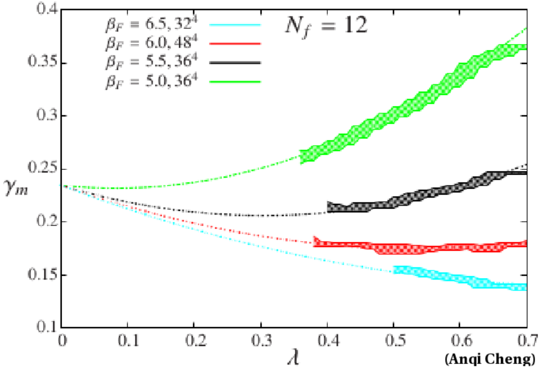
Only retain results free from finite-volume effects

All systems have $m \leq 0.0025$ and $\rho(0) = 0$

- Strong dependence on irrelevant gauge coupling $\beta_F \simeq 12/g_0^2$
- γ_{eff} increasing with λ is a sort of "backward flow" at strong coupling

$\gamma_m^\star(\lambda)$ from eigenmodes for $N_F = 12$

Extrapolate $\lim_{\lambda\to 0} \gamma_{\text{eff}}(\lambda)$ to find $\gamma_m^\star$ at conformal fixed point in IR limit



Focus on largest volumes for couplings closest to $g_\star^2$ (in this scheme)

Joint quadratic extrapolation predicts $\gamma_m^\star = 0.235(27)$

Uncertainty dominated by $\lambda \to 0$ extrapolation

A single fit for some range of $\lambda > 0$ would give a precise result but generally not $\gamma_m^\star$ at the $\lambda \to 0$ IR fixed point

Checkpoint

1. **Running coupling** indicates IR fixed point (zero in β function)
   New development: Improved continuum extrapolation
   (arXiv:1404.0984 & arXiv:1410.5886)

2. **Dirac operator eigenmode scaling**
   predicts mass anomalous dimension $\gamma_m^\star = 0.235(27)$
   New development: Scale-dependent $\gamma_{\text{eff}}(\lambda)$ and IR extrapolation
   (arXiv:1301.1355, arXiv:1311.1287 & work in progress)

3. **Finite-size scaling** predicts $\gamma_m^\star = 0.235(15)$ and $\gamma_g^\star \approx -0.5$

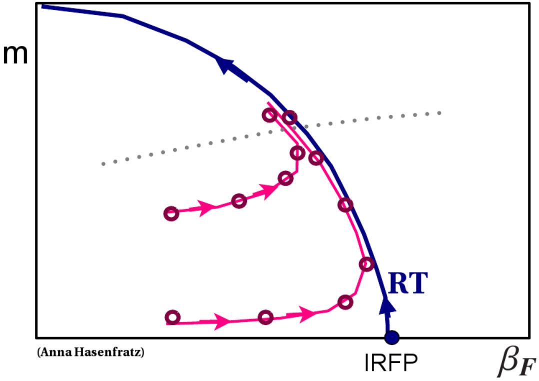
Finite-size scaling

(arXiv:1401.0195)

Wilson RG picture of finite-size scaling

Fermion mass m is relevant coupling; gauge coupling β_F is irrelevant

End up at same point on renormalized trajectory (RT)
by increasing m while decreasing amount of RG flow (L)



Universal flow along RT

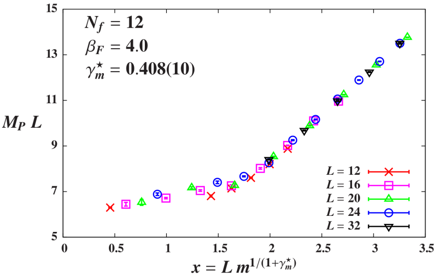
Correlation functions
depend on scaling variable

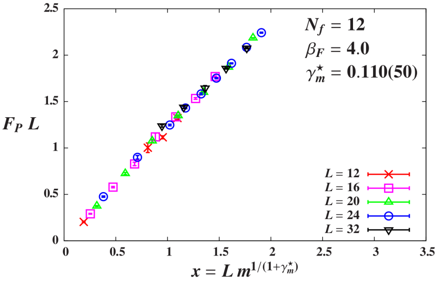
$$x \equiv L\, m^{1/(1+\gamma_m^\star)}$$

Assuming RG flow
quickly reaches RT

Finite-size scaling for $N_F = 12$

Correlation lengths only depend on scaling variable $x \equiv L\, m^{1/(1+\gamma_m^\star)}$
$\Longrightarrow$ Predict $\gamma_m^\star$ by optimizing "curve collapse" where $\xi_H^{-1} L = f_H(x)$



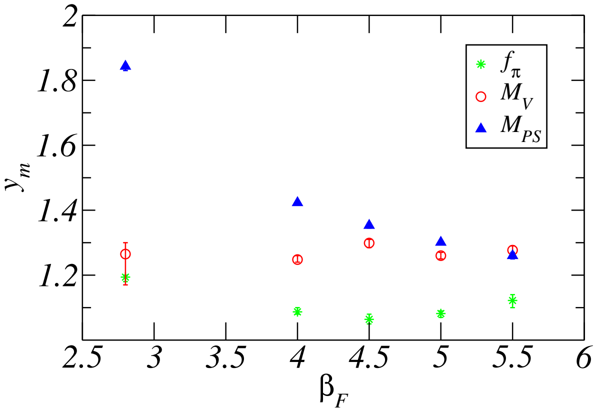


Despite reasonable curve collapse
different observables prefer very different $\gamma_m^\star$

Non-universal $\gamma_m^\star$ is inconsistent with conformal scaling

Inconsistent finite-size scaling results for $N_F = 12$

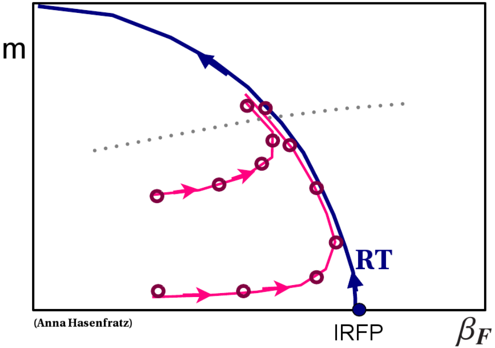
Different observables and (irrelevant) β_F prefer very different $\gamma_m^\star$



How to distinguish between two possible interpretations?
1) Near-marginal gauge coupling $\Longrightarrow$ significant corrections to scaling
2) The theory does not have a conformal fixed point

New development: Approximate corrections to scaling

How to distinguish between two possible interpretations?



If gauge coupling runs slowly
RG flow may not reach RT

$\Longrightarrow$ No universal behavior

Leading correction to scaling:

$$\xi_H^{-1} L = f_H(x, g m^{\omega})$$

where $\omega = -\gamma_g^\star / (1 + \gamma_m^\star)$

Two-loop $\overline{\mathrm{MS}}$: small $\omega \approx 0.2$

Not practical to extract both $\gamma_m^\star$ and $\gamma_g^\star$ from curve collapse

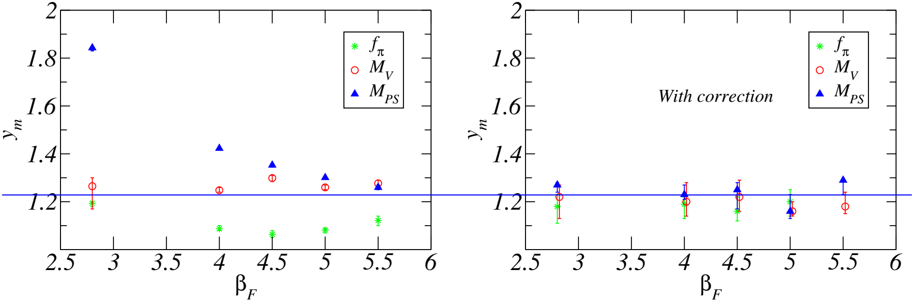
Instead approximate $f_H(x, g m^{\omega}) \approx f_H(x) \left[1 + c_g m^{\omega}\right]$

Consistent corrected scaling for $N_F = 12$

With approximate corrections to scaling $\xi_H^{-1} L = f_H(x) \left[1 + c_g m^\omega\right]$

different observables and β_F predict consistent $\gamma_m^\star$

Quality of curve collapse also improves (not surprising)



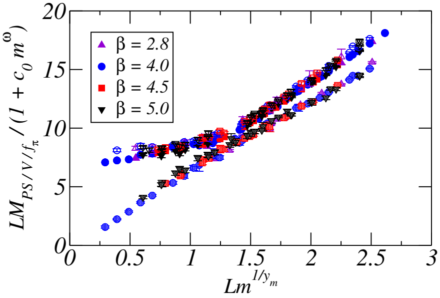
Carry out combined analyses of multiple data sets

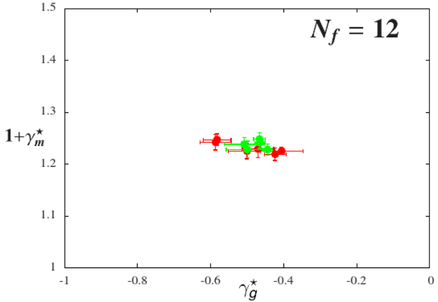
to better constrain $\gamma_m^\star$ and $\gamma_g^\star$...

Consistent corrected scaling for $N_F = 12$

With approximate corrections to scaling $\xi_H^{-1} L = f_H(x) \left[1 + c_g m^\omega\right]$ different observables and β_F predict consistent $\gamma_m^\star$

Combined analyses of multiple data sets better constrain $\gamma_m^\star$ and $\gamma_g^\star$





Result from green points: $\gamma_m^\star = 0.235(15)$ and $\gamma_g^\star \approx -0.5$

Recapitulation: Going beyond QCD on the lattice

Lattice studies of strongly coupled systems beyond QCD are theoretically interesting and phenomenologically important

Multiple independent methods produce a consistent picture of IR conformality for 12-flavor SU(3) gauge theory

1. **Running coupling** indicates IR fixed point (zero in β function)
   New development: Improved continuum extrapolation
   (arXiv:1404.0984 & arXiv:1410.5886)

2. **Dirac operator eigenmode scaling**
   predicts mass anomalous dimension $\gamma_m^\star = 0.235(27)$
   New development: Scale-dependent $\gamma_{\text{eff}}(\lambda)$ and IR extrapolation
   (arXiv:1301.1355, arXiv:1311.1287 & work in progress)

3. **Finite-size scaling** predicts $\gamma_m^\star = 0.235(15)$ and $\gamma_g^\star \approx -0.5$
   New development: Corrections to scaling from nearly marginal g
   (arXiv:1401.0195)

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Methods can be applied to other systems, further tested and refined
(e.g., studying lattice universality of anomalous dimensions)

Thank you!

Thank you!

Collaborators

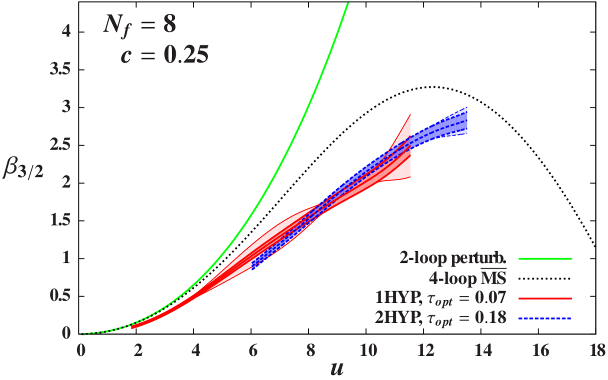
Anqi Cheng, Anna Hasenfratz, Yuzhi Liu, Gregory Petropoulos, Aarti Veernala

Funding and computing resources

Supplement: Discrete β function for $N_F = 8$

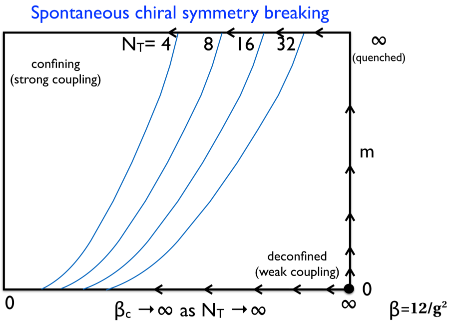
Continuum extrapolated $\beta_s(g_c^2)$ with scale change $s = 3/2$
increases monotonically for $g_c^2 \lesssim 14$

Although β_s is even smaller than IR-conformal four-loop $\overline{\mathrm{MS}}$ prediction
any IR fixed point must be at stronger coupling



Supplement: Thermal transitions to identify SχSB

May distinguish between chirally broken and IR-conformal cases from scaling $\Delta\beta_F$ of finite-temperature transitions as N_T increases



Plots show transitions and some RG flow lines in space of fermion mass m and gauge coupling β_F

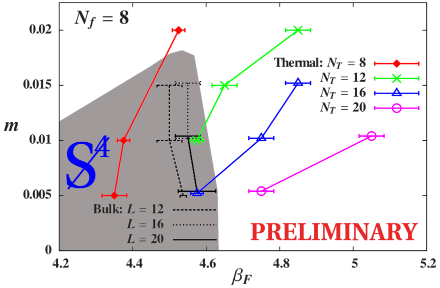
Contrast only clear near critical surface at $m = 0$

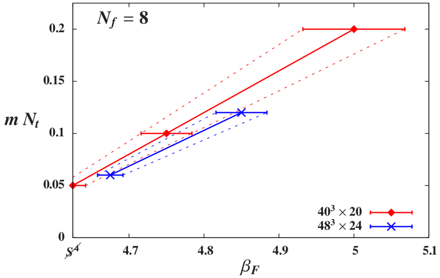
Supplement: Search for $N_F = 8$ spontaneous χSB

QCD-like scaling at large-m does not persist as m decreases

Thermal transitions run into lattice phase before reaching chiral limit

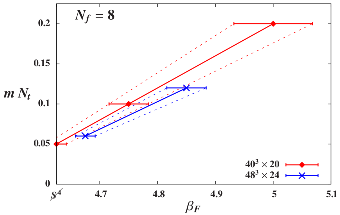
Even large lattice volumes up to $48^3 \times 24$ are insufficient to establish spontaneous chiral symmetry breaking

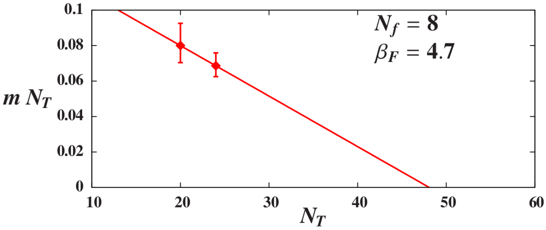




Supplement: Search for $N_F = 8$ spontaneous χSB

Extrapolating $m \to 0$ at fixed $\beta_F = 4.7$ suggests $N_T \gtrsim 48$ needed to observe spontaneous chiral symmetry breaking

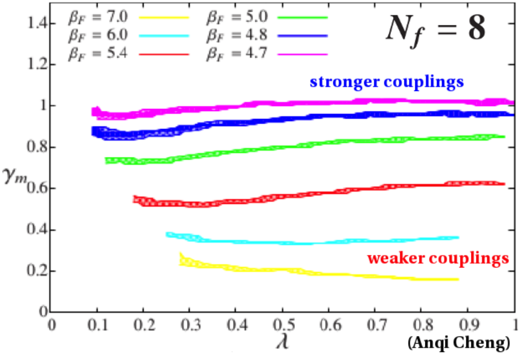




This behavior is extremely different from QCD but not sufficient to establish IR conformality

Supplement: $\gamma_{\text{eff}}(\lambda)$ from eigenmodes for $N_F = 8$

Fit $\nu(\lambda) \propto \lambda^{1+\alpha}$ in a limited range of λ to find $1 + \gamma_{\text{eff}}(\lambda) = \dfrac{4}{1+\alpha(\lambda)}$



$\nu(\lambda)$ computed stochastically

Fit ranges included in error bands

Only retain regions where all volumes overlap

All systems have $m = 0$ and $\rho(0) = 0$

Behaves very differently compared to $N_F = 12$ (and compared to QCD)

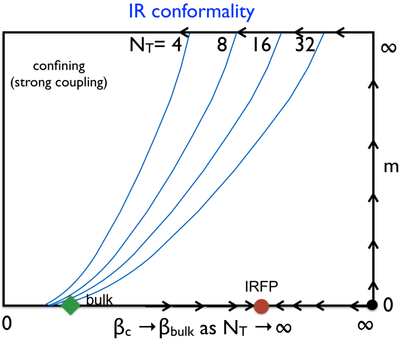
γ_{eff} appears to run very slowly across a wide range of scales

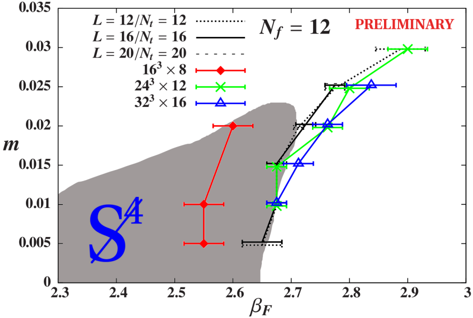
Backup: Thermal transitions for $N_F = 12$

Behave as expected for an IR-conformal system

Accumulate at zero-temperature bulk transition for small enough m

$N_T = 12$ and $N_T = 16$ transitions are indistinguishable





Backup: A bit about the Wilson flow

Evolution of gauge links $U(x,\mu)$ in a "flow time" t:

$$\frac{d}{dt}V_t(x,\mu) = -g_0^2\left[\frac{\delta}{\delta V_t(x,\mu)}S_W(V_t)\right]V_t(x,\mu),$$

where $V_{t=0}(x,\mu) = U(x,\mu)$ and S_W is the Wilson gauge action

$$S_W(U) = \frac{2N}{g_0^2}\sum_{\{P\}}\mathrm{ReTr}\left[1 - P(U)\right]$$

$$P_{x,\mu\nu}(U) = U_{x,\mu}U_{x+\widehat{\mu},\nu}U^\dagger_{x+\widehat{\nu},\mu}U^\dagger_{x,\nu}$$

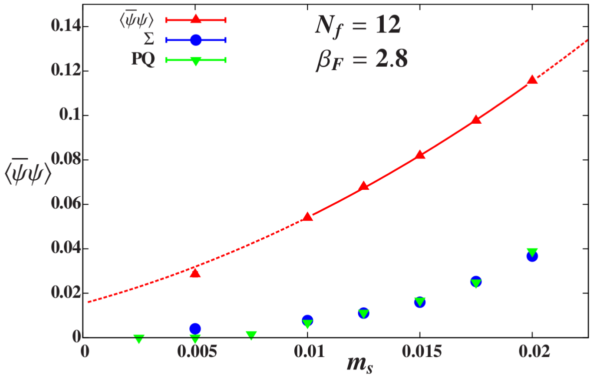
Solution:

$$V_t(x,\mu) = \exp\left[-tg_0^2\frac{\delta}{\delta U(x,\mu)}S_W(U)\right]U(x,\mu)$$

$\Longrightarrow$ numerical integration of **infinitesimal stout smearing steps**

Backup: $\langle\overline{\psi}\psi\rangle$ in three ways for $N_F = 12$

The chiral condensate directly probes chiral symmetry,
but is explicitly broken by non-zero fermion mass on lattice



"Direct" $\langle\overline{\psi}\psi\rangle$ uses $m_{valence} = m_{sea}$

Σ measured from $m_v = 0$ eigenmodes

Partially quenched with $m_v \to 0$

Minimal example of sensitivity to method:
Same quantity extracted from same gauge field configurations

Backup: Fermion mass dependence of $\langle\overline{\psi}\psi\rangle$

$\langle\overline{\psi}\psi\rangle$ depends on both valence mass m_v and sea mass m_s

For massless Dirac operator, $\rho(\lambda)$ depends only on m_s

$$\langle\overline{\psi}\psi\rangle_{m_v;\ m_s} = m_v \int \frac{\rho(\lambda, m_s)}{\lambda^2 + m_v^2} d\lambda + m_v^5 \int \frac{\rho(\lambda, m_s)}{\left(\lambda^2 + m_v^2\right)\lambda^4} d\lambda + \gamma_1 m_v \Lambda^2 + \gamma_2 m_v + \mathcal{O}\left(1/\Lambda\right)$$

where $\Lambda = a^{-1}$ is the UV cutoff (Leutwyler & Smilga)

Quadratic UV divergence complicates chiral extrapolation

Can address with partially-quenched ($m_v \neq m_s$) measurements, to extrapolate $m_v \to 0$ with fixed m_s

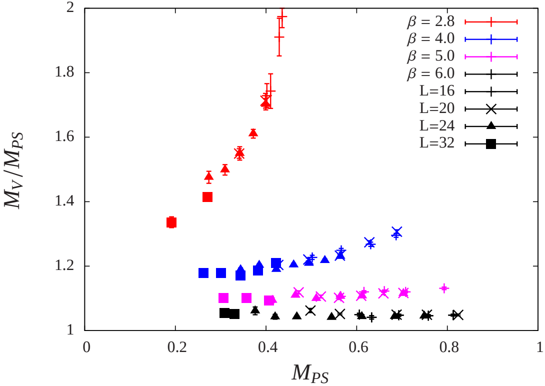
Can also remove m_v dependence via $\Sigma_{m_s} = \pi\rho(0, m_s) = \langle\overline{\psi}\psi\rangle_{m_v=0;\ m_s}$

It is a good check that these two approaches agree!

Backup: Dependence on gauge coupling for $N_F = 12$

Look at simple ratio M_V/M_P
plotted against relevant parameter (fermion mass $m \rightsquigarrow M_P$)

Even though β_F is formally irrelevant
it has significant effects for $M_P \gtrsim 0.2a^{-1}$



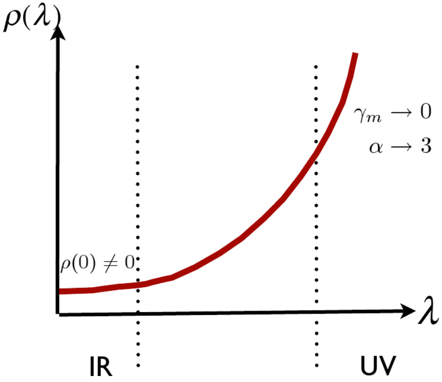
Backup: $\gamma_{\text{eff}}(\lambda)$ for chirally broken systems

λ defines an energy scale

Fitting $\nu(\lambda) \propto \lambda^{1+\alpha(\lambda)}$ accesses $1 + \gamma_{\text{eff}}(\lambda) = \frac{4}{1+\alpha(\lambda)}$ at that scale

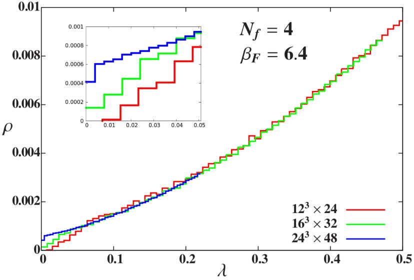
For chirally broken systems
UV: Asymp. freedom $\Rightarrow \gamma_{\text{eff}}(\lambda) \to 0$
corresponding to $\alpha(\lambda) \to 3$

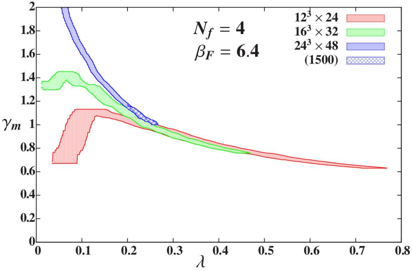
IR: $\langle\overline{\psi}\psi\rangle \propto \rho(0) > 0 \implies \alpha(\lambda) \to 0$
would produce $\gamma_{\text{eff}}(\lambda) \to 3$
but $\rho(\lambda)$ no longer $\propto \lambda^{\alpha}$



Ideally monitor evolution from perturbative UV to chirally broken IR

Backup: Finite-volume effects in $\gamma_{\text{eff}}(\lambda)$ from $\nu(\lambda)$

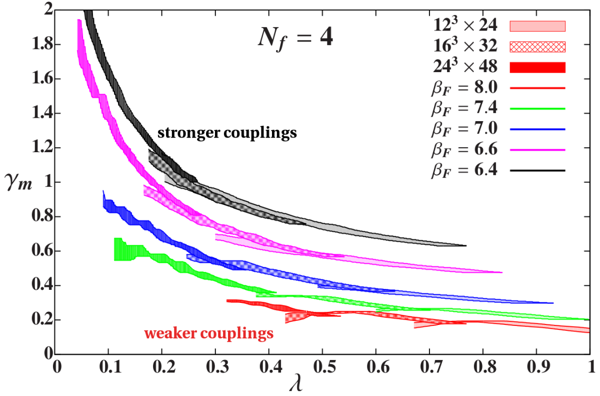




- As discussed above, $\langle\overline{\psi}\psi\rangle \propto \rho(\lambda \to 0) > 0 \implies \gamma_{\text{eff}} \nearrow 3$, but scaling $\rho(\lambda) \propto \lambda^\alpha$ breaks down in this situation
- Finite-volume effects can produce a “gap” with $\rho(0) = 0$
  This is a different breakdown of the scaling, leading to $\gamma_{\text{eff}} \searrow 0$
- Both of these effects are unphysical;
  we remove the finite-volume transients from most γ_{eff} plots

Backup: $\gamma_{\text{eff}}(\lambda)$ for QCD-like $N_F = 4$

Fit $\nu(\lambda) \propto \lambda^{1+\alpha}$ in a limited range of λ to find $1 + \gamma_{\text{eff}}(\lambda) = \dfrac{4}{1+\alpha(\lambda)}$



1000 eigenvalues on each volume

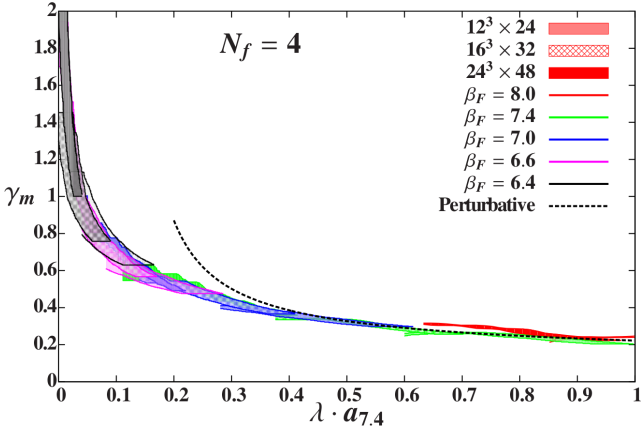
Fit ranges included in error bands

Only retain results free from finite-volume effects

$m = 0$ except for chirally broken systems at $\beta_F = 6.6$ and 6.4 where $\gamma_{\text{eff}} \nearrow 2$, becoming unphysically large

Backup: Rescaled $\gamma_{\text{eff}}(\lambda)$ for QCD-like $N_F = 4$

- Rescale $\lambda \to \left(\frac{a_{7.4}}{a}\right)^{1+\gamma_{\text{eff}}(\lambda)} \lambda$ to plot with lattice spacing fixed
- Relative lattice spacings from gradient flow & MCRG matching
- Match to one-loop perturbation theory at $\lambda \cdot a_{7.4} = 0.8$



Universal curve from χSB to asymp. freedom

Strong test of method & control over systematics

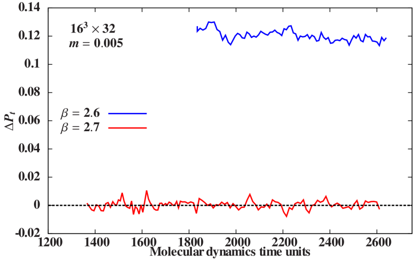
Backup: Order parameters for $\not S^4$ phase

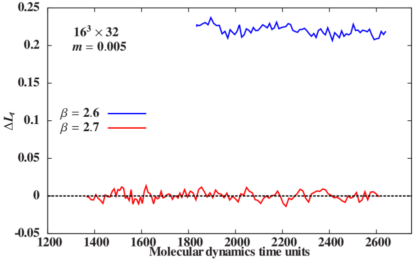
Staggered lattice actions possess exact single-site shift symmetry which is spontaneously broken in a novel lattice phase we encountered

Order parameters (any or all μ)

$$\Delta P_\mu = \langle \text{ReTr}\,\square_n - \text{ReTr}\,\square_{n+\mu} \rangle_{n_\mu \text{ even}}$$

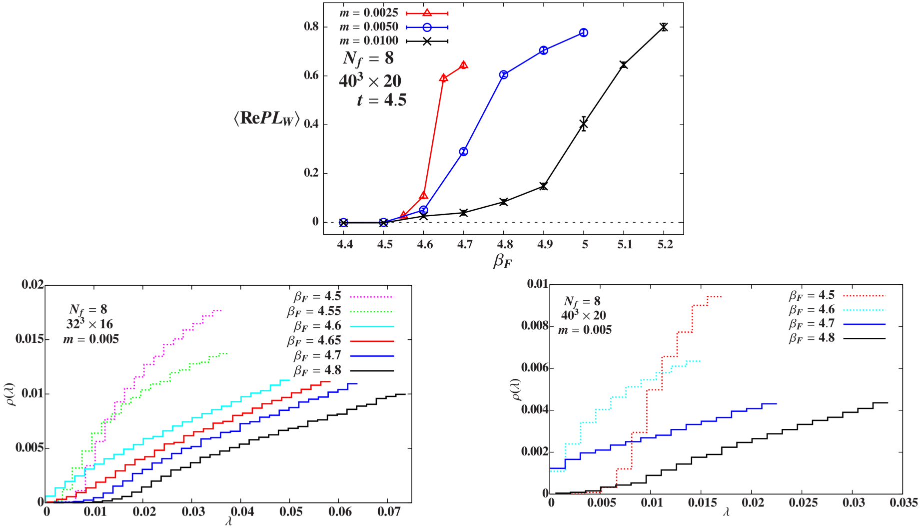
$$\Delta L_\mu = \left\langle \alpha_{\mu,n}\overline{\chi}_n U_{\mu,n}\chi_{n+\mu} - \alpha_{\mu,n+\mu}\overline{\chi}_{n+\mu} U_{\mu,n+\mu}\chi_{n+2\mu} \right\rangle_{n_\mu \text{ even}}$$





$\not S^4$ has never been seen before, but is clear in our data

Backup: Sample $N_F = 8$ transition signals



Need $N_T = 20$ to observe chirally broken phase at $m = 0.005$