Numerical Simulations of $\mathcal{N} = 4$ Supersymmetric Yang–Mills

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arXiv:1405.0644 with Simon Catterall, Poul Damgaard, Tom DeGrand, Joel Giedt

Lattice fields and lattice action: exact supersymmetry vs. stable numerical calculations

—All fields transform in the adjoint representation of gauge group U(N)

—Gauge & scalar fields combined into **five** complexified links $\mathcal{U}_a$ with field strength $\mathcal{F}_{ab}$

—Fermion field components grouped into singlet η, vector ψ_a and anti-symmetric tensor χ_{ab}

$$S = \frac{N}{\lambda_{\text{lat}}} \sum_x \left[-\overline{\mathcal{F}}_{ab}\mathcal{F}_{ab} + \frac{1}{2}\left(\overline{\mathcal{D}}_a^{(-)}\mathcal{U}_a\right)^2 - \chi_{ab}\mathcal{D}_{[a}^{(+)}\psi_{b]} - \eta\overline{\mathcal{D}}_a^{(-)}\psi_a - \frac{1}{4}\epsilon_{abcde}\chi_{de}\overline{\mathcal{D}}_c^{(-)}\chi_{ab}\right]$$

$$+ \mu^2 \sum_{x,\ a}\left(\frac{1}{N}\text{Tr}\left[\overline{\mathcal{U}}_a\mathcal{U}_a\right] - 1\right)^2 + \kappa\sum_{\mathcal{P}}\left|\det\mathcal{P} - 1\right|^2 \qquad (\mathcal{P} \text{ is plaquette})$$

—First line exactly preserves a single supersymmetry $\mathcal{Q}$, other 15 broken

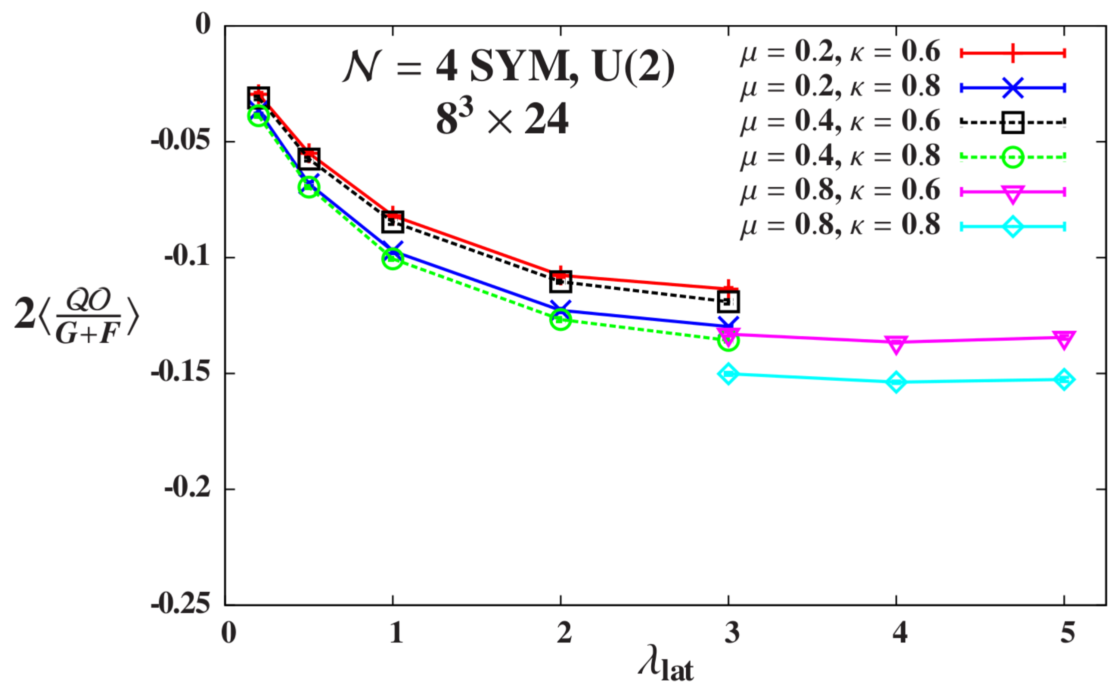
—μ term regulates flat directions, stabilizes continuum limit, acts like bosonic mass

—κ term approximately reduces U(N) $\longrightarrow$ SU(N), suppressing U(1) confinement lattice phase

Supersymmetry breaking from μ and κ

—Exact $\mathcal{Q} \Longrightarrow$ Ward identity $\langle\mathcal{Q}\mathcal{O}\rangle = 0$

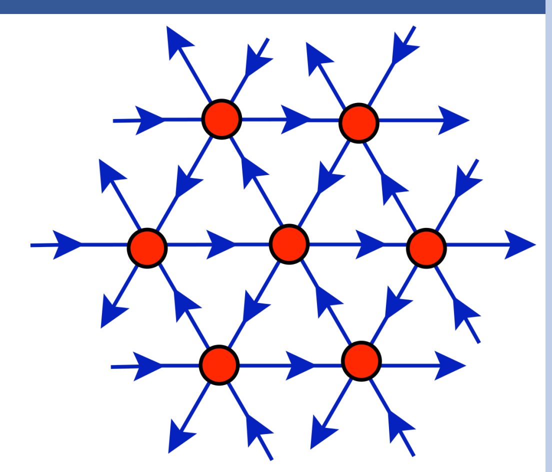
—Ward identity violations from non-zero μ, κ suggest $\mathcal{O}(10\%)$ supersymmetry breaking



Discretization on A_4^* lattice

5 links symmetrically span 4d

Analog of 2d triangular lattice



Non-orthogonal links

$\Longrightarrow$ continuum $\lambda = \lambda_{\text{lat}}/\sqrt{5}$

A_4^* lattice has S_5 point group symmetry

S_5 irreducible representations of lattice fields

$\longrightarrow$ continuum SO(4) euclidean Lorentz irreps.

$$\mathcal{U}_a = \mathbf{4} \oplus \mathbf{1} \longrightarrow U_\mu, \Phi$$

$$\psi_a = \mathbf{4} \oplus \mathbf{1} \longrightarrow \psi_\mu, \overline{\eta}$$

$$\chi_{ab} = \mathbf{6} \oplus \mathbf{4} \longrightarrow \chi_{\mu\nu}, \overline{\psi}_\mu$$

Is there a sign problem?

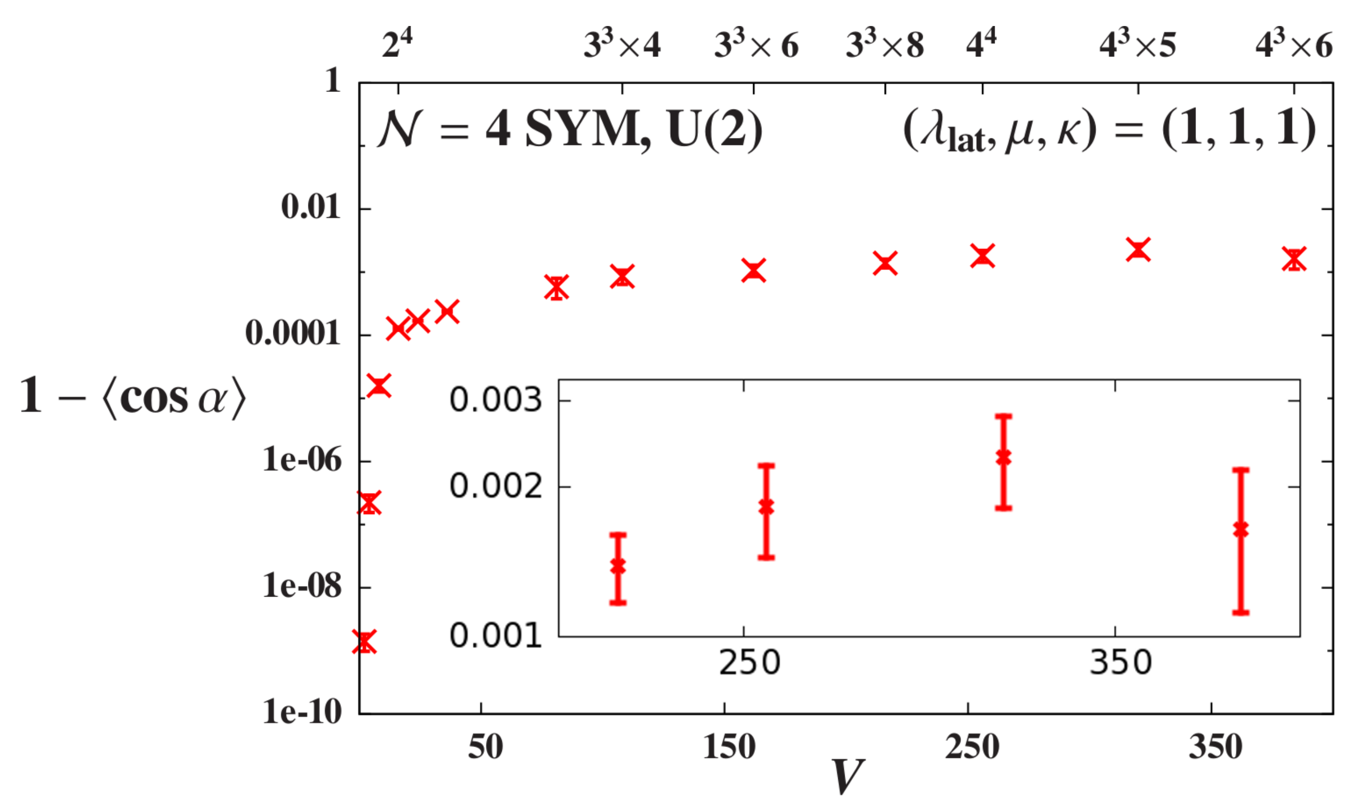
—**Complex** pfaffian $P = |P|e^{i\alpha}$ from fermions

—Our "phase-quenched" calculations ignore $e^{i\alpha}$

—We measure P to be nearly real and positive

$\Longrightarrow 1 - \langle\cos\alpha\rangle \ll 1$

—Fluctuations aren't growing with volume

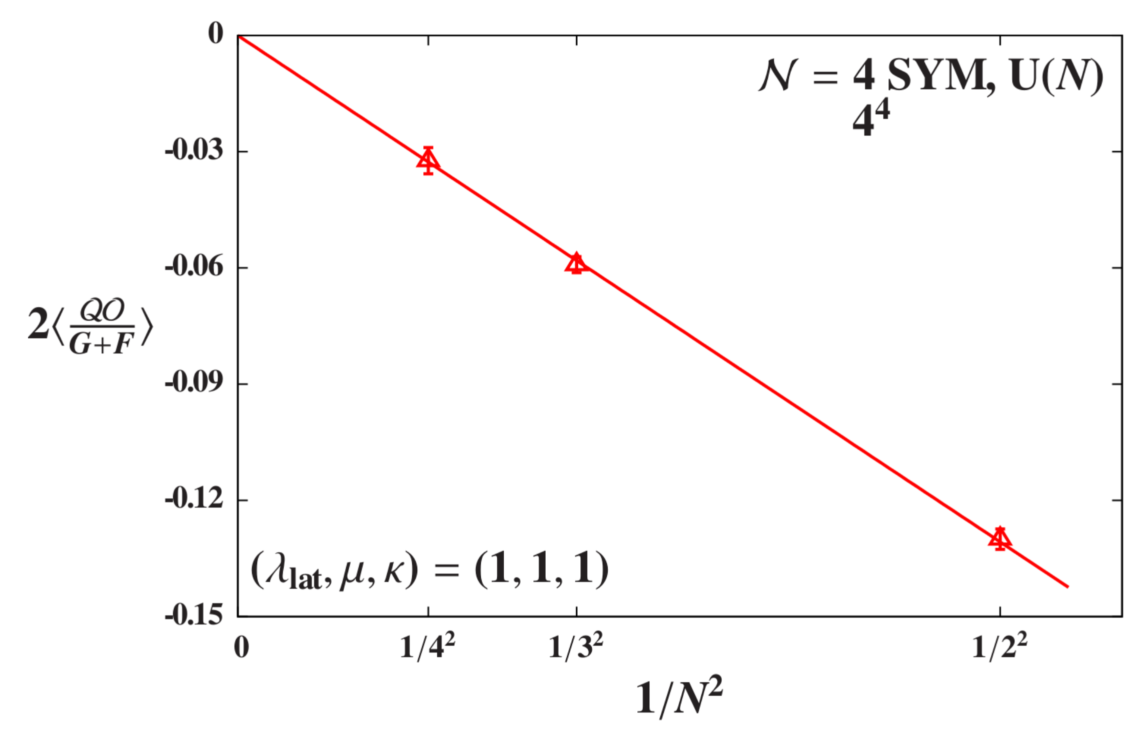


Towards the large-N limit

—Important for contact with continuum theory

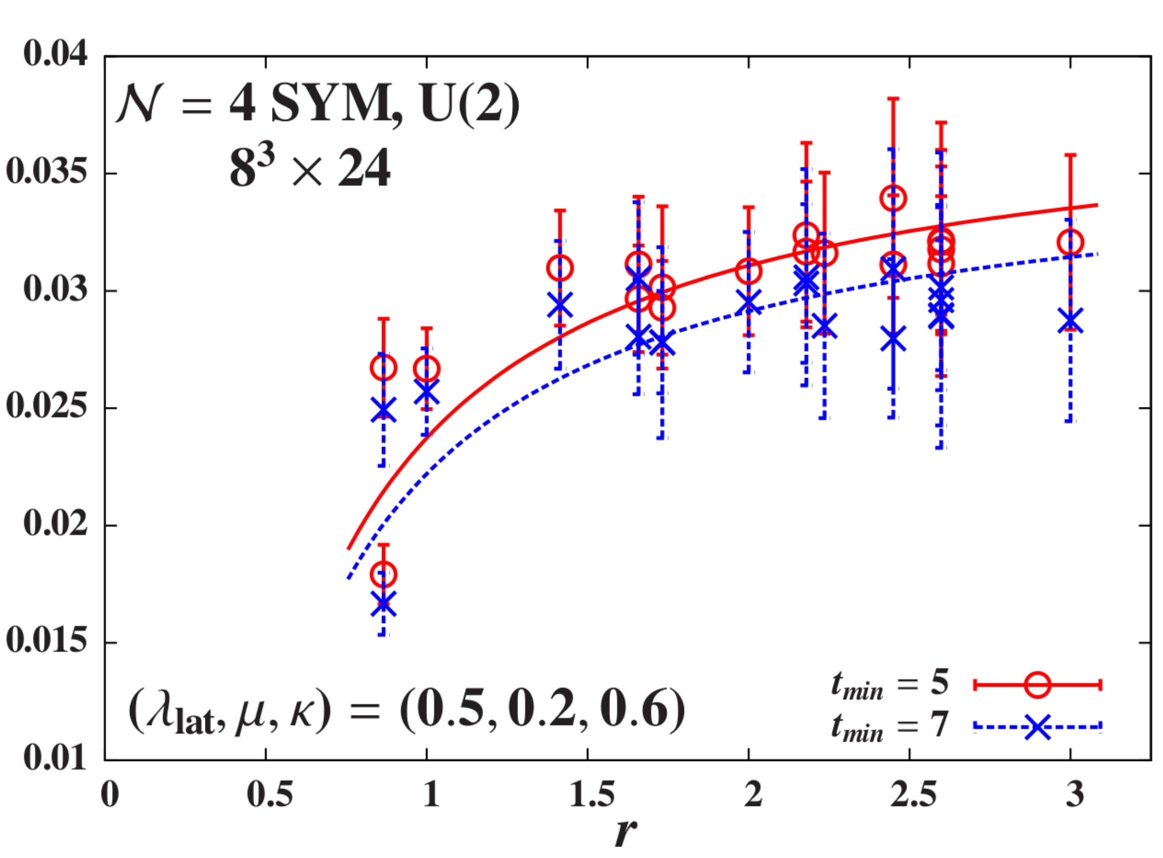
—Challenge: computational costs grow $\propto N^5$

—Benefit: supersymmetry breaking $\propto 1/N^2$



Static potential is coulombic at both weak and strong coupling

$V(r) = A - \frac{C}{r} \longrightarrow$ Coulomb coefficients in agreement with perturbation theory, $C = \lambda_{\text{lat}}/(4\pi\sqrt{5})$



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