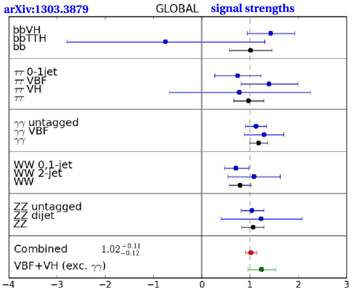
From Lattice Strong Dynamics to Electroweak Phenomenology

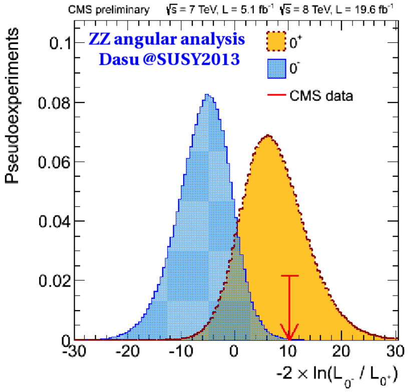
David Schaich, 4 November 2013

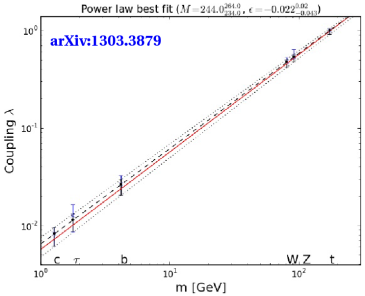
PRL 106:231601 (2011) [1009.5967]
PRD 85:074505 (2012) [1201.3977]
and work in progress with the Lattice Strong Dynamics Collaboration

See also: USQCD white paper 1309.1206

- Higgs boson
- Standard-model-like
- No BSM discovered







Suppose new strong dynamics $\longrightarrow$ composite Higgs

- This scenario **is** disfavored, but hard to rule out (non-perturbative!)
  USQCD white paper arXiv:1309.1206 reviews some possibilities
- Would expect many new composite states... around $4\pi v \simeq 3$ TeV

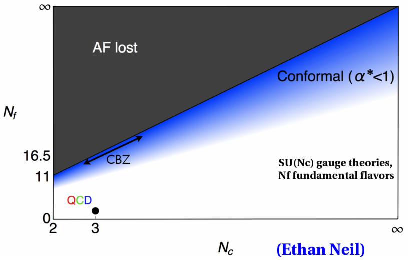
Complementary approach: low-energy effective theory

New strong dynamics fix Low-Energy Constants of effective theory

If heavy composite states can't be directly observed at the LHC, first signs of new physics may appear through these LECs

Many strongly-coupled systems produce similar effective theories with range of LEC values

Non-perturbative lattice calculations can explore possibilities



Scale setting & electroweak effective theory

(Appelquist & Wu, PRD 48:3235, 1993)

—Integrating out resonances around $4\pi v$ scale gives chiral lagrangian
—Dynamical d.o.f. are Goldstones π^a to be eaten by W and Z,
appear through matrix field $U \equiv \exp\left[\, 2iT^a\pi^a/F \,\right]$

$$\mathcal{L}_{LO} = \frac{F^2}{4}\mathrm{Tr}\left[D_\mu U^\dagger D^\mu U\right] + \frac{F^2 B}{2}\mathrm{Tr}\left[m\left(U + U^\dagger\right)\right]$$

—Decay constant F sets electroweak scale, W & Z masses

$F = v = 246$ GeV in simplest case (one electroweak doublet)

—Chiral condensate $\langle\overline{\psi}\psi\rangle \propto F^2 B$ involved in fermion mass generation

Caveat

125 GeV is not around $4\pi v \Longrightarrow$ work in progress to extend $\mathcal{L}_\chi$

Higher-Order Low-Energy Constants

With $T \equiv U\tau_3 U^\dagger$ and $V_\mu \equiv (D_\mu U)\,U^\dagger$, next-to-leading order includes
oblique corrections $\quad S \propto \alpha_1, \quad T \propto \beta_1, \quad U \propto \alpha_8$
triple gauge vertices and dominant contributions to WW scattering:

$$\mathcal{L}_1 = \frac{\alpha_1}{2} g_1 g_2 B_{\mu\nu} \mathrm{Tr}\,(TW^{\mu\nu}) \qquad \mathcal{L}_2 = \frac{i\alpha_2}{2} g_1 B_{\mu\nu} \mathrm{Tr}\,(T\,[V^\mu, V^\nu])$$

$$\mathcal{L}_3 = i\alpha_3 g_2 \mathrm{Tr}\,(W_{\mu\nu}\,[V^\mu, V^\nu]) \qquad \mathcal{L}_4 = \alpha_4 \left\{\mathrm{Tr}\,(V_\mu V_\nu)\right\}^2$$

$$\mathcal{L}_5 = \alpha_5 \left\{\mathrm{Tr}\,(V_\mu V^\mu)\right\}^2 \qquad \mathcal{L}_6 = \alpha_6 \mathrm{Tr}\,(V_\mu V_\nu)\,\mathrm{Tr}\,(TV^\mu)\,\mathrm{Tr}\,(TV^\nu)$$

$$\mathcal{L}_7 = \alpha_7 \mathrm{Tr}\,(V_\mu V^\mu)\,\mathrm{Tr}\,(TV_\mu)\,\mathrm{Tr}\,(TV^\nu) \qquad \mathcal{L}_8 = \frac{\alpha_8}{4} g_2^2 \left\{\mathrm{Tr}\,(TW_{\mu\nu})\right\}^2$$

$$\mathcal{L}_9 = \frac{i\alpha_9}{2} g_2 \mathrm{Tr}\,(TW_{\mu\nu})\,\mathrm{Tr}\,(T\,[V^\mu, V^\nu]) \qquad \mathcal{L}_{10} = \frac{\alpha_{10}}{2} \left\{\mathrm{Tr}\,(TV_\mu)\,\mathrm{Tr}\,(TV_\nu)\right\}^2$$

$$\mathcal{L}_{11} = \alpha_{11} g_2 \epsilon^{\mu\nu\rho\lambda} \mathrm{Tr}\,(TV_\mu)\,\mathrm{Tr}\,(V_\nu W_{\rho\lambda}) \qquad \mathcal{L}_1' = \frac{\beta_1}{4} g_2^2 F^2 \left\{\mathrm{Tr}\,(TV_\mu)\right\}^2$$

Outline for this talk: *S* parameter and WW scattering,
work with Lattice Strong Dynamics Collaboration

Lattice Strong Dynamics Collaboration

Argonne	Meifeng Lin, Heechang Na, James Osborn
Berkeley	Sergey Syritsyn
Boston	Richard Brower, Michael Cheng, Claudio Rebbi, Evan Weinberg, Oliver Witzel
Colorado	Ethan Neil
Livermore	Chris Schroeder, Pavlos Vranas, Joe Wasem
NVIDIA	Ron Babich, Mike Clark
Syracuse	DS
UC Davis	Joseph Kiskis
U Wash.	Mike Buchoff, Saul Cohen
Yale	Thomas Appelquist, George Fleming, Gennady Voronov

Exploring the range of possible phenomena in strongly-coupled gauge theories

IBM Blue Gene/L @Livermore

“Kraken” Cray XT5 @Oak Ridge

USQCD “Ds” cluster @Fermilab

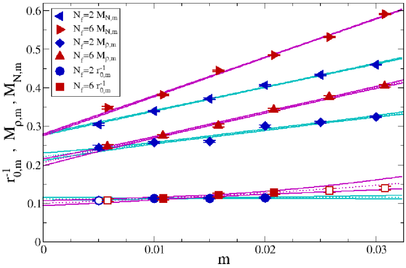
Results to be shown are from
state-of-the-art lattice calculations

$\mathcal{O}$(100M core-hours) invested overall

Many thanks to DOE (through USQCD & Livermore), NSF (through XSEDE)

Lattice Strong Dynamics strategy

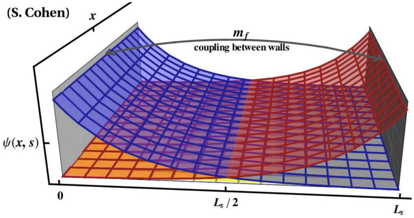
- SU(3) gauge theory,
  $N_F = 2 \longrightarrow 6 \longrightarrow 8 \longrightarrow 10$ fund.
  (only $N_F = 2$ and 6 today)



- Systematically explore trends
  as N_F increases from QCD

- Match $m \to 0$ IR scales to allow
  more direct comparisons

- $32^3 \times 64 \times 16$ domain wall fermions
  $\to$ good chiral & flavor symmetries
  (but expensive!)

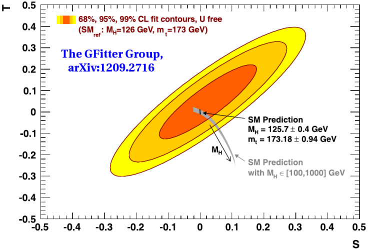


The S parameter

(Peskin & Takeuchi, 1991)

Oblique correction to vacuum polarization of neutral EW gauge bosons

S defined to vanish for the standard model with $M_H = 125$ GeV



Experiment: $S = 0.03 \pm 0.10$

- Z decay widths and asymmetries
- Neutrino scattering cross sections
- M_W, M_Z
- Atomic parity violation

—Scaling up QCD (and imposing $M_H = 125$ GeV) predicts $S \approx 0.43$
—Lattice calculations can predict S for strong dynamics beyond QCD

S on the lattice

$$S = 4\pi N_D \lim_{Q^2 \to 0} \frac{d}{dQ^2} \Pi_{V-A}(Q^2) - \Delta S_{SM}(M_H)$$

$$\mathcal{L}_\chi \ni \frac{\alpha_1}{2} g_1 g_2 B_{\mu\nu} \mathrm{Tr}\left[T W^{\mu\nu}\right] \longrightarrow \Pi_{V-A}(Q^2)$$

1. $\Pi_{V-A}(Q^2)$ is transverse component of vacuum polarization tensor

$$\Pi^{\mu\nu}_{V-A}(Q) = Z \sum_x e^{iQ\cdot(x+\widehat{\mu}/2)} \mathrm{Tr}\left[\left\langle \mathcal{V}^{\mu a}(x) V^{\nu b}(0) \right\rangle - \left\langle \mathcal{A}^{\mu a}(x) A^{\nu b}(0) \right\rangle\right]$$

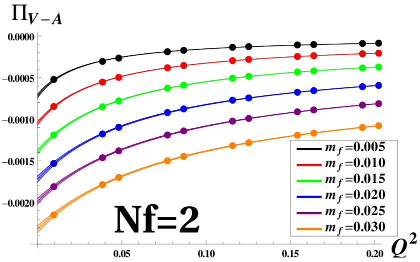
(correlators mix conserved and local domain wall currents for efficiency)

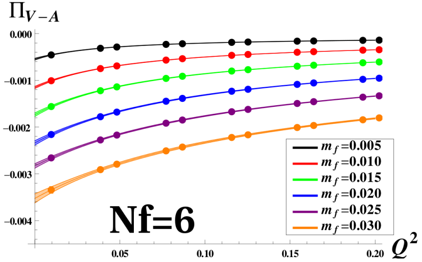
2. $N_D \geq 1$ is the number of doublets with chiral electroweak couplings
3. $\Delta S_{SM}(M_H)$ subtracted so that $S = 0$ in the standard model

   Removes three eaten Goldstones, depends on Higgs mass

Polarization function data and fits, $N_F = 2$ and $N_F = 6$

$$S = 4\pi N_D \lim_{Q^2 \to 0} \frac{d}{dQ^2} \Pi_{V-A}(Q^2) - \Delta S_{SM}(M_H)$$





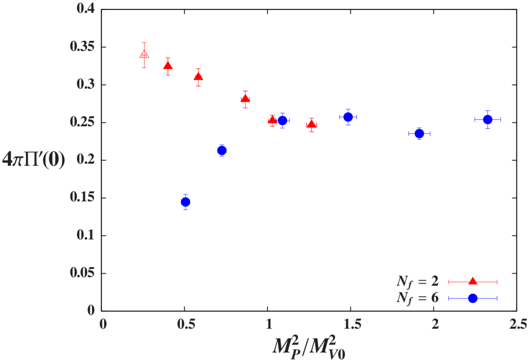
Uncorrelated fits to "Padé-(1, 2)" rational function (cf. Aubin et al.)

$$\Pi_{V-A}(Q^2) = \frac{a_0 + a_1 Q^2}{1 + b_1 Q^2 + b_2 Q^4} = \frac{\sum_{m=0}^{1} a_m Q^{2m}}{\sum_{n=0}^{2} b_n Q^{2n}}$$

(motivated by meson pole dominance and Weinberg sum rules / OPE)

Can already see contrast between $N_F = 2$ and $N_F = 6$...

Fit results for $\Pi'_{V-A}(0)$, $N_F = 2$ and $N_F = 6$



Vertical axis: $4\pi\Pi'_{V-A}(0)$

where

$$\Pi'(0) = \lim_{Q^2\to 0} \frac{d}{dQ^2}\Pi(Q^2)$$

$$S = 4\pi N_D \Pi'_{V-A}(0) - \Delta S_{SM}$$

Horizontal axis: M_P^2/M_{V0}^2 gives a more physical comparison than m

$$M_{V0} \equiv \lim_{m\to 0} M_V \text{ is matched between } N_F = 2 \text{ and } N_F = 6$$

Expect agreement in the quenched limit $M_P^2 \to \infty$

From slopes to S for $M_H = 125$ GeV

$$S = 4\pi N_D \lim_{Q^2 \to 0} \frac{d}{dQ^2} \Pi_{V-A}(Q^2) - \Delta S_{SM}(M_H)$$

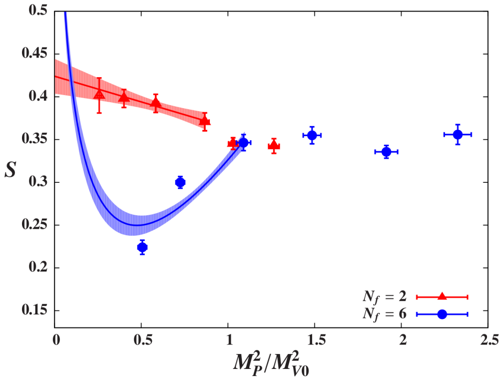
1. N_D doublets with chiral electroweak couplings contribute to S
   Scaled-up QCD often considers maximum $N_D = N_F/2$
   but only $N_D \geq 1$ is required for electroweak symmetry breaking

2. $\Delta S_{SM} = \frac{1}{4} \int_{4M_P^2}^{\infty} \frac{ds}{s} \left[1 - \left(1 - \frac{M_{V0}^2}{s}\right)^3 \Theta(s - M_{V0}^2)\right] - \frac{1}{12\pi} \log\left(\frac{M_{V0}^2}{M_H^2}\right)$

   Integral diverges logarithmically as $M_P^2 \to 0$
   to cancel contribution of three eaten modes

   First term assumes $M_H \sim M_{V0} \sim$ TeV;
   second term corrects for $M_H = 125$ GeV $\ll$ TeV

S parameter, $N_F = 2$ and $N_F = 6$



Direct $N_F = 2$ result

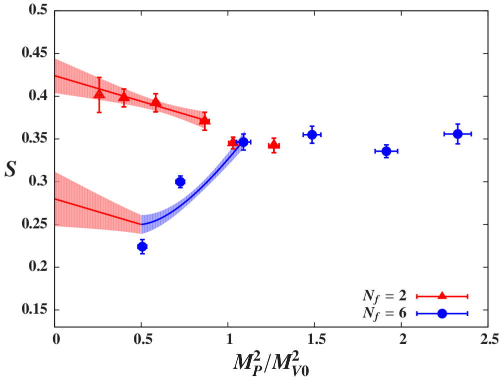
$$\lim_{M_P^2 \to 0} S = 0.42(2)$$

matches scaled-up QCD

Linear fit to light points ($M_P \lesssim M_{V0}$) guides the eye,
accounts for any chiral logs remaining after $\Delta S_{SM}(M_H)$

$$S = A + B\frac{M_P^2}{M_{V0}^2} + \frac{1}{12\pi}\left(\frac{N_F}{2} - 1\right)\log\left(\frac{M_{V0}^2}{M_P^2}\right) \quad \text{for } N_D = 1$$

Phenomenologically-relevant chiral limit



Direct $N_F = 2$ result

$$\lim_{M_P^2 \to 0} S = 0.42(2)$$

matches scaled-up QCD

Significant $N_F = 6$ reduction

- Lattice calculation involves $N_F^2 - 1$ degenerate pseudoscalars
- Only three massless Goldstones eaten by W and Z,

  $N_F^2 - 4$ must be given non-zero masses

Imagine freezing those $N_F^2 - 4$ masses at the blue curve's minimum,

and taking only three to zero mass

Connection to effective theory approach

The effective theory predicts the functional form of $\Pi_{V-A}(Q^2)$,
which we could have fit directly to determine $\alpha_1 = -S/16\pi^2$

Instead we used a rational function ansatz and had to subtract ΔS_{SM}

Advantages of effective theory analysis:

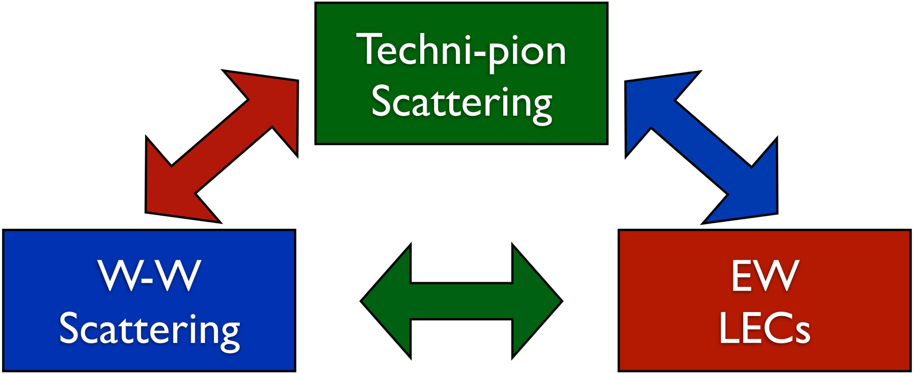
1. Eliminates model dependence of Q^2 rational function ansatz
2. Incorporates $\Delta S_{SM}(M_H)$ directly into fit
   (resolving potential ambiguity from radiative corrections to M_H)

Drawback: the analysis becomes more involved

Let's consider the simpler case of WW scattering to see how it goes

WW scattering from the lattice: The Big Picture

—WW scattering guaranteed to contain information about EWSB
—Very direct probe (though **not** easiest) at LHC
—On the lattice, restricted to **low-energy** scattering



(M. Buchoff)

WW scattering from the lattice: EFT matching

—Hadronic chiral lagrangian has $m > 0$ and $g = 0$
—Electroweak chiral lagrangian has $m = 0$ and $g > 0$
—Both reduce to same form in the limit $m \to 0$ and $g \to 0$

Hadronic EFT

EW EFT

$m_d \to 0$

$p^2 \ll M_{ds}^2, M_{ss}^2$

$g, g' \to 0$

$p^2 \ll M_{ds}^2, M_{ss}^2$

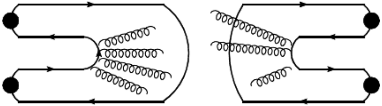
$$\frac{f^2}{4}\mathrm{tr}(\partial_\mu U^\dagger \partial^\mu U) + \alpha_5 \left[\mathrm{tr}(\partial_\mu U^\dagger \partial^\mu U)\right]^2 + \alpha_4 \left[\mathrm{tr}(\partial_\mu U^\dagger \partial_\nu U)\right]^2$$

Pseudoscalar scattering on the lattice: Target

"Simplest, cleanest, and best" scattering process

We focus on S-wave scattering of identical charged pseudoscalars (the $I = 2$ channel for $N_F = 2$)

- Other isospin channels (e.g., $I = 0$) can involve fermion-line-disconnected diagrams



Extremely expensive to evaluate on lattice

- Other spin channels (e.g., D-wave) have smaller signals, require higher precision

—We want to extract LECs ℓ_1 and ℓ_2 related to α_4 and α_5 in $\mathcal{L}_\chi$
—These hide in the scattering length a_{PP}

Pseudoscalar scattering on the lattice: Procedure

Maiani & Testa, 1990

No asymptotically non-interacting "in" and "out" states in euclidean spacetime
$\longrightarrow$ Lehmann–Symanzik–Zimmermann scattering formalism inapplicable

In a finite volume, measure energy of two-pseudoscalar state E_{PP}, projecting each correlator onto zero momentum for S-wave scattering

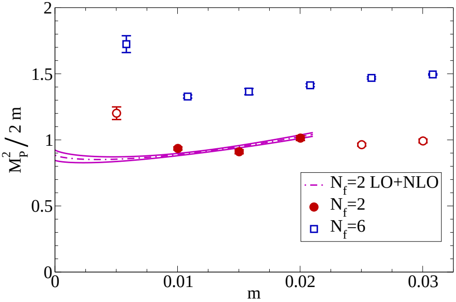
Access scattering phase shift δ from energy shift ΔE_{PP} (Lüscher, 1986)

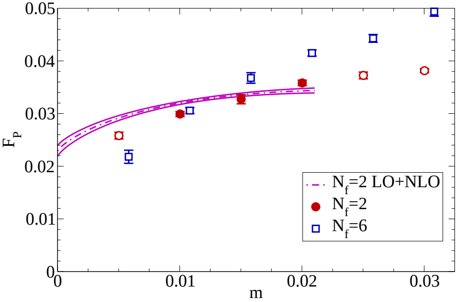
$$\Delta E_{PP} = E_{PP} - 2M_P = 2\sqrt{|\vec{k}|^2 + M_P^2} - 2M_P$$

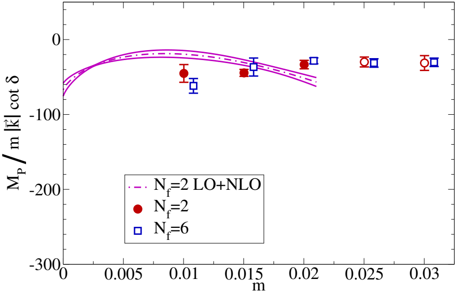
$$|\vec{k}| \cot\delta = \frac{1}{\pi L}\left[\sum_{\vec{\jmath}\neq 0}^{\Lambda_J} \frac{1}{|\vec{\jmath}|^2 - |\vec{k}|^2 L^2/(4\pi^2)} - 4\pi\Lambda_j\right]$$

Low-energy **scattering length** is $a_{PP} = \dfrac{1}{|\vec{k}|\cot\delta} + \mathcal{O}\left(\dfrac{|\vec{k}|^2}{M_P^2}\right)$

Joint chiral fit to M_P^2, F_P, $\langle\overline{\psi}\psi\rangle$ and $M_P a_{PP}$







—$a_{PP} \approx 1/|\vec{k}|\cot\delta$

—$\langle\overline{\psi}\psi\rangle$ in backup slide

—Only $N_F = 2$ fit feasible

—Fit restricted to solid points, $0.01 \le m_f \le 0.02$

—$\chi^2/\mathrm{dof} = 83/6$

$N_F = 2$ NLO contribution to WW scattering

—Chiral fit predicts sum of hadronic LECs $\ell_1 + \ell_2$
—EFT matching discussed above relates this to the sum $\alpha_4 + \alpha_5$
—Matching involves one-loop standard model calculation

$$\alpha_4 + \alpha_5 = \left(3.34 \pm 0.17^{+0.08}_{-0.71}\right) \times 10^{-3} - \frac{1}{128\pi^2}\left[\log\left(\frac{M_H^2}{v^2}\right) + \mathcal{O}(1)_{SM}\right]$$

(dominant systematic error from chiral fit range)

Context for our $N_F = 2$ result

Unitarity bounds [hep-ph/0604255]:

$$\alpha_4 + \alpha_5 \geq 1.14 \times 10^{-3} \qquad \alpha_4 \geq 0.65 \times 10^{-3}$$

Expected LHC bounds [hep-ph/0606118]: (99% CL; 100/fb; 14 TeV)

$$-7.7 < \alpha_4 \times 10^3 < 15 \qquad -12 < \alpha_5 \times 10^3 < 10$$

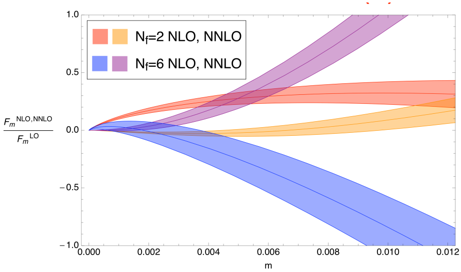
Complications for $N_F > 2$

- As for the S parameter, only charge one chiral doublet d
  Here we take the other $N_F - 2$ to be electroweak singlets s,
  leading to $N_F^2 - 4$ pseudoscalars with masses M_{ds} and M_{ss}

- Hadronic chiral perturbation theory (χPT) now involves **9** LECs
  with more complicated relations to α_4 and α_5

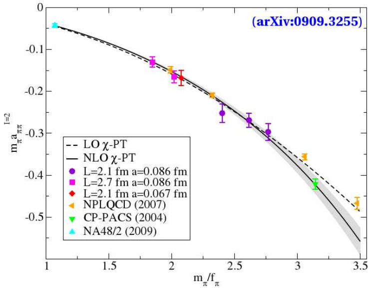
- Higher-order terms in χPT
  increase with N_F

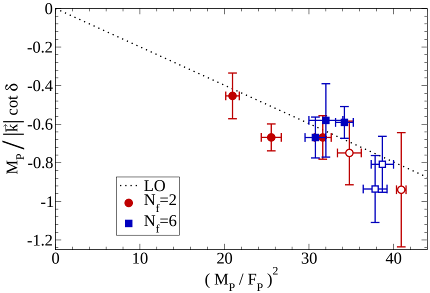
  Leads to smaller
  radius of convergence



Strategy: Reorganize chiral expansion

Replace low energy constants B and F by measured M_P and F_P
Expansion parameter is M_P^2/F_P^2, leading order is $M_P a_{PP} = -\frac{M_P^2}{16\pi F_P^2}$





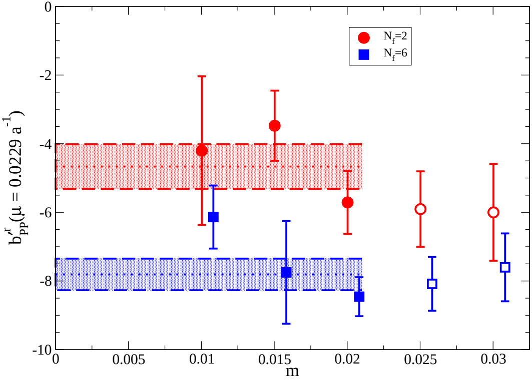
—An old story in QCD (Weinberg, 1966)
—Somewhat controversial: leading-order relation persists well beyond expected radius of convergence
—As a result, we can directly compare $N_F = 2$ and $N_F = 6$ LECs:
$N_F = 6$ scattering length only slightly smaller, but chiral logs differ...

Possible enhancement of WW scattering for $N_F = 6$

Combined LEC b'_{PP} must increase from $N_F = 2$ to $N_F = 6$,
to get similar a_{PP} despite different chiral logs

$$b'_{PP} = -256\pi^2 \left[L_0 + 2L_1 + 2L_2 + L_3 - 2L_4 - L_5 + 2L_6 + L_8\right]$$

contains α_4 and α_5, but we aren't able to isolate them



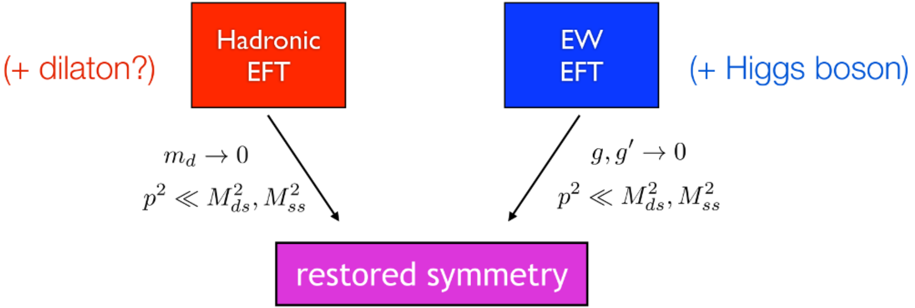
$b'_{PP} = -4.67 \pm 0.65^{+1.08}_{-0.05}$ (2f); $b'_{PP} = -7.81 \pm 0.46^{+1.23}_{-0.56}$ (6f)

Wrap-up & ongoing work: extend $\mathcal{L}_\chi$ with light Higgs

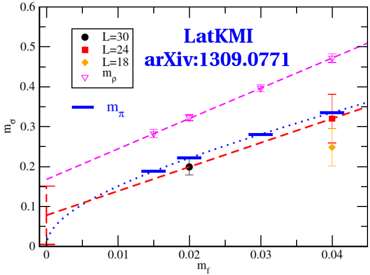
Effective theory approach helps lattice calculations of LECs
make contact with electroweak phenomenology (S, WW scattering)

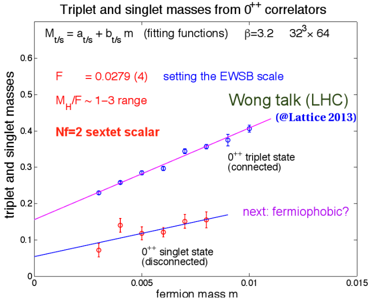
Matching EFTs provides rigor but requires complicated analyses

Now we have another complication: a Higgs too light to integrate out
Work in progress to include in extended effective theory



Backup: Light Higgs from lattice calculations



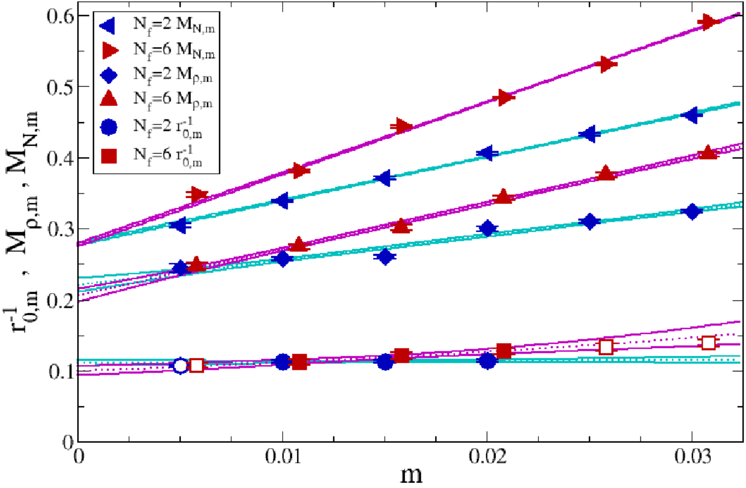


LatKMI Collaboration
SU(3) with $N_F = 8$ fundamental
$\frac{M_H}{v} = 4(4)$

Lattice Higgs Collaboration
SU(3) with $N_F = 2$ sextet
(two-index symmetric)
$\frac{M_H}{v} = 2(1)$

arXiv:1211.1083 claims radiative corrections
could shift $M_H \sim 600$ GeV $\longrightarrow$ 125 GeV (that is, $\frac{M_H}{v} \sim 2 \longrightarrow 0.5$)

Backup: Matching IR scales in the chiral limit

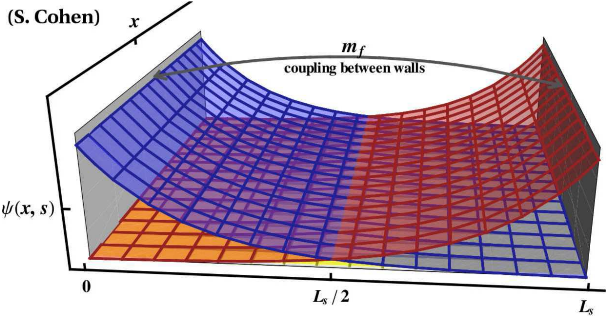


Vector mass, nucleon mass, and inverse Sommer scale

all match at 10% level between $N_F = 2$ and $N_F = 6$

$M_{V0} =$ 0.216(2) [2f]; 0.199(3) [6f]; (0.170(3) [8f] & 0.148(22) [10f])

Backup: More on domain wall fermions



- Domain wall fermions add fifth dimension of length L_s,
  a significant computational expense
- Exact chiral symmetry at finite lattice spacing in the limit $L_s \to \infty$,
  with nice continuum-like currents and flavor symmetries
- At finite $L_s = 16$, "residual mass" $m_{res} \ll m_f$; $m = m_f + m_{res}$
  $10^5 m_{res} = 2.6$ [2f]; 82 [6f]; 268 [8f]; 170 [10f]

Backup: Electroweak vacuum polarization functions

$$= ig_1 g_2 \cos\theta_w \sin\theta_w \Pi_{ee} \delta_{\mu\nu} + \ldots$$

$$= ig_1 g_2 \left(\Pi_{3e} - \sin^2\theta_w \Pi_{ee}\right) \delta_{\mu\nu} + \ldots$$

$$= \frac{ig_1 g_2}{\cos\theta_w \sin\theta_w} \left(\Pi_{33} - 2\sin^2\theta_w \Pi_{3e} + \sin^4\theta_w \Pi_{ee}\right) \delta_{\mu\nu} + \ldots$$

$$= ig_2^2 \Pi_{11} \delta_{\mu\nu} + \ldots$$

$$\Pi_{VV} = 2\Pi_{3e} \qquad \Pi_{AA} = 4\Pi_{33} - 2\Pi_{3e}$$

$$S = 4\pi N_D \lim_{Q^2 \to 0} \frac{d}{dQ^2}\left[\Pi_{VV}(Q^2) - \Pi_{AA}(Q^2)\right] - \Delta S_{SM}(M_H)$$

Backup: Scaling up QCD gives $S \gtrsim 0.4$

$N_F \geq 2$ fermions in fundamental rep of SU(N) for $N \geq 3$,
with $1 \leq N_D \leq N_F/2$ doublets given chiral electroweak charges

$$S \simeq 0.3 \frac{N_F}{2} \frac{N}{3} + \frac{N_D - 1}{12\pi} \log\left(\frac{M_V^2}{M_P^2}\right) + \frac{1}{12\pi} \log\left(\frac{\sim \text{TeV}^2}{M_H^2}\right)$$

1. Resonance contribution uses QCD phenomenology to model $R(s)$

$$4\pi \lim_{Q^2 \to 0} \frac{d}{dQ^2} \Pi_{V-A}(Q^2) = \frac{1}{3\pi} \int_0^\infty \frac{ds}{s} \left[R_V(s) - R_A(s)\right]$$

(essentially vector meson dominance with large-N scaling)

2. Chiral-log contribution based on leading-order chiral pert. theory
3. 125 GeV Higgs contributes $\sim$0.1 (leading-order estimate)

May be subtlety regarding M_H (cf. arXiv:1211.1083)
for strong sector in isolation (no EW or radiative corrections)

Backup: More on current correlators

$$S = 4\pi N_D \lim_{Q^2\to 0} \frac{d}{dQ^2}\Pi_{V-A}(Q^2) - \Delta S_{SM}(M_H)$$

$$\Pi^{\mu\nu}_{V-A}(Q) = Z\sum_x e^{iQ\cdot(x+\widehat{\mu}/2)}\mathrm{Tr}\left[\left\langle \mathcal{V}^{\mu a}(x)V^{\nu b}(0)\right\rangle - \left\langle \mathcal{A}^{\mu a}(x)A^{\nu b}(0)\right\rangle\right]$$

$$\Pi^{\mu\nu}(Q) = \left(\delta^{\mu\nu} - \frac{\widehat{Q}^\mu\widehat{Q}^\nu}{\widehat{Q}^2}\right)\Pi(Q^2) - \frac{\widehat{Q}^\mu\widehat{Q}^\nu}{\widehat{Q}^2}\Pi^L(Q^2) \qquad \widehat{Q} = 2\sin(Q/2)$$

- Renormalization constant Z evaluated non-perturbatively
  Chiral symmetry of domain wall fermions $\Longrightarrow Z = Z_A = Z_V$
  $Z = 0.85$ [2f]; 0.73 [6f]; 0.70 [8f]; 0.71 [10f]

- Conserved currents $\mathcal{V}$ and $\mathcal{A}$ ensure that lattice artifacts cancel…

Backup: Conserved and local domain wall currents

Conserved currents:

$$\mathcal{V}^a_\mu(x) = \sum_{s=0}^{L_s-1} j^a_\mu(x,s) \qquad \mathcal{A}^a_\mu(x) = \sum_{s=0}^{L_s-1} \mathrm{sign}\left(s - \frac{L_s-1}{2}\right) j^a_\mu(x,s)$$

$$j^a_\mu(x,s) = \overline{\Psi}(x+\widehat{\mu},s)P_{+\mu}\tau^a U^\dagger_{x,\mu}\Psi(x,s) - \overline{\Psi}(x,s)P_{-\mu}\tau^a U_{x,\mu}\Psi(x+\widehat{\mu},s)$$

where $P_{\pm\mu} \equiv \dfrac{1\pm\gamma_\mu}{2}$

Local currents:

$$V^a_\mu(x) = \overline{q}(x)\gamma_\mu\tau^a q(x) \qquad A^a_\mu(x) = \overline{q}(x)\gamma_\mu\gamma_5\tau^a q(x)$$

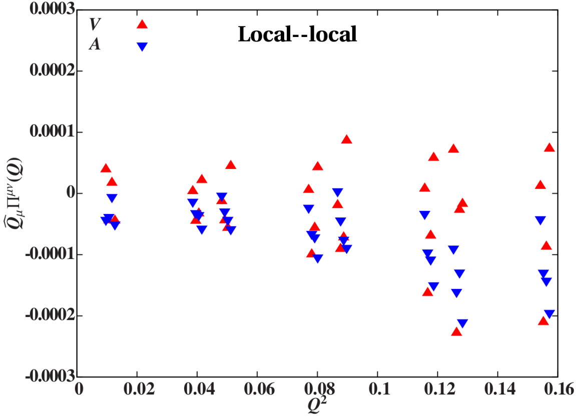
$$q(x) = \frac{1-\gamma_5}{2}\Psi(x,0) + \frac{1+\gamma_5}{2}\Psi(x,L_s-1)$$

Backup: Non-conservation of local currents

$$\Pi^{\mu\nu}_{V-A}(Q) = Z \sum_x e^{iQ\cdot(x+\widehat{\mu}/2)} \mathrm{Tr}\left[\left\langle \mathcal{V}^{\mu a}(x) V^{\nu b}(0)\right\rangle - \left\langle \mathcal{A}^{\mu a}(x) A^{\nu b}(0)\right\rangle\right]$$

Local currents are simply $\overline{q}\gamma_\mu q$, defined on the domain walls

No Ward identity: $\widehat{Q}_\mu[\sum_x e^{iQ\cdot x}\left\langle V^a_\mu(x) V^a_\nu(0)\right\rangle] \neq 0$

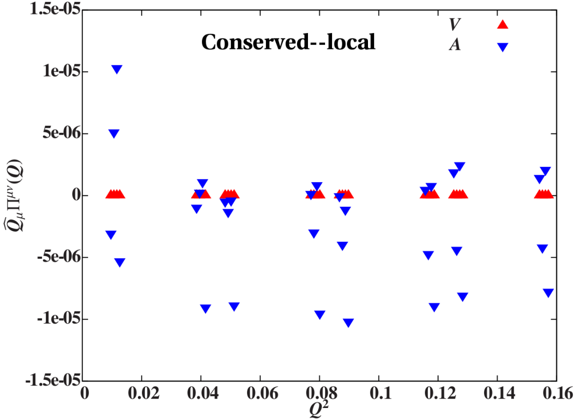


Backup: Ward identity for conserved currents

$$\Pi^{\mu\nu}_{V-A}(Q) = Z \sum_x e^{iQ\cdot(x+\widehat{\mu}/2)} \mathrm{Tr}\left[\left\langle \mathcal{V}^{\mu a}(x) V^{\nu b}(0)\right\rangle - \left\langle \mathcal{A}^{\mu a}(x) A^{\nu b}(0)\right\rangle\right]$$

Conserved currents are point-split, summed over fifth dimension

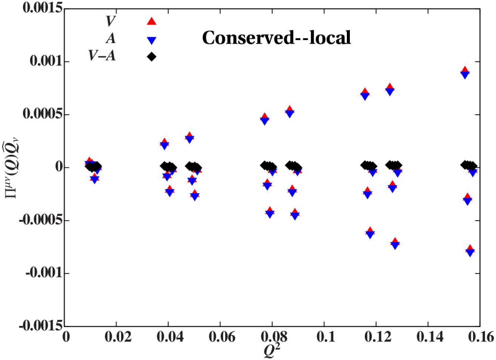
Obey Ward identity, PCAC: $\widehat{Q}_\mu[\sum_x e^{iQ\cdot(x+\widehat{\mu}/2)} \left\langle \mathcal{V}^a_\mu(x) V^a_\nu(0)\right\rangle] = 0$



Backup: Lattice artifacts cancel in mixed correlators

Plot shows divergence of local current in each correlator,

$$\text{e.g.,} \quad \left[\sum_x e^{iQ\cdot(x+\widehat{\mu}/2)} \left\langle \mathcal{V}^a_\mu(x) V^a_\nu(0) \right\rangle \right] \cdot \widehat{Q}_\nu \neq 0$$

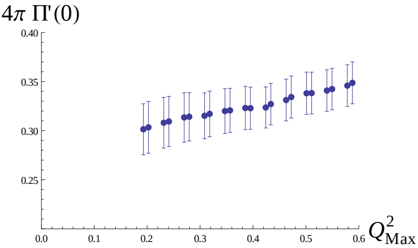


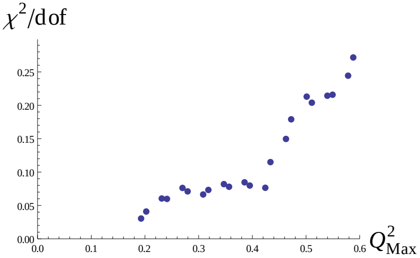
Cancellation seems due to conserved currents forming exact multiplet, also possible with overlap — even staggered (Y. Aoki @ Lattice 2013)

Backup: Padé fit Q^2-range dependence

Uncorrelated fits to "Padé-(1, 2)" rational function,

$$\Pi_{V-A}(Q^2) = \frac{a_0 + a_1 Q^2}{1 + b_1 Q^2 + b_2 Q^4} = \frac{\sum_{m=0}^{1} a_m Q^{2m}}{\sum_{n=0}^{2} b_n Q^{2n}}$$



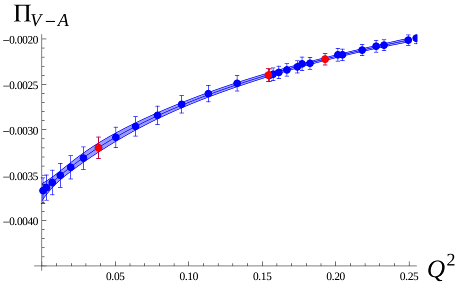


Results reported above use $Q^2_{Max} = 0.4$

Backup: Twisted boundary conditions for $\Pi_{V-A}(Q^2)$

Twisted boundary conditions (TwBCs)

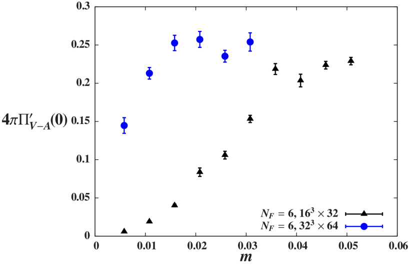
- Introduce external abelian field (add phase at lattice boundaries)
- Allows access to arbitrary Q^2, not just lattice modes $2\pi n/L$

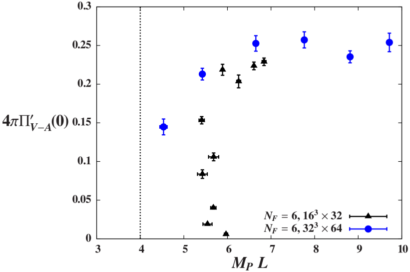


- Correlations $\Longrightarrow$ TwBCs do not improve Padé fit results for slope
- May help fits to chiral perturbation theory,
  where we need both small M_P and small Q^2

Backup: Spurious $S \to 0$ from finite volume effects

Compare $N_F = 6$ results on $16^3 \times 32$ and $32^3 \times 64$ lattice volumes:





$L = 16$ results crash to zero as $m \longrightarrow 0$,

attributable to volume-induced parity doubling

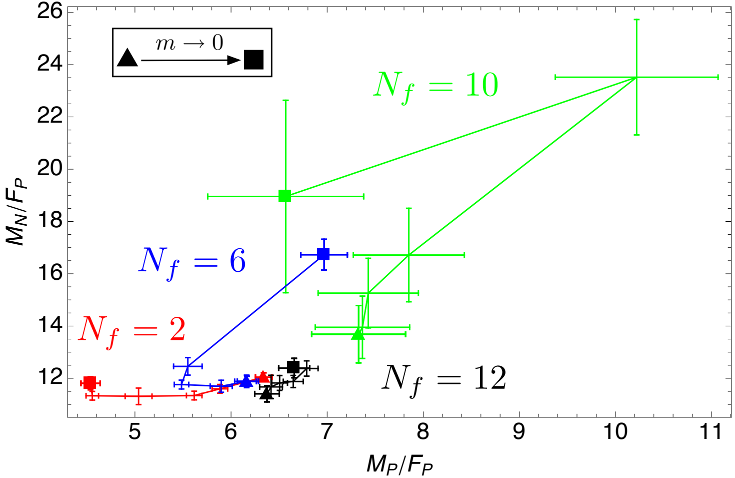
In tandem, M_P freezes around $M_P L \approx 5.5$

$L = 32$ results show no such distortion of the spectrum,

but we want more quantitative control over finite-volume effects

Backup: Finite-volume diagnostic plot

Expect finite-volume effects to push points up and to the right

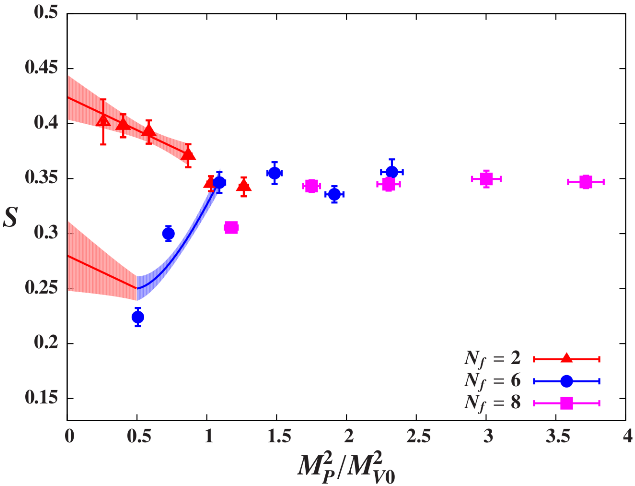


Finite-volume effects may be significant in lightest $N_F = 6$ point

($N_F = 12$ data from Fodor *et al.*, PLB 703:348 (2011) [1104.3124])

Backup: 8f results for S parameter

Assuming $M_{V0} > 0$

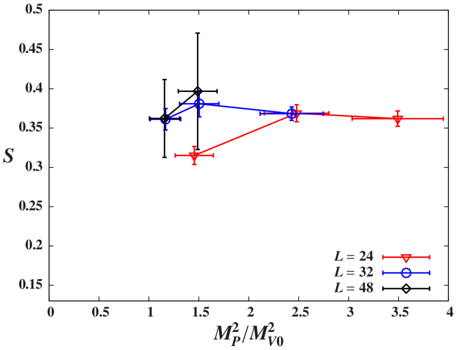


Expect agreement in the quenched limit $M_P^2 \to \infty$

10f analysis requires careful treatment of frozen topological charge

Backup: $N_F = 8$ domain wall on staggered

—USBSM is investigating $N_F = 8$ on volumes up to $64^3 \times 128$
(initial results recently released in arXiv:1310.7006)
—I am carrying out mixed-action measurements for S
—May provide more information about finite-volume effects



Lightest $24^3 \times 48$ point shows clear finite-volume effects

$N_F = 6$ reduction began only for $M_P^2 \lesssim M_{V0}^2$

Also working on calculation with staggered fermions

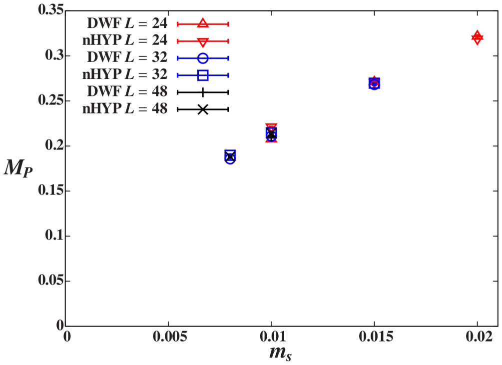
Currently measuring smaller masses on $48^3 \times 96$, generating $64^3 \times 128$

Backup: Mixed action procedure (LHPC, arXiv:0705.4295)

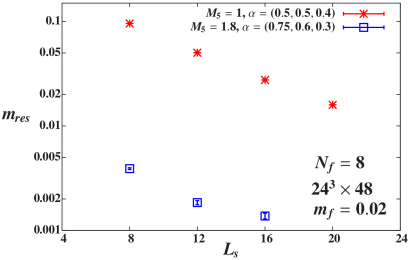
- HYP smear to reduce m_{res} and get renormalization factors $Z \sim 1$
- Tune domain wall height M_5 and length L_s of fifth direction so that residual chiral symmetry breaking $m_{res} \ll m$
- Tune bare valence mass m_f so that M_P matches unitary value

$M_5 = 1.8$ and $L_s = 16$
$\longrightarrow Z_V \approx Z_A \approx 1.08$
$\longrightarrow m_{res} \approx 0.001 \lesssim m_f/13$

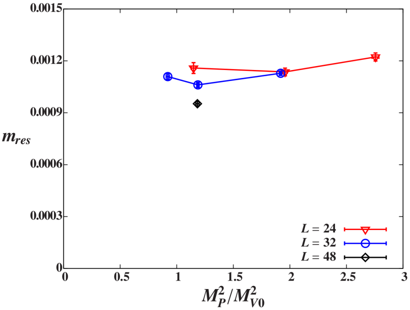
Need $m_f > m_s$ to match M_P:
$$1.7 \lesssim m/m_s \lesssim 2.05$$
where $m \equiv m_f + m_{res}$

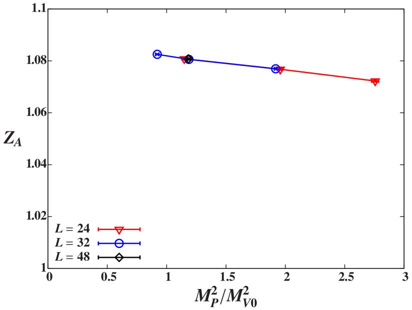


Backup: Valence domain wall m_{res} and Z_A



- Want $m_{res} \ll m$
- Want $Z_A \sim 1$





Backup: NLO chiral expansions

For general N_F, $A = 2 - N_F + 2N_F^2 + N_F^3$

$$M_P a_{PP} = -\frac{2mB}{16\pi F^2}\left\{1 + \frac{2mB}{(4\pi F)^2}\left[b_{PP} - 2\frac{N_F - 1}{N_F^2} + \frac{A}{N_F^2}\log\left(\frac{2mB}{\mu^2}\right)\right]\right\}$$

$$M_P^2 = 2mB\left\{1 + \frac{2mB}{(4\pi F)^2}\left[b_M + \frac{1}{N_F}\log\left(\frac{2mB}{\mu^2}\right)\right]\right\}$$

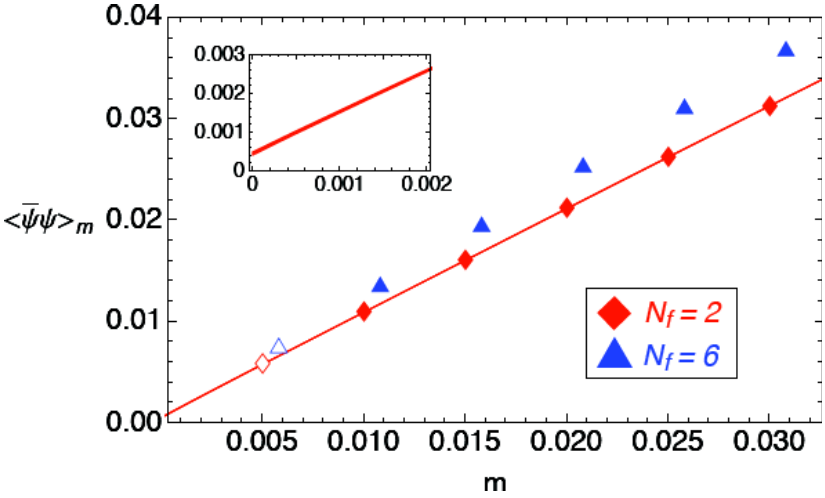
$$F_P = F\left\{1 + \frac{2mB}{(4\pi F)^2}\left[b_F - \frac{N_F}{2}\log\left(\frac{2mB}{\mu^2}\right)\right]\right\}$$

$$\langle\overline{\psi}\psi\rangle = \frac{F^2 2mB}{2m}\left\{1 + \frac{2mB}{(4\pi F)^2}\left[b_C - \frac{N_F^2 - 1}{N_F}\log\left(\frac{2mB}{\mu^2}\right)\right]\right\}$$

- LECs b are all linear combinations of low-energy constants L_i
- LECs' dependence on scale μ cancels the corresponding logs
- b_C includes "contact term" $m\Lambda^2 \sim m/a^2$
- NNLO M_P^2 coefficients enhanced by N_F^2

(arXiv:0910.5424)

Backup: Chiral condensate with chiral fit



Joint NNLOχPT fit to $N_F = 2$ F_P, M_P^2, $\langle\overline{\psi}\psi\rangle$

Linear term clearly dominant

Backup: Reorganized chiral expansion for $M_P a_{PP}$

Solve chiral expansions for measured M_P and F_P
replace low-energy constants $2mB$ and F by M_P^2/F_P^2:

$$M_P a_{PP} = -\frac{M_P^2}{16\pi F_P^2}\left\{1 + \frac{M_P^2}{16\pi^2 F_P^2}\left[b'_{PP} - 2\frac{N_f - 1}{N_f^2} + 2\frac{1 - N_f + N_f^2}{N_f^2}\log\left(\frac{M_P^2}{\mu^2}\right)\right]\right\}$$

Now $b'_{PP} = -256\pi^2\left[L_0 + 2L_1 + 2L_2 + L_3 - 2L_4 - L_5 + 2L_6 + L_8\right]$
No explicit factors of N_f in b'_{PP},
all N_f dependence due to dynamics affecting LECs L_i
Unable to untangle L_i to recover $\ell_1, \ell_2 \longrightarrow \alpha_4, \alpha_5$

Backup: Chiral perturbation theory for $\Pi_{V-A}(Q^2)$

$\Pi_{V-A}(Q^2)$ in hadronic χPT:

$$\Pi_{V-A}(M_{dd}^2, Q^2) = -F_P^2 - Q^2 \left[8L_{10}^r(\mu) + \frac{1}{24\pi^2}\left\{\log\left[\frac{M_{dd}^2}{\mu^2}\right] + \frac{1}{3} - H\left(\frac{4M_{dd}^2}{Q^2}\right)\right\}\right]$$

$$H(x) = (1+x)\left[\sqrt{1+x}\log\left(\frac{\sqrt{1+x}-1}{\sqrt{1+x}+1} + 2\right)\right]$$

Match with $S = -16\pi^2\alpha_1$ in electroweak chiral lagrangian:

$$S(\mu, M_{ds}) = \frac{1}{12\pi}\left[-192\pi^2\left(L_{10}^r(\mu) + \frac{1}{384\pi^2}\left\{\log\left[\frac{M_{ds}^2}{\mu^2}\right] + 1\right\}\right) + \log\left[\frac{\mu^2}{M_H}\right] - \frac{1}{6}\right].$$