Composite dark matter on the lattice
— the effective Higgs interaction —

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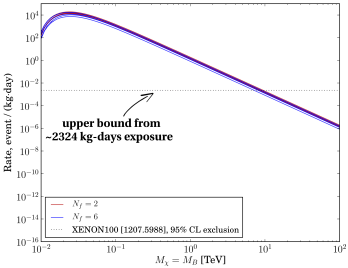
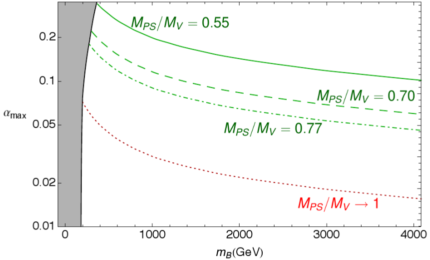
24 March 2014

PRD 88:014502 (2013) [arXiv:1301.1693]
arXiv:1402.6656 (submitted to PRD) and work in progress
with the Lattice Strong Dynamics Collaboration

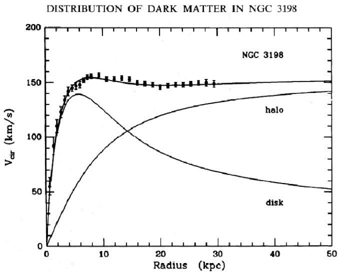
Overview

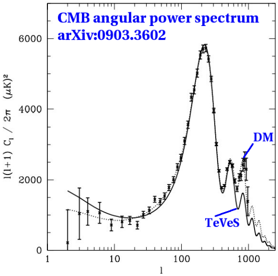
- Motivation for composite dark matter
- Purpose and strategy of lattice calculations
- Direct-detection bounds from electromagnetic form factors (PRD 88:014502)
- The effective Higgs interaction and bounds from direct detection (arXiv:1402.6656)
- Future directions and open questions



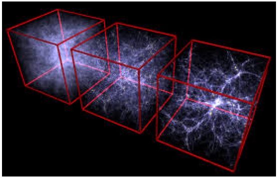


Evidence for dark matter



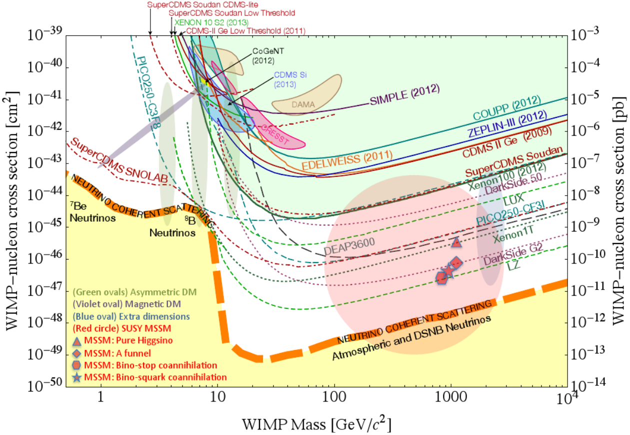


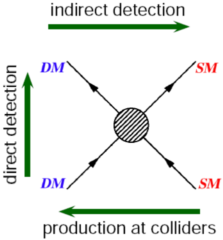
- Rotation curves of galaxies & clusters
- Gravitational lensing
- Structure formation
- Cosmological backgrounds



Of course, these are all gravitational effects. . .

Non-gravitational searches for dark matter





Similar lack of clear signals in indirect-detection and collider searches

Even so, we dark matter to interact with standard model fields. . .

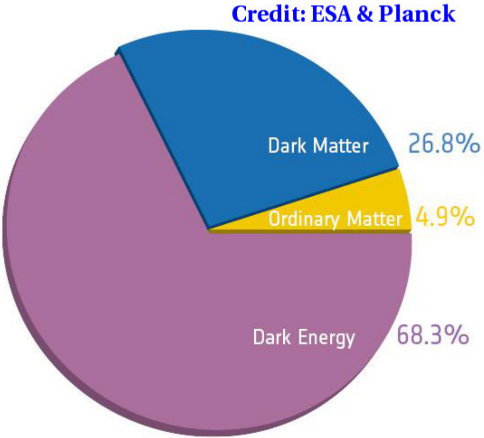
Motivation for non-gravitational interactions

$$\frac{\Omega_{DM}}{\Omega_{SM}} \approx 5$$

Why not 10^5 or 10^{-5}?

Suggests DM–SM coupling

Common feature of thermal
and asymmetric mechanisms
for relic density generation



If relic density relies on coupling to standard model fields,
such interactions must obey current experimental constraints
Composite dark matter models are a natural way to achieve this

Generic composite dark matter properties

Hypothesize a new confining gauge force in the dark sector

- Dark matter particle is **electroweak-neutral** "dark baryon"
  with charged constituent fermions
- Constituents & resonances provide interactions in early universe
  $\longrightarrow$ Oberved Ω_{DM} as thermal relic, through asymmetry, or both
- **Stability** from analog of baryon number conservation
  Automatic for SU(N) gauge theories with $N > 2$
- Aside: Composite dark matter would be self-interacting
  Possible solution to potential problems (e.g., core/cusp),
  but unclear whether this is actually needed, so let's move on

How would experiments detect electroweak-neutral composite DM?

Direct detection of composite dark matter

Electroweak-neutral $\Longrightarrow$ two spin-independent contributions:

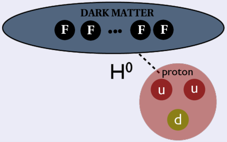
Photon exchange via electromagnetic form factors

Interactions suppressed by powers of confinement scale $\Lambda_{DM} \sim M_{DM}$

- **Dimension 5:** Magnetic moment $\longrightarrow \left(\overline{\psi}\sigma^{\mu\nu}\psi\right) F_{\mu\nu}/\Lambda$
- **Dimension 6:** Charge radius $\longrightarrow \left(\overline{\psi}\psi\right) v_\mu \partial_\nu F_{\mu\nu}/\Lambda^2$
- **Dimension 7:** Polarizability $\longrightarrow \left(\overline{\psi}\psi\right) F^{\mu\nu} F_{\mu\nu}/\Lambda^3$

Higgs boson exchange via scalar form factor

Effective Higgs interaction of composite DM may be produced by constituent fermions



Scalar form factor gives $\langle B | m_\psi \overline{\psi}\psi | B\rangle$ (σ term)

All form factors arise non-perturbatively $\Longrightarrow$ lattice calculations

Lattice Strong Dynamics Collaboration

L — S
D

Argonne	James Osborn
Brookhaven	Meifeng Lin
Boston	Rich Brower, Michael Cheng, Claudio Rebbi, Evan Weinberg, Oliver Witzel
Colorado/RBRC	Ethan Neil
INT	Mike Buchoff
Livermore	Evan Berkowitz, Enrico Rinaldi, Chris Schroeder, Pavlos Vranas
RBRC	Sergey Syritsyn
Syracuse	DS
UC Davis	Joe Kiskis
Yale	Tom Appelquist, George Fleming, Gennady Voronov

with special guest Graham Kribs (Oregon)

Exploring the range of possible phenomena in strongly-coupled gauge theories

Strategy for composite DM lattice studies

1. Magnetic moment and charge radius in SU(3) gauge theory
2. Effective Higgs interaction in SU(4) gauge theory
3. Polarizability in SU(4) gauge theory (underway)

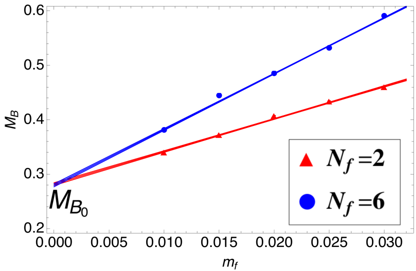
Results to be shown are from state-of-the-art lattice calculations

IBM Blue Gene/Q @Livermore

USQCD "Ds" cluster @Fermilab

$\mathcal{O}$(100M core-hours) invested overall
Many thanks to DOE, through Livermore & USQCD

SU(3) dark matter model



Initial explorations re-analyze the existing lattice ensembles I told you about in the fall

- SU(3) gauge group (like QCD)
- $32^3 \times 64$ lattices with domain wall fermions
- **Compare** $N_F = 2$ or 6 degenerate flavors, with fixed confinement scale $\Lambda \sim M_{B_0}$
- **Scan** range of fermion masses m_F
  Unlike QCD fermions are relatively heavy, $0.55 \lesssim M_{PS}/M_V \lesssim 0.75$

Also unlike QCD, fermions are all SU(2)$_L$ singlets

This ain't technicolor !

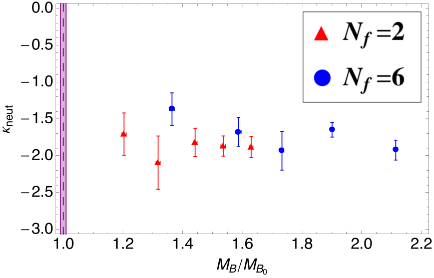
$Q = Y$, assign half of fermions $Q_{\text{P}} = 2/3$, other half $Q_{\text{M}} = -1/3$

DM candidate is electroweak-neutral "dark baryon" $B = $ PMM

Form factors for dark matter direct detection

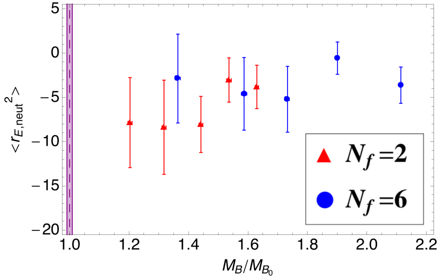
Magnetic moment κ

$(\overline{\psi}\sigma^{\mu\nu}\psi)\ F_{\mu\nu}/\Lambda$ (dim-5)



Charge radius $\langle r^2 \rangle$

$(\overline{\psi}\psi)\ v_\mu\partial_\nu F^{\mu\nu}/\Lambda^2$ (dim-6)



Results show little dependence on N_F or on m_F

In "QCD units", κ comparable to neutron's $\kappa_N = -1.91$,

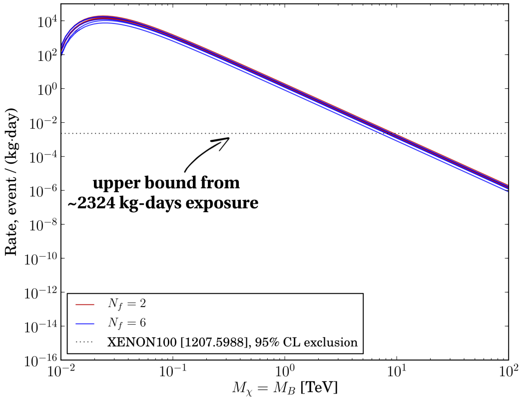
$\langle r^2 \rangle$ smaller than $\langle r^2 \rangle_N \approx -38$, due to our larger M_{PS}/M_V

Insert into the usual calculations to predict scattering rates...

Predicted XENON100 event rate

PRD 88:014502

Form factor results predict the number of events XENON100 would observe for these models given dark matter mass M_B

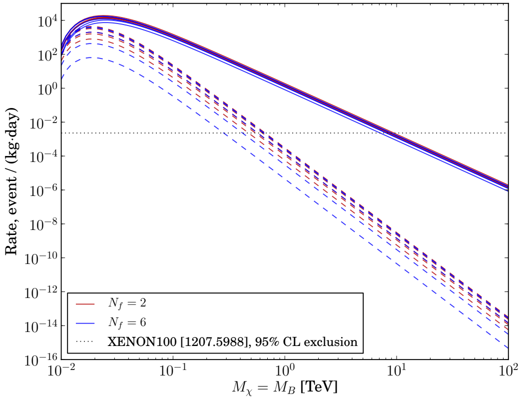


XENON100 results (PRL 109:181301, 2012) exclude $M_B \lesssim 10$ TeV

All ten ensembles overlaid — little dependence on N_F or on m_F

Magnetic moment κ dominates for $M_B \gtrsim 25$ GeV

—Dashed lines show charge radius $\langle r^2 \rangle$ contribution to full rate
—Suppressed by $1/M_B^2$ relative to magnetic moment contribution

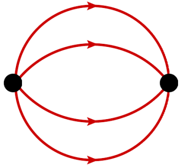


XENON100 results
only sensitive to $\langle r^2 \rangle$
for $M_B \lesssim 0.5$ TeV

Estimate $M_B \lesssim \mathcal{O}(0.01)$ TeV
sensitive to polarizability

$\kappa = 0$ automatically for SU(N) gauge theories with even N...

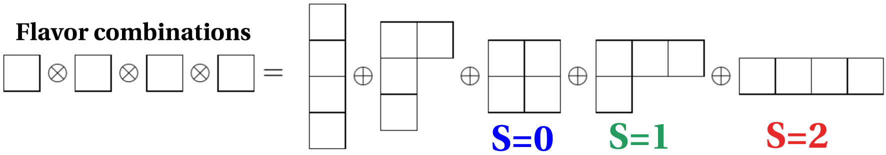
SU(4) bosonic dark baryons



Generate quenched SU(4) lattice ensembles

Lattice volumes up to $64^3 \times 128$,
several lattice spacings to check systematic effects

Again consider relatively heavy fermions $\longrightarrow 0.5 \lesssim M_{PS}/M_V \lesssim 0.9$



Dark matter candidate is spin-zero baryon $\longrightarrow$ no magnetic moment

Interested in models with at least two flavors to anti-symmetrize
Those with custodial SU(2) global symmetry $\longrightarrow$ no charge radius

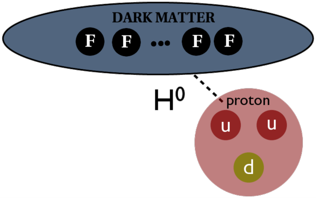
Effective Higgs interaction

Without magnetic moment and charge radius,
leading e.m. interaction is dim-7 polarizability $(\overline{\psi}\psi)\, F^{\mu\nu} F_{\mu\nu}/\Lambda^3$

With $M_H = 125$ GeV, Higgs boson exchange may dominate spin-independent cross section

$$\sigma_H^{(SI)} \propto \left| \frac{\mu_{B,N}}{M_H^2} y_\psi \langle B|\overline{\psi}\psi|B\rangle y_q \langle N|\overline{q}q|N\rangle \right|^2$$

For example, rules out "quirky" dark matter



For quarks, $y_q = m_q/v \Longrightarrow y_q \langle N|\overline{q}q|N\rangle \propto \frac{M_N}{v} \frac{\langle N|m_q\overline{q}q|N\rangle}{M_N}$

Dark constituent fermion masses are more complicated

If ψ transform in vector-like electroweak rep.,
can have mix of bare mass and Yukawa coupling to Higgs

Parametrizing the effective Higgs interaction

Write $y_\psi = \alpha\ m_\psi/v \longrightarrow y_\psi\langle B|\overline{\psi}\psi|B\rangle = \frac{M_B}{v}\ \alpha\ \frac{\langle B|m_\psi\overline{\psi}\psi|B\rangle}{M_B}$

with Higgs coupling parameterized by $\alpha \equiv \left.\frac{v}{m_\psi}\frac{\partial m_\psi(h)}{\partial h}\right|_{h=v}$

Limits: $m_\psi(h) \propto h$ (as for quarks) $\longrightarrow$ maximum $\alpha = 1$
m_ψ purely vector-like $\longrightarrow \alpha = 0$, no effective Higgs interaction

Example: 4-flavor model with $\mathrm{SU}(2)_L\times\mathrm{SU}(2)_R$ global symmetry

—Higgs vev preserves diagonal $\mathrm{SU}(2)_C$
—Two flavors transform as an EW doublet
others as two EW singlets
—All have charge $Q = T_{3L} + T_{3R} = \pm\frac{1}{2}$
—Mass is $m_\psi(h) = m_V + yh$

Field	$\mathrm{SU}(4)_D$	$\mathrm{SU}(2)_{L,R}$
ψ_1	$\mathbf{4}$	$\mathbf{2}$, $\mathbf{1}$
ψ_2	$\overline{\mathbf{4}}$	$\overline{\mathbf{2}}$, $\mathbf{1}$
ψ_3	$\mathbf{4}$	$\mathbf{1}$, $\mathbf{2}$
ψ_4	$\overline{\mathbf{4}}$	$\mathbf{1}$, $\overline{\mathbf{2}}$

from $\mathcal{L}_m = m_V\,(\psi_1\psi_2 + \psi_3\psi_4) + y\,\left(\psi_1 H\psi_4 + \psi_2 H^\dagger\psi_3\right) + \text{h.c.}$

—Leads to $\alpha = [1 + m_V/(yv)]^{-1}$ with appropriate limits $0 \le \alpha \le 1$

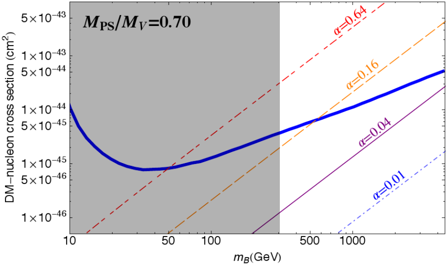
Results for Higgs-exchange cross section

Next compute the sigma term: $$y_\psi \langle B|\overline{\psi}\psi|B\rangle = \frac{M_B}{v}\,\alpha\,\frac{\langle B|m_\psi\overline{\psi}\psi|B\rangle}{M_B}$$

Use Feynman–Hellmann theorem $$\frac{\langle B|m_\psi\overline{\psi}\psi|B\rangle}{M_B} = \frac{m_\psi}{M_B}\frac{\partial M_B}{\partial m_\psi}$$

Calculate M_B for several m_ψ to find $0.15 \lesssim \frac{m_\psi}{M_B}\frac{\partial M_B}{\partial m_\psi} \lesssim 0.34$

—Predictions still depend on dark matter mass, on M_{PS}/M_V, and on α
—Compare with LUX bounds: (PRL 112:091303, 2014)

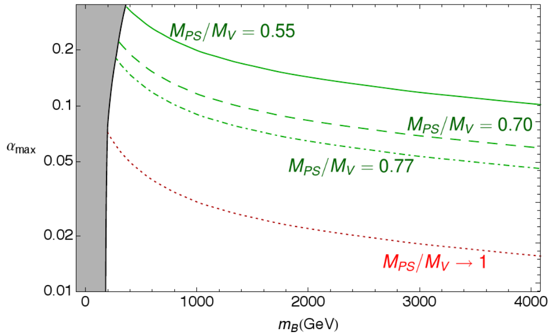


Shaded region is LEP bound for charged pseudoscalar meson $\Pi^\pm$:
$m_\Pi \gtrsim 100$ GeV from $\tau^+\tau^- + \not{E}_T$

LHC should eventually be sensitive to $m_\Pi \sim 300$ GeV

Bounds on effective Higgs coupling

Cross section results predict
maximum effective Higgs coupling allowed by LUX



Maximum coupling α
depends on M_{PS}/M_V
and dark matter mass

LEP bound more significant
for smaller m_ψ & M_{PS}

Bottom line: Effective Higgs interaction tightly constrained,
$\alpha \lesssim 0.3$ means fermion masses must be mainly vector-like

Next step: electromagnetic polarizability

Even for a model with no magnetic moment, no charge radius and no effective Higgs interaction...

Polarizability α_E produces unavoidable interaction $\left(\overline{\psi}\psi\right) F^{\mu\nu} F_{\mu\nu} / \Lambda^3$

Method: Compute energy shift due to background electric field $\mathcal{E}$

$$E_B(\mathcal{E}) = M_B + \alpha_E \mathcal{E}^2/2 + \mathcal{O}\left(\mathcal{E}^4\right)$$

Challenges of polarizability calculation

—Large statistics (and control over systematics) to extract small signal
—Large lattice volumes required to reach small $\mathcal{E} \propto 2\pi/L^2$

Vanishing magnetic moment and charge radius
$\Longrightarrow$ dramatic simplifications compared to QCD

Recapitulation

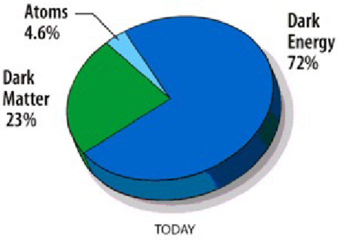
- **Composite dark matter** is well-motivated
  and phenomenologically interesting
- **Lattice calculations** required to study non-perturbative physics
- **Direct detection** sensitive to magnetic moment, charge radius
  and effective Higgs interaction
- **Polarizability** produces unavoidable interaction

Many more directions can be explored in the future

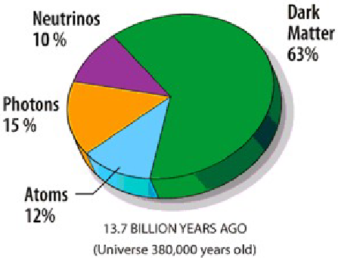
—Bounds on custodial symmetry breaking from resulting charge radius
—BB elastic scattering to study self-interactions
—ΠB elastic scattering affects thermal relic density
—B–$\overline{B}$ oscillation to estimate indirect detection signals
(annihilation not directly accessible on lattice)

Backup: Dark matter density in cosmological history

$$\frac{\Omega_{DM}}{\Omega_{baryons}} \approx \frac{\Omega_{DM}}{\Omega_{SM}} \approx 5 \text{ now}$$



$$\frac{\Omega_{DM}}{\Omega_{baryons}} \approx 5 \text{ at recombination}$$



Simply because both are "matter" and evolve in the same way

Backup: Two roads to natural asymmetric dark matter

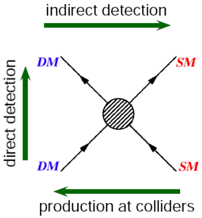
Basic idea: Dark matter relic density related to baryon asymmetry

$$\rho_D \approx 5\rho_B$$
$$\implies M_D n_D \approx 5 M_B n_B$$

- $n_D \sim n_B \quad \implies \quad M_D \sim 5$ GeV
  High-dimensional interactions relate baryon# and DM# violation
- $M_D \gg M_B \quad \implies \quad n_B \gg n_D \sim \exp\left[-M_D/T_s\right]$
  Sphaleron transitions above $T_s \sim 200$ GeV distribute asymmetries

Both require coupling between standard model and dark matter

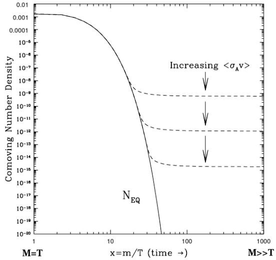
Backup: Thermal freeze-out for relic density



$T \gtrsim M_{DM}$: DM $\longleftrightarrow$ SM
Thermal equilibrium

$T \lesssim M_{DM}$: DM $\longrightarrow$ SM
Rapid depletion of Ω_{DM}

Hubble expansion $\longrightarrow$ dilution
leads to freeze-out



Requires coupling between standard model and dark matter

Mass and coupling of pure thermal relic are related: $\dfrac{M_{DM}}{100\text{ GeV}} \sim 200\alpha$

(The "WIMP miracle" is $\alpha \sim \alpha_{EW} \sim 0.01 \Longrightarrow M \sim 200\text{ GeV} \sim v$)

Thermal relic suppressed by **strong** coupling, easy for composite DM

Backup: Electroweak-charged composite dark matter

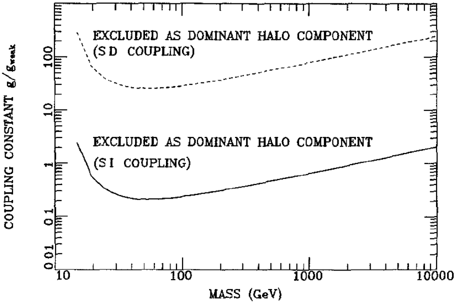
SU(N) "dark baryons" are bosons if N even, Dirac fermions if N odd

Therefore a net electroweak charge

$\Longrightarrow$ Unsuppressed tree-level Z-exchange interaction with nuclei

$\Longrightarrow$ Spin-independent cross section $\sigma \sim 10^{-38}$ cm^2

$\Longrightarrow$ Ruled out decades ago (Example: Ahlen et al., 1987)



Neutralinos are Majorana fermions, so evade this bound

Backup: Form factor calculations on the lattice

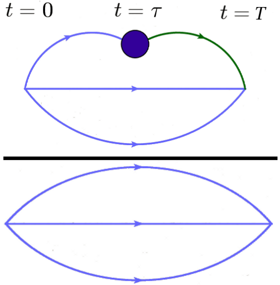
With $q = p' - p$ and $Q^2 = -q^2 > 0$,

$$\langle B(p')|\overline{\psi}\gamma^\mu\psi|B(p)\rangle = \overline{U}(p')\left[F_1^\psi(Q^2)\gamma^\mu + F_2^\psi(Q^2)\frac{i\sigma^{\mu\nu}q_\nu}{2M_B}\right]U(p)$$

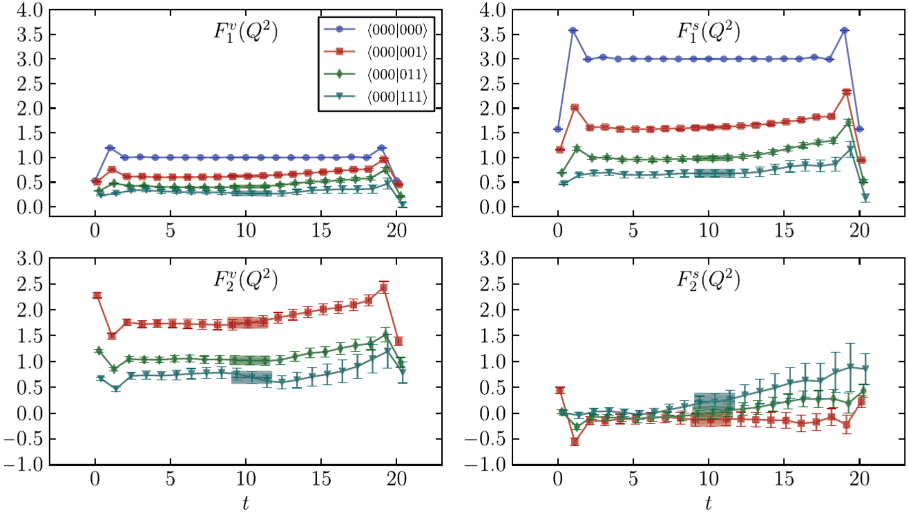
$$\kappa \equiv F_2(0) \qquad \left\langle r^2\right\rangle = \int d^3r\left[r^2\,\rho(r)\right] \equiv -6\,\frac{dF_1(Q^2)}{dQ^2}\bigg|_{Q^2=0} + \frac{3\kappa}{2M_B^2}$$

$$\begin{aligned} R_{\mathcal{O}}\left(\tau, T, p, p'\right) \\ &\longrightarrow \langle B(p')|\mathcal{O}|B(p)\rangle \\ &+ \mathcal{O}\left(e^{-\Delta\tau}\right) + \mathcal{O}\left(e^{-\Delta T}\right) \\ &+ \mathcal{O}\left(e^{-\Delta(T-\tau)}\right) \end{aligned}$$

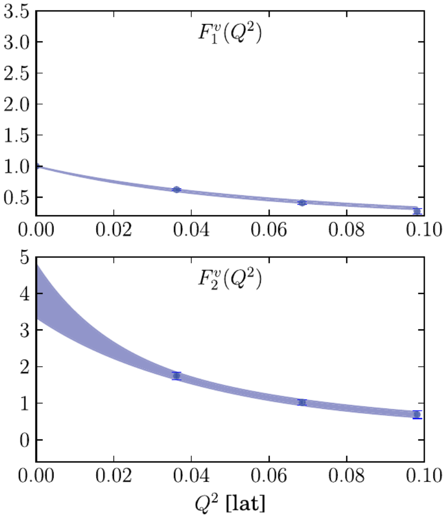
$R(\tau, T, p, p') \sim$



Backup: Form factor ratio plateaus, $N_F = 2$



Backup: Form factor dipole (and linear) fits, $N_F = 2$



Fit to $F(Q^2) = A/(1 + BQ^2)$ except for isoscalar Pauli form factor which is close to zero

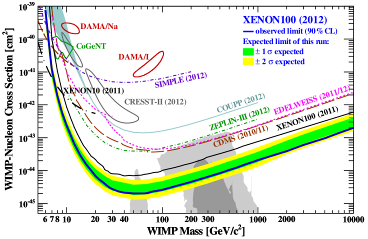
Backup: Event rate calculations and lattice input

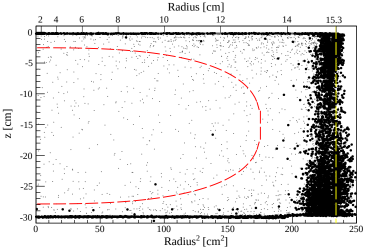
$$\text{Rate} = \frac{M_{detector}}{M_T}\frac{\rho_{DM}}{M_B}\int_{E_{min}}^{E_{max}} dE_R \mathcal{A}cc(E_R)\left\langle v_{DM}\frac{d\sigma}{dE_R}\right\rangle_f$$

$$\frac{d\sigma}{dE_R} = \frac{\overline{|\mathcal{M}_{SI}|^2} + \overline{|\mathcal{M}_{SD}|^2}}{16\pi\left(M_B + M_T\right)^2 E_R^{max}} \qquad E_R^{max} = \frac{2M_B^2 M_T v_{col}^2}{\left(M_B + M_T\right)^2}$$

$$\frac{\overline{|\mathcal{M}_{SI}|^2}}{e^4\left[ZF_c(Q)\right]^2} = \left(\frac{M_T}{M_B}\right)^2\left[\frac{4}{9}M_B^4\left\langle r^2\right\rangle^2 + \frac{\kappa^2\left(M_T + M_B\right)^2\left(E_R^{max} - E_R\right)}{M_T E_R}\right]$$

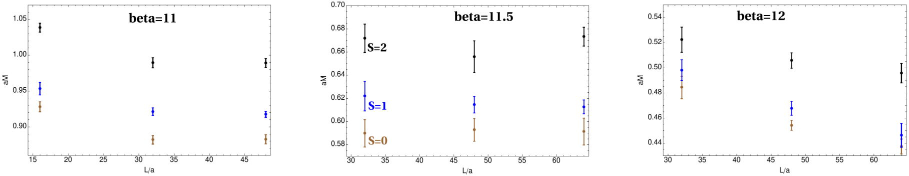
$$\overline{|\mathcal{M}_{SD}|^2} = e^4\frac{2}{3}\left(\frac{J+1}{J}\right)\left[\left(A\frac{\mu_T}{\mu_n}\right)F_s(Q)\right]^2\kappa^2$$



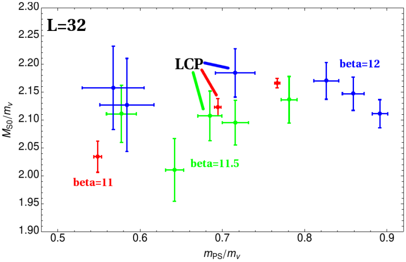


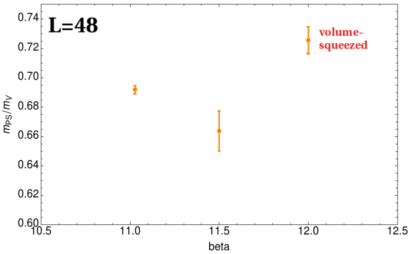
Backup: Volume and discretization effects for SU(4)

Baryon masses vs. L at fixed coupling β and fermion mass m_ψ:



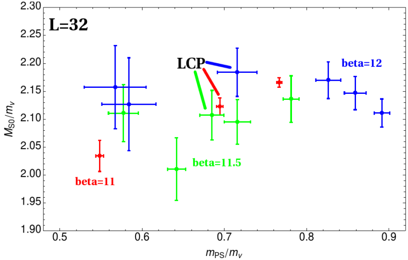
Edinburgh-style plot of $\frac{M_{S0}}{M_V}$ vs. $\frac{M_{PS}}{M_V}$ and line of constant physics (LCP):

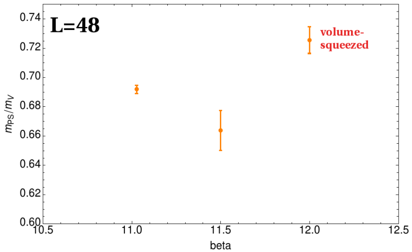




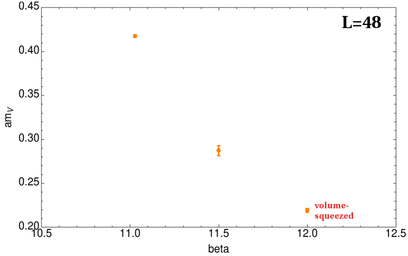
Backup: Volume and discretization effects for SU(4)

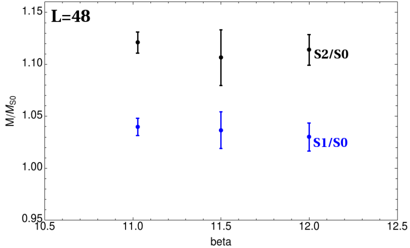
Edinburgh-style plot of $\frac{M_{S0}}{M_V}$ vs. $\frac{M_{PS}}{M_V}$ and line of constant physics (LCP):





Lattice spacing and discretization effects for $\frac{M_{S2,S1}}{M_{S0}}$ on LCP:





Backup: More Higgs-exchange results

$$\sigma_H^{(SI)} \propto \left| y_\psi \langle B | \overline{\psi}\psi | B \rangle \right|^2$$

Matrix element $\propto \dfrac{\partial M_B}{\partial m_\psi}$

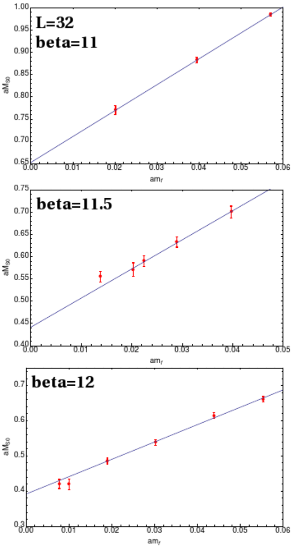
(Feynman–Hellmann)

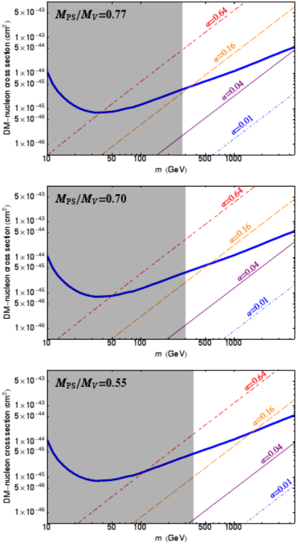
We find

$$0.15 \lesssim \frac{m_\psi}{M_B}\frac{\partial M_B}{\partial m_\psi} \lesssim 0.34$$

Compare with QCD

$$0.04 \lesssim \frac{m_q}{M_N}\frac{\partial M_N}{\partial m_q} \lesssim 0.08$$





Backup: Feynman–Hellmann theorem

- $m_\psi \overline{\psi}\psi$ is the only term in the hamiltonian that depends on m_ψ
$$\longrightarrow \langle B|\frac{\partial H}{\partial m_\psi}|B\rangle = \langle B|\overline{\psi}\psi|B\rangle$$
- But $\langle B|H|B\rangle = M_B$ is just the baryon mass, so

$$\begin{aligned}
\frac{\partial M_B}{\partial m_\psi} &= \frac{\partial}{\partial m_\psi}\langle B|H|B\rangle \\
&= \langle\frac{\partial B}{\partial m_\psi}|H|B\rangle + \langle B|H|\frac{\partial B}{\partial m_\psi}\rangle + \langle B|\frac{\partial H}{\partial m_\psi}|B\rangle \\
&= M_B\langle\frac{\partial B}{\partial m_\psi}|B\rangle + M_B\langle B|\frac{\partial B}{\partial m_\psi}\rangle + \langle B|\frac{\partial H}{\partial m_\psi}|B\rangle \\
&= M_B\frac{\partial}{\partial m_\psi}\langle B|B\rangle + \langle B|\overline{\psi}\psi|B\rangle \\
&= \langle B|\overline{\psi}\psi|B\rangle
\end{aligned}$$

Backup: Large-N predictions for SU(4) baryons

Rotor spectrum for spin-J large-N baryons:

$$M(N,J) = NM_0 + C + B\frac{J(J+1)}{N} + \mathcal{O}\left(\frac{1}{N^2}\right)$$

—Match SU(3) and SU(4) pseudoscalar and vector meson masses

—Fit M_0, C and B with nucleon, Δ and spin-0 baryon masses
$\longrightarrow$ predictions for $S = 1, 2$ baryons (diamonds)