Lattice Strong Dynamics
Turning it up to ten

David Schaich, 19 April 2012

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LSD Collaboration

Broad Outline

- LSD Philosophy and Program (brief review)
- Lattice gauge theory with ten light fermions:

  Background Lattice Issues Results

Lattice Strong Dynamics Collaboration

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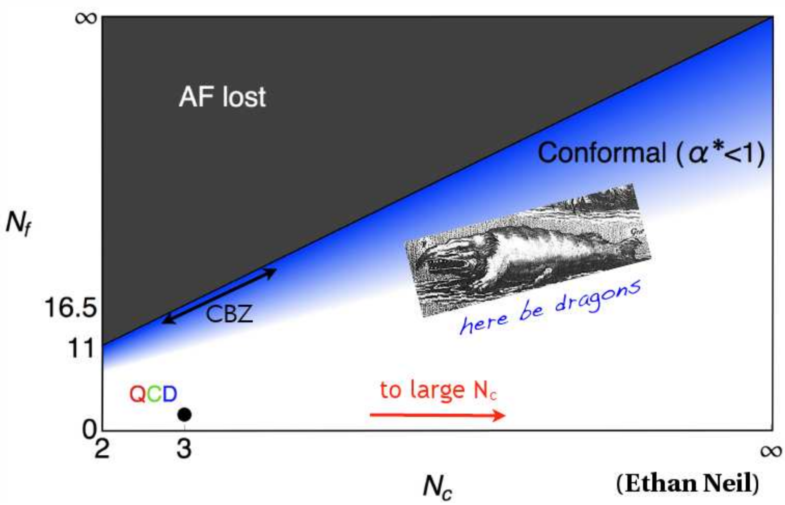
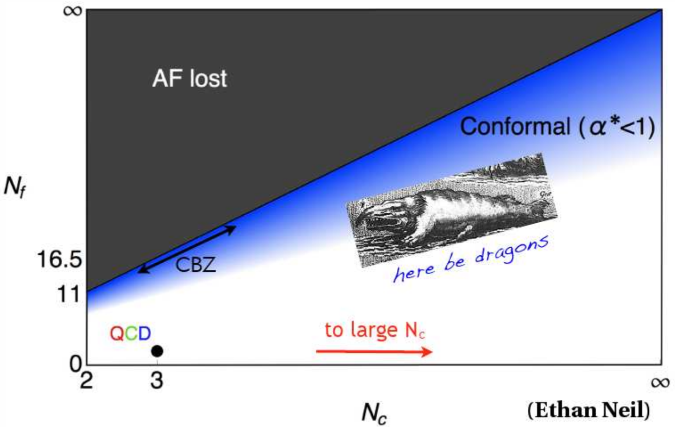
UC Davis	Joseph Kiskis
U Wash.	Saul Cohen
Yale	Thomas Appelquist, George Fleming, Meifeng Lin, Gennady Voronov

Performing non-perturbative studies of strongly interacting theories likely to produce observable signatures at the Large Hadron Collider

What is the range of possible behavior of strongly-coupled systems?

SU(N_c) gauge theories with N_f massless fundamental fermions
(similar picture for other representations)

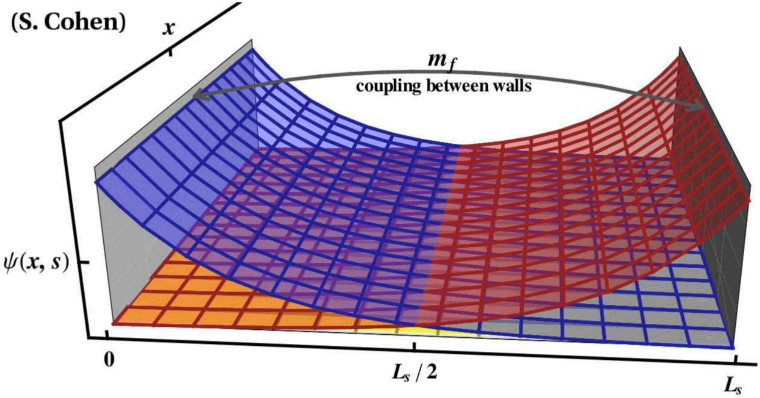




Dragons

- Large "theory space" – N_c, N_f, fermion representation
- Coupling runs more slowly $\Longrightarrow$ lattice artifacts can be more severe
- (Potentially) widely separated scales overflow the lattice volume
- We don't know the answer $\Longrightarrow$ systematic effects harder to assess

Domain wall fermions address (some) systematics



- Form a fifth dimension from L_s copies of the 4d gauge fields
- Exact chiral symmetry at finite lattice spacing in the limit $L_s \to \infty$
- At finite L_s, "residual mass" $m_{res} \ll m_f$; $\quad m = m_f + m_{res}$
- $32^3 \times 64$ with $L_s = 16$: **significant computational expense**

  $m_{res} \approx 2.6 \times 10^{-5}$ [2f]; $\quad 82 \times 10^{-5}$ [6f]; $\quad 170 \times 10^{-5}$ [10f]

100s of millions of core-hours on clusters and supercomputers

Livermore Nat’l Lab; USQCD (DOE); XSEDE (NSF); etc.

Lattice Strong Dynamics projects

Strategy

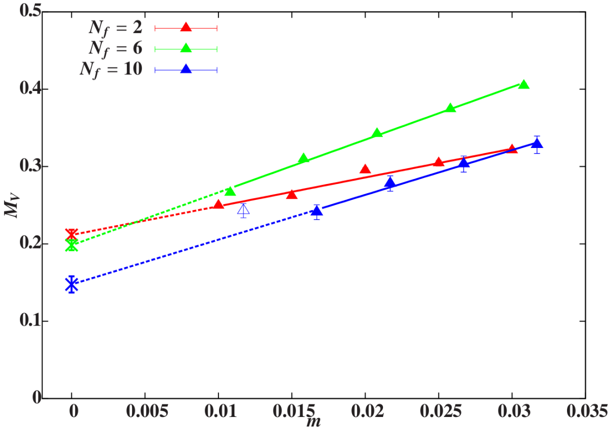
- Focus on QCD-like analyses, using lattice QCD as baseline
- Explore trends for increasing $N_f = 2 \longrightarrow 6 \longrightarrow 10$
- Attempt to match IR scale(s) for more direct comparison
- Use domain wall fermions for good chiral and flavor symmetries

Goals and results

- For $N_f = 6$, we studied the spectrum, S parameter, WW scattering and baryonic form factors
- Simultaneously (in 2009!), we started working on $N_f = 10$, planning to carry out a similar program
- The $N_f = 10$ project encountered many issues (some anticipated)

Attempt to match IR scales (vector meson mass)

Fix coupling $\beta = 6/g^2$, consider a range of fermion masses m

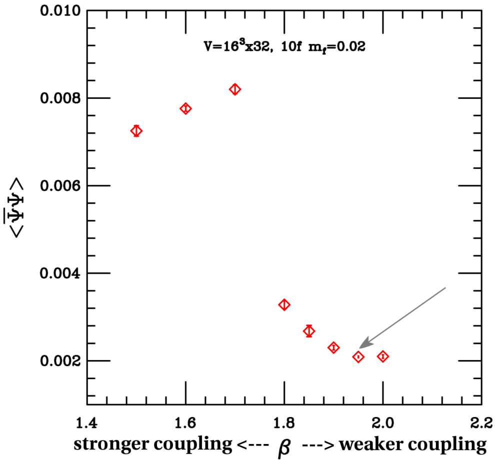


For $N_f = 10$, $M_{V0} \equiv \lim_{m \to 0} M_V$ seems too small

$\Longrightarrow$ We should have used a stronger coupling

Issue: Strong-coupling lattice artifacts

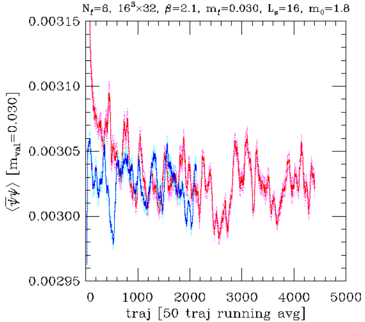
At strong couplings, lattice artifacts change behavior of the system

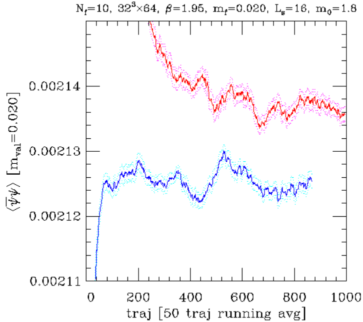


Visible effects in $\langle\overline{\psi}\psi\rangle$ at any stronger coupling than we used

Issue: Thermalization and autocorrelations

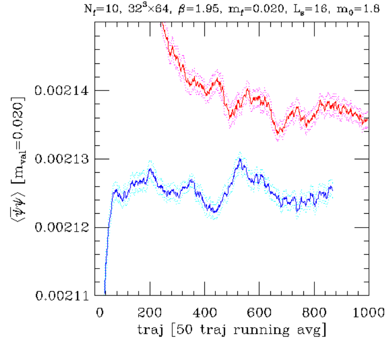
- We generate a Markov chain of gauge field configurations
  $\Longrightarrow$ Nearby links in the chain are correlated
- From initial state, system thermalizes to equilibrium distribution
- Independent measurements require autocorrelations to die off

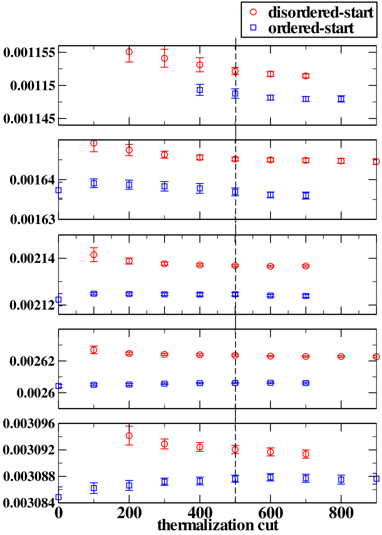




Independent ensembles starting from either random or ordered states
$\Longrightarrow$ Find thermalization time from convergence to equilibrium

Issue: Thermalization and autocorrelations

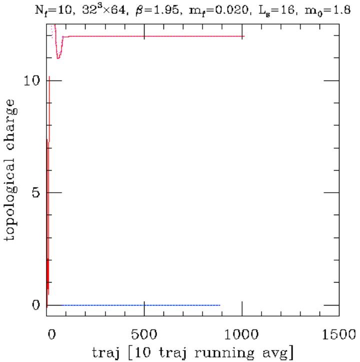


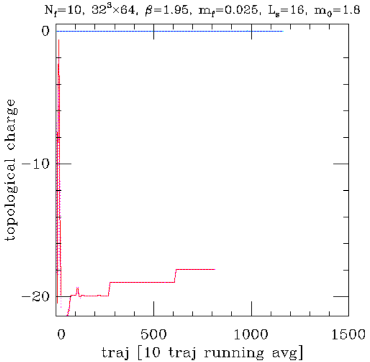


- Random and ordered initial states
- Goal: convergence to equilibrium
  $\Longrightarrow$ thermalization time
- Result: two independent samples

Issue: Fixed topology

Topological observables typically have the longest autocorrelations





Topological charge is (nearly) fixed throughout each Markov chain
$Q = 0$ from ordered starts, $Q \neq 0$ (sometime large) for random starts
$\Longrightarrow$ Could this explain why the ensembles don't converge?

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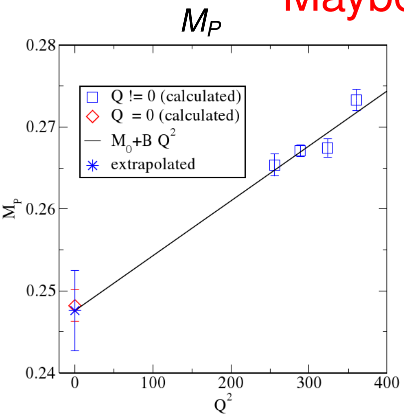
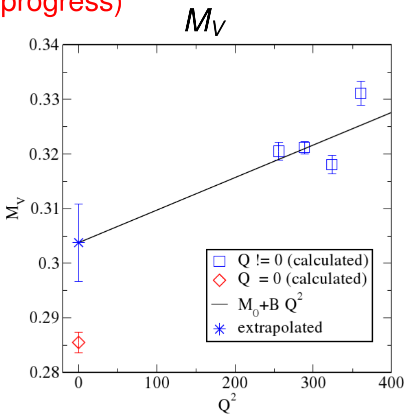
$\Longrightarrow$ Could this explain why the ensembles don't converge?

Maybe (work in progress)

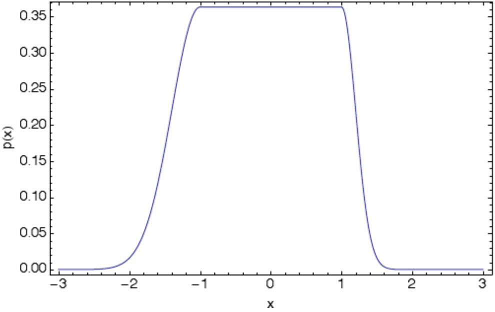




Can conclude true distribution centered between the two ensembles
More robust analysis requires topological susceptibility

Combination procedure

- Only two independent samples of the statistical distribution
- Topological effects $\Longrightarrow$ distribution centered between the samples
- Estimate width of distribution from **maximum** difference between the two samples for each fermion mass



(Ethan Neil)

Model: Half-Gaussian distributions on either side, uniform in the middle

Resulting combined data agree with simple weighted averages, but have larger error bars

Combination procedure

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- Topological effects $\Longrightarrow$ distribution centered between the samples
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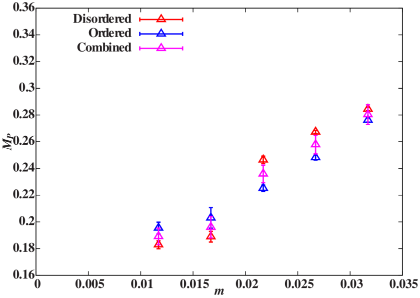
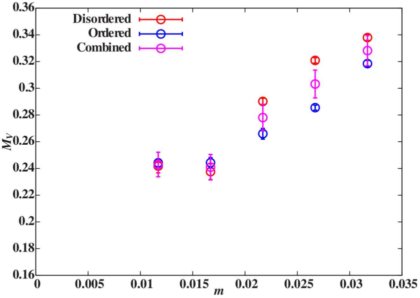
M_P



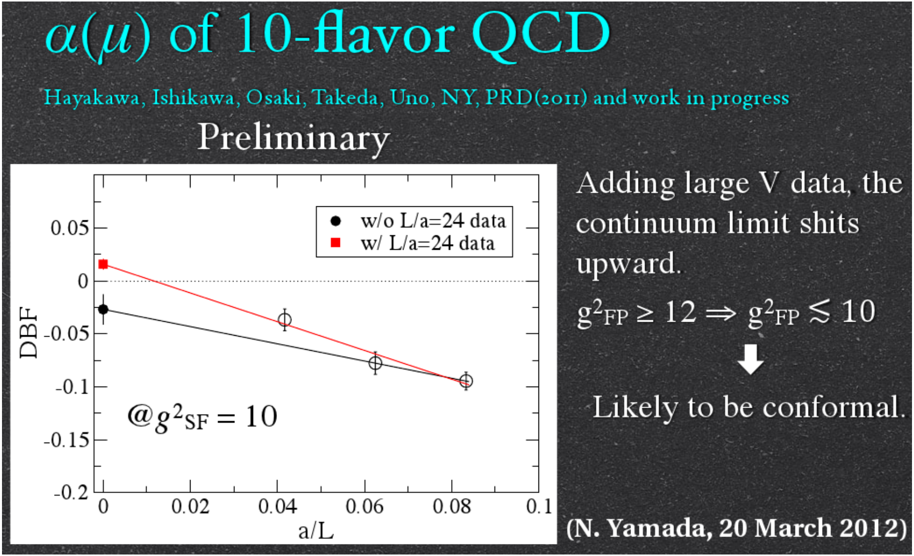
M_V



Resulting combined data agree with simple weighted averages, but have larger error bars

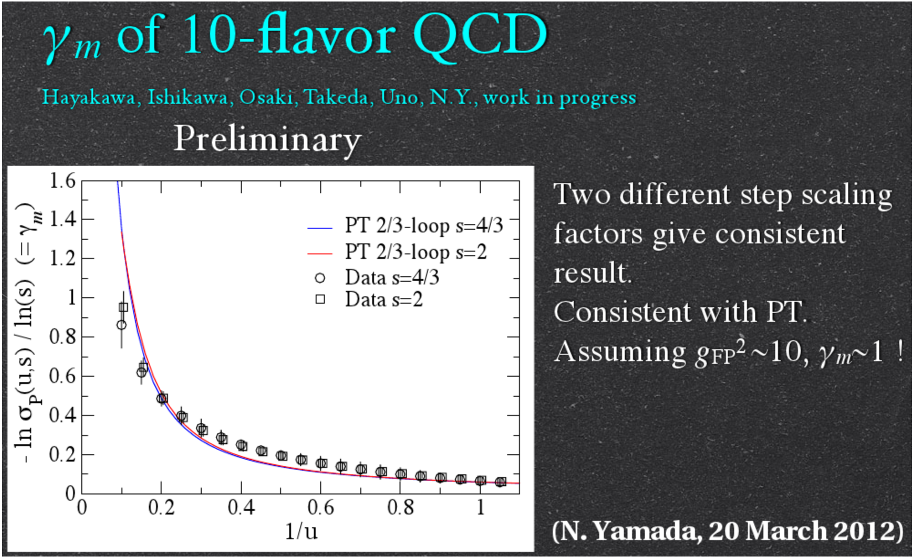
Meanwhile, in Japan...

Indication of strongly-coupled IR fixed point for $N_f = 10$

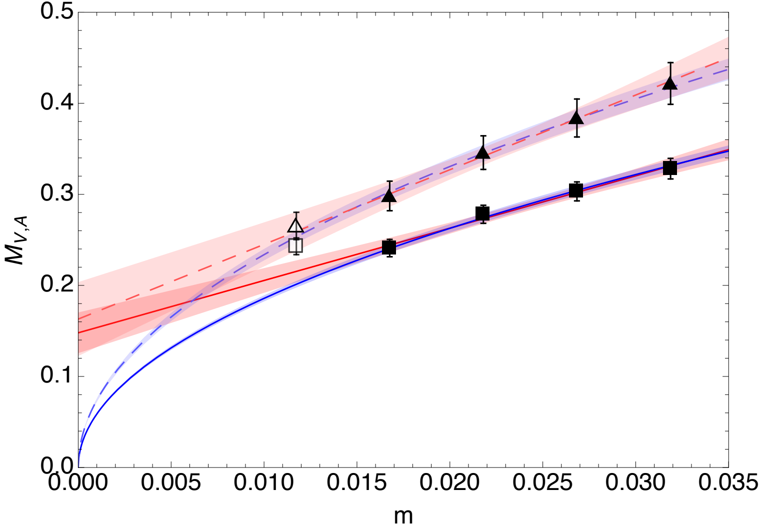


Meanwhile, in Japan. . .

Indication of large mass anomalous dimension $\gamma_m \sim 1$ for $N_f = 10$



Issue: How to decide IR conformality from spectrum?



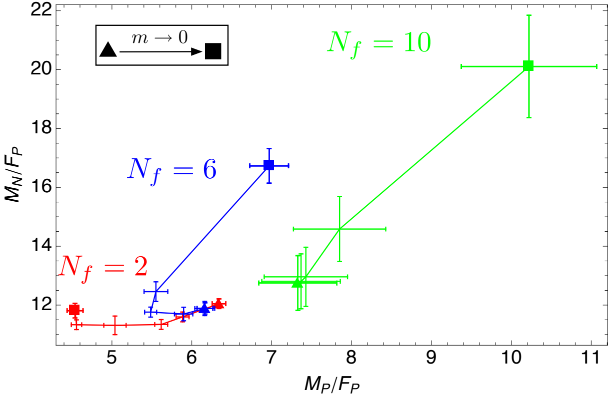
(M_A, M_V)

Compare $M = C + Bm$ with $M = Bm^{1/(1+\gamma_m)} \rightarrow B\sqrt{m}$ for $\gamma_m = 1$
Fit only to solid points ($m_f \geq 0.015$) to control finite-volume effects...

Issue: Finite-volume effects

Range of accessible masses determined by lattice volume

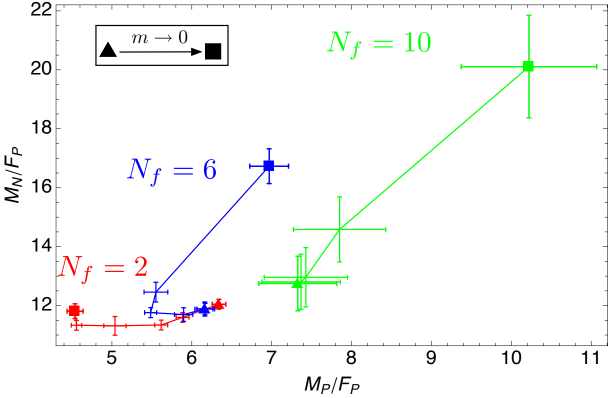
If masses get too small, finite-volume effects significant



In this diagnostic, finite-volume effects push points up and to the right by increasing the masses but decreasing F_P

Expectations

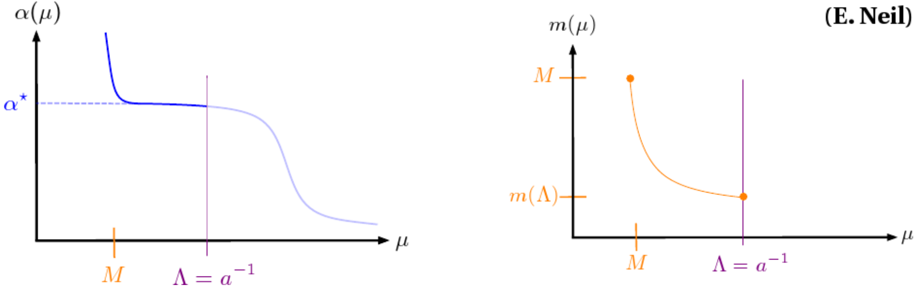
QCD-like: $M_P \to 0$ as $m \to 0$, while $F_P > 0$ and $M_N > 0$ (cf. $N_f = 2$)
IR conformal: All $\propto m^{1/[1+\gamma_m]} \Longrightarrow$ ratios should stay roughly constant



$N_f = 10$ ratios are roughly constant: consistent with IR-conformality
as well as QCD-like dynamics with large m and finite-volume effects

Mass-deformed IR-conformal spectrum analysis

Conformality explicitly broken by lattice spacing, volume, fermion mass



- IR fixed point governs physics up to lattice cutoff $\Lambda = a^{-1}$
- Small fermion mass $m(\Lambda) = m$ at cutoff runs according to $\gamma_\star$
- Fermions screen out around $m(M) = M$, inducing confinement
  All masses and decay constants scale $\sim m^{1/(1+\gamma_\star)}$

A slowly-running theory will look IR-conformal for *m* too large

Mass-deformed IR-conformal spectrum analysis

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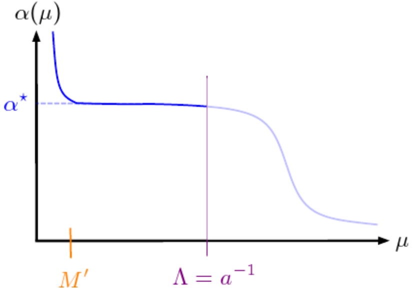

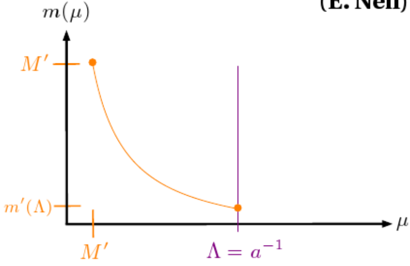

(E. Neil)

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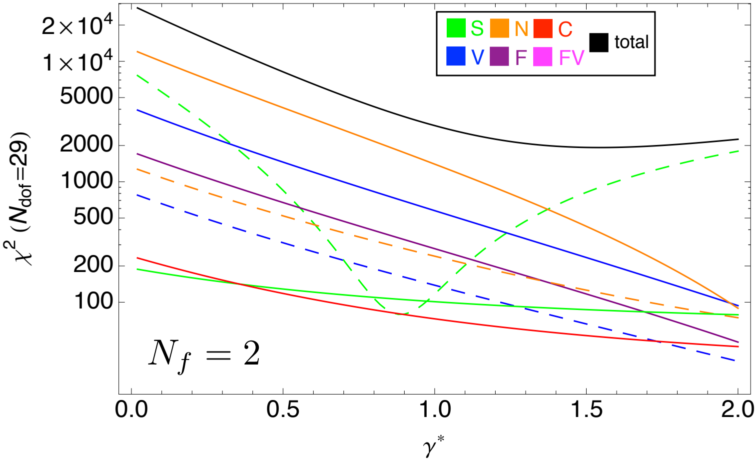
A slowly-running theory will look IR-conformal for m too large

Conformal fit χ^2 vs. γ_m, $N_f = 2$

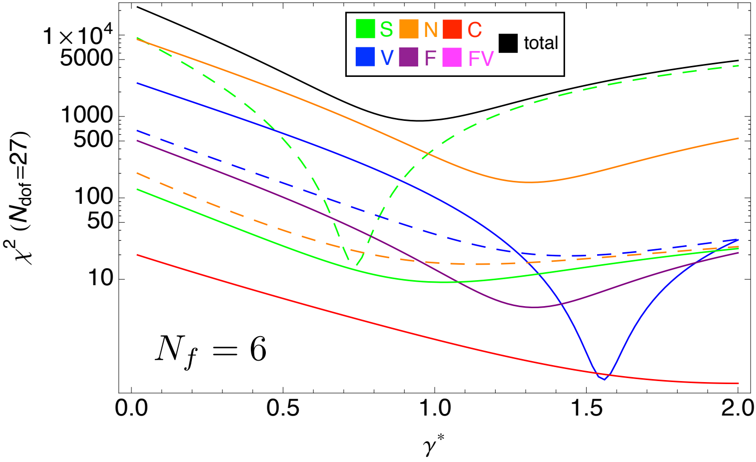
As N_f increases, minima develop and move to smaller γ_m



$N_f = 2$ is QCD; only M_P shows a minimum: $\gamma_m \approx 1 \Longrightarrow M_P \sim m^{1/2}$

Conformal fit χ^2 vs. γ_m, $N_f = 6$

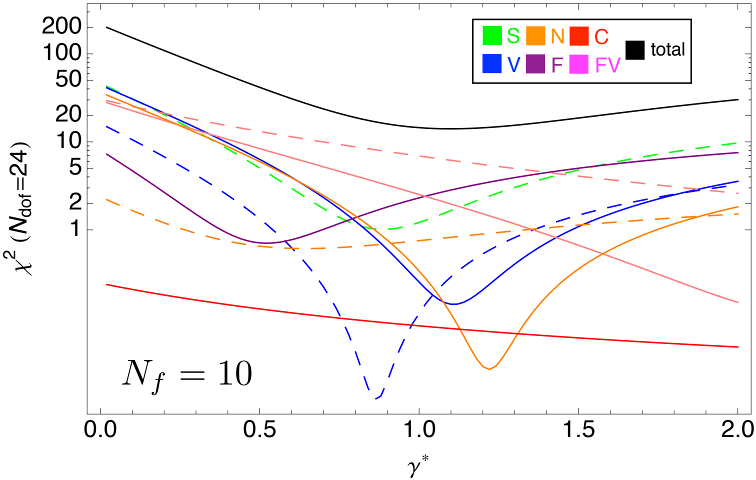
As N_f increases, minima develop and move to smaller γ_m



$N_f = 6$ is QCD-like; minima around $\gamma_m \approx 1.5$ are spurious

Conformal fit χ^2 vs. γ_m, $N_f = 10$

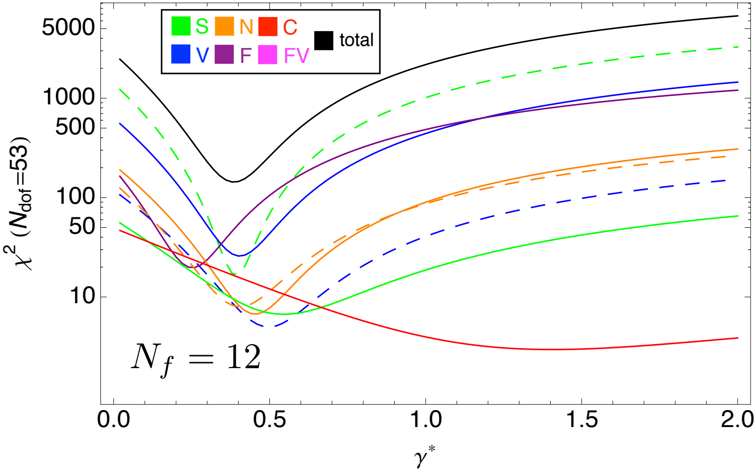
As N_f increases, minima develop and move to smaller γ_m



$m_f \geq 0.015$; Relatively small χ^2 may be due to conservative error bars

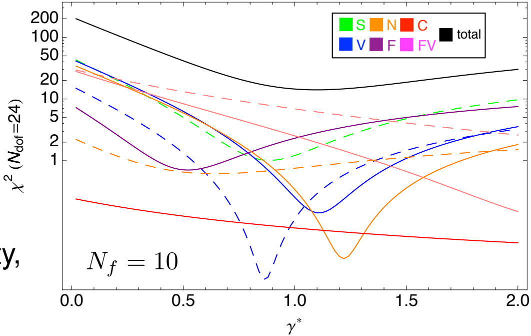
Conformal fit χ^2 vs. γ_m, $N_f = 12$ comparison

As N_f increases, minima develop and move to smaller γ_m



$N_f = 12$ data from Fodor *et al.*, PLB 703:348 (2011) [1104.3124]

$N_f = 10$ fit results



$N_f = 10$ spectrum appears consistent with IR-conformality, $\gamma_m \approx 1$

Global fit with $m_f \geq 0.015$: $\gamma_m = 0.999(11)_{\text{stat}}$ $\chi^2/dof = 16/24$

Restricting to $m_f \geq 0.02$: $\gamma_m = 0.988(17)_{\text{stat}}$ $\chi^2/dof = 5/15$

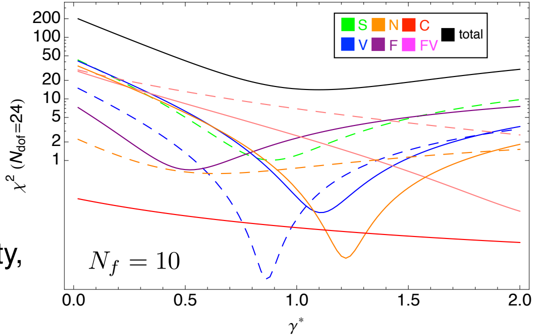
Compare quality of joint NLO chiral fits to M_P, F_P and $\langle\overline{\psi}\psi\rangle$

$m_f \geq 0.015$: $\chi^2/dof = 176/7$ $m_f \geq 0.02$: $\chi^2/dof = 85/4$

However, NLO chiral fit needs $m \lesssim 0.005$ to converge

$\Longrightarrow$ Cannot rule out spontaneous chiral symmetry breaking

Conclusions and next steps



$N_f = 10$ spectrum appears consistent with IR-conformality, $\gamma_m \approx 1$

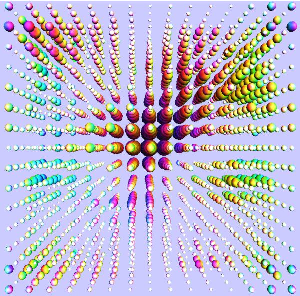
What is to be done?

- Improve analysis of topological effects
- Investigate smaller volumes to understand finite-volume effects and perform finite-volume scaling analyses
- Explore different couplings on smaller volumes

It is not clear whether more expensive analyses will be worthwhile (*S* parameter, WW scattering, form factors)

Backup: Hybrid Monte Carlo algorithm

1. Generate random “momenta” with gaussian distribution
2. Molecular dynamics evolution through fictitious MD “time” to produce new four-dimensional field configuration
3. Use MD discretization errors in Metropolis accept/reject step



Numerically evaluate observables from the defining functional integral

$$\langle\mathcal{O}\rangle = \frac{\int \mathcal{D}U \;\; \mathcal{O}(U) \;\; e^{-S(U)}}{\int \mathcal{D}U \;\; e^{-S(U)}}$$

U: four-dimensional field configurations
S: action giving probability distribution e^{-S}

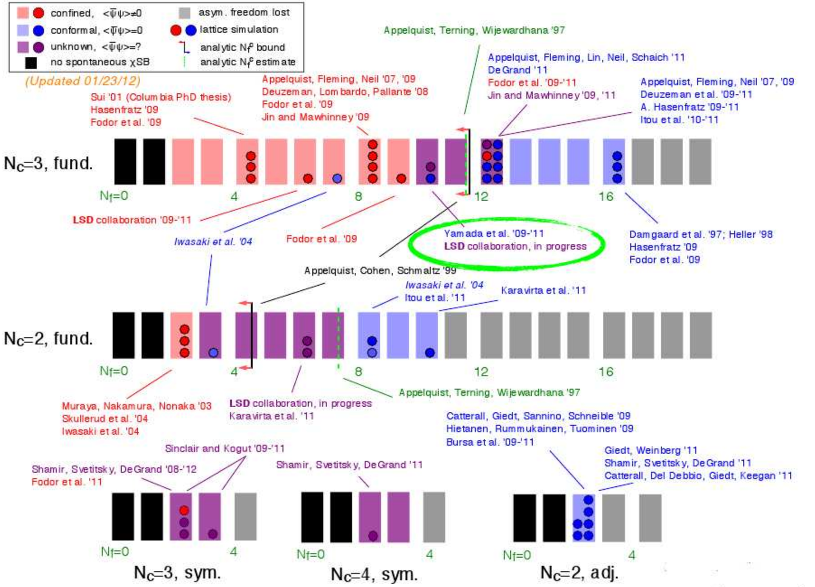
Backup: Domain wall Dirac operator

$$D^W_{x,y}(M_5) = (4 - M_5)\delta_{x,y} - \frac{1}{2}\Big[(1+\gamma^\mu)U^\dagger_{x,\mu}\delta_{x,y+\mu} + (1-\gamma^\mu)U_{x,\mu}\delta_{x+\mu,y}\Big]$$

$$D_{s,s'}(m) = \Big[D^W(M_5) + 1\Big]\delta_{s,s'} + P_L\left[(1+m)\delta_{s,L_s-1}\delta_{s',0} - \delta_{s+1,s'}\right] + P_R\left[(1+m)\delta_{s,0}\delta_{s',L_s-1} - \delta_{s,s'+1}\right]$$

$$D(m) = \begin{pmatrix} D^W+1 & -P_L & 0 & \cdots & mP_R \\ -P_R & D^W+1 & -P_L & \cdots & 0 \\ 0 & -P_R & D^W+1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ mP_L & 0 & 0 & \cdots & D^W+1 \end{pmatrix}$$

$P_L = \frac{1}{2}(1-\gamma_5)$, $P_R = \frac{1}{2}(1+\gamma_5)$; $M_5 < 2$ is height of domain wall



A Conformal Window Roadmap

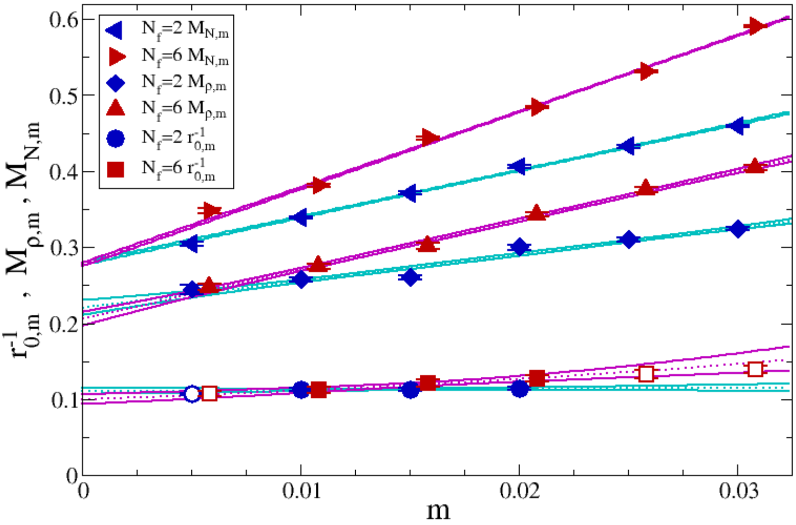
(Ethan Neil)

Light Scalar

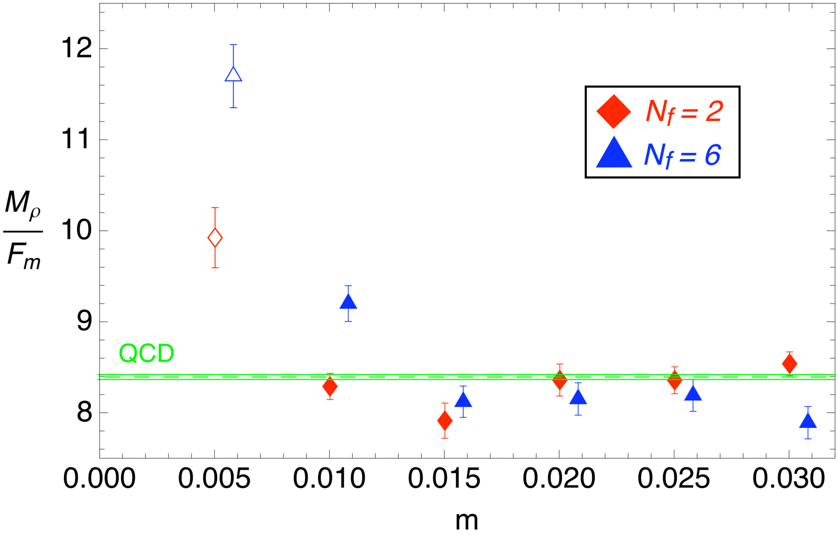
(Michael Cheng)

- LHC may have discovery-level (5σ) signal of m=125 GeV resonance by end of 2012.
- Is this SM Higgs? Still need to verify all decay channels...
- Technicolor can produce fake Higgs signals Martin, Phys. Rev. D84 (2011)
- Light scalar in low-scale technicolor Delgado, et. al., Phys. Lett. B696 (2011)
- Light Dilaton in walking theories? Appelquist, Bai, Phys. Rev. D82 (2010)
- Accessing 0^{++} state in QCD a major challenge. Nature of lightest 0^{++} state $\sigma(600)$ is still unknown.
- Evaluation of fermionic correlation function requires quark-line disconnected diagrams
- Mixing between scalar meson, glueballs, meson-meson bound states.
- Challenging, but important to investigate.

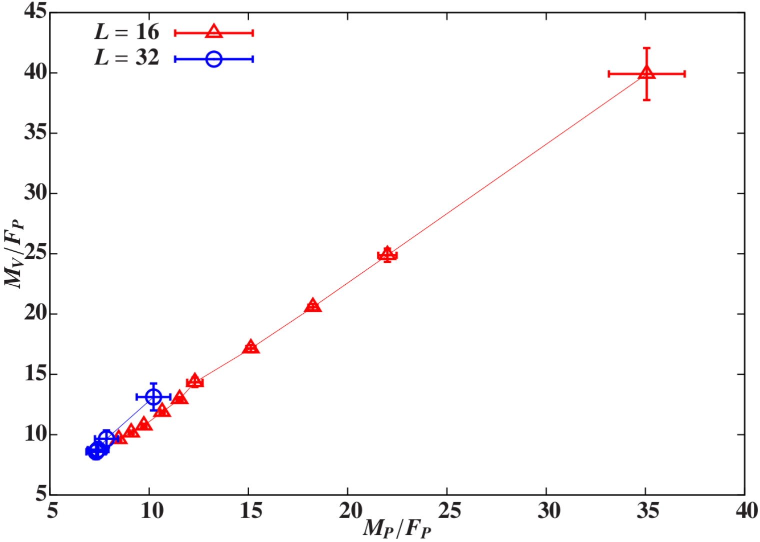
Backup: Other IR scales for $N_f = 2$ and 6



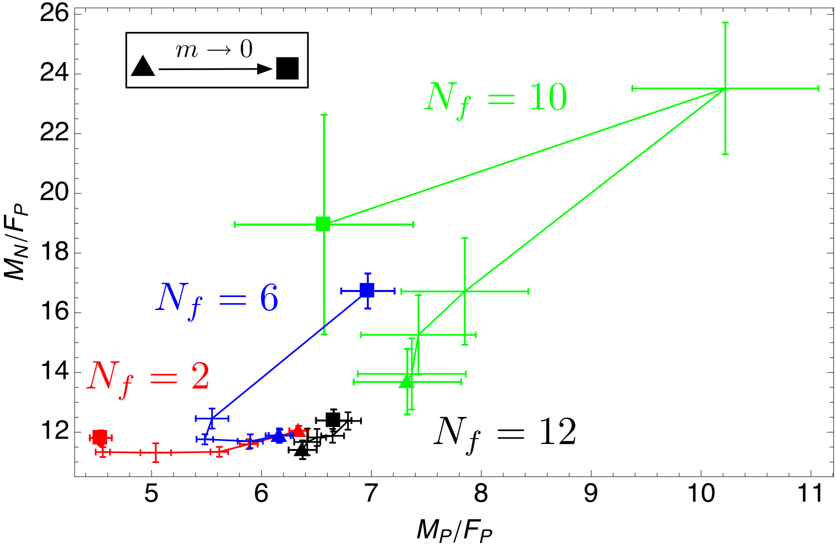
Backup: M_V/F_P compared to QCD



Backup: Edinburgh plot including $L = 16$

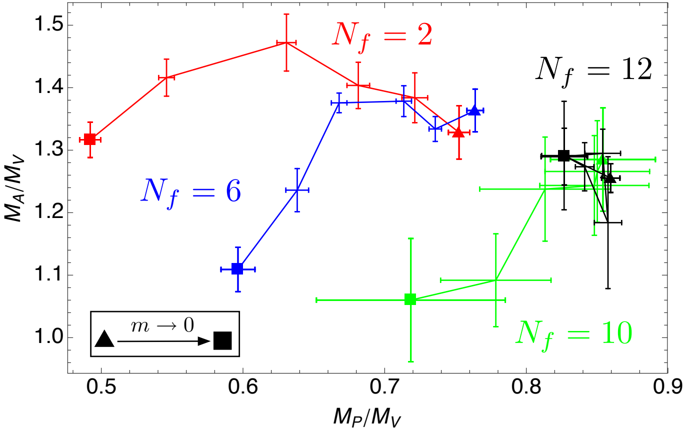


Backup: Edinburgh plot including $N_f = 12$



$N_f = 12$ data from Fodor *et al.*, PLB 703:348 (2011) [1104.3124]

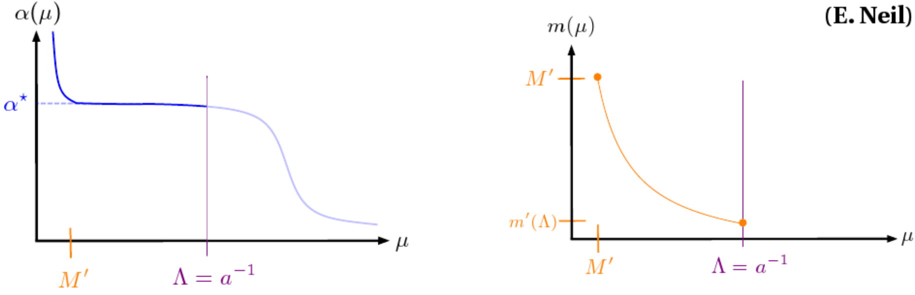
Backup: Edinburgh-style plot for M_A/M_V vs. M_P/M_V



Edinburgh-style plot illustrates (spurious?) parity doubling,
less change in M_P/M_V as N_f increases

$N_f = 12$ data from Fodor *et al.*, PLB 703:348 (2011) [1104.3124]

Mass-deformed IR-conformal spectrum analysis



- Leading order: $M_X = C_X m^{1/(1+\gamma_\star)}$
- Higher order: $M_X = C_X m^{1/(1+\gamma_\star)} + D_X m$
- Finite volume: $M_X = C_X M \left[1 + \frac{z_X}{ML}\right] + D_X m$
- $\langle \overline{\psi}\psi \rangle = A_C m + B_C m^{[(3-\gamma_\star)/(1+\gamma_\star)]} + C_C m^{[3/(1+\gamma_\star)]} + D_C m^3$

For now, we neglect higher-order and finite-volume corrections

A slowly-running theory will look IR-conformal for m too large

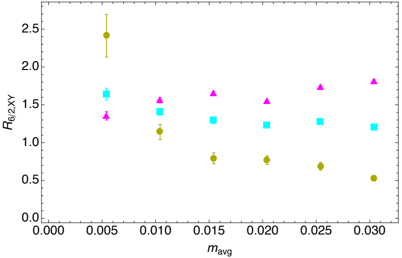
Backup: Condensate enhancement ratios

Three dimensionless ratios all approach $\left\langle \overline{\psi}\psi \right\rangle / F_P^3$ in the chiral limit:

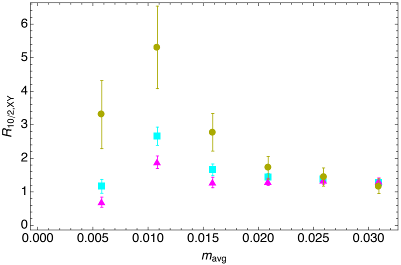
$$X^{(FM)} = \frac{M_P^2}{2mF_P} \qquad X^{(CM)} = \frac{\left(M_P^2/2m\right)^{3/2}}{\left\langle \overline{\psi}\psi \right\rangle^{1/2}} \qquad X^{(FM)} = \frac{\left\langle \overline{\psi}\psi \right\rangle}{F_P^3}$$

Condensate enhancement from "ratios of ratios": $\quad R_{N_1/N_2}^{(AB)} = \frac{X_{N_f=N_1}^{AB}}{X_{N_f=N_2}^{AB}}$

$N_f = 6/N_f = 2$



$N_f = 10/N_f = 2$

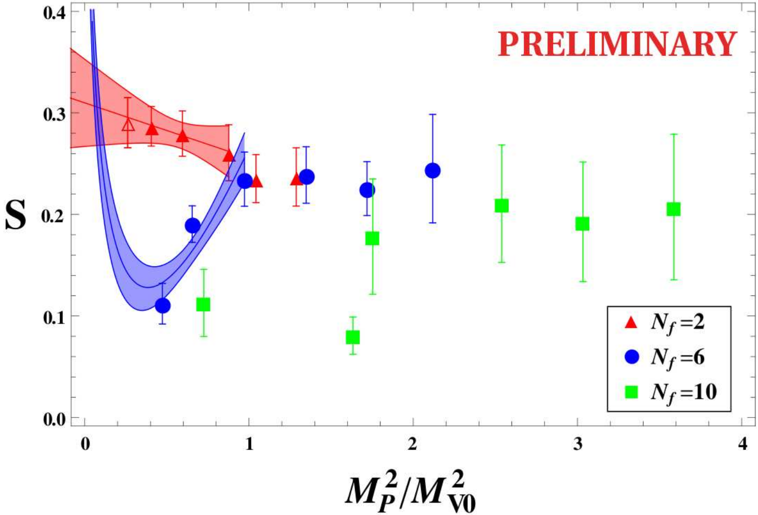


Ordering CM < FM < CF consistent with IR conformality for $\gamma_m \approx 1$

Also consistent with large finite-volume effects

Backup: 10f results for S parameter

NB: assumes $M_{V0} > 0$



10f finite-volume effects set in for $M_P^2 \approx 1.6 M_{V0}^2$

Expect (and observe) naïve scaling for $M_P^2 > M_{V0}^2$

Backup: Spurious $S \to 0$ from finite-volume effects

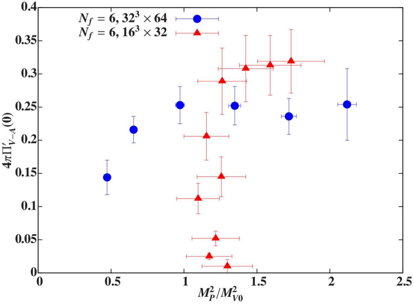
If m too small compared to L, system deconfines

$\implies$ chiral symmetry restored, parity doubling

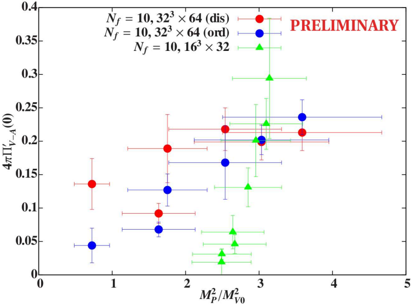
$$4\pi\Pi'_{V-A}(0) = \frac{1}{3\pi}\int_0^\infty \frac{ds}{s}\left[R_V(s) - R_A(s)\right] \longrightarrow 0$$

Also clearly distorts spectrum

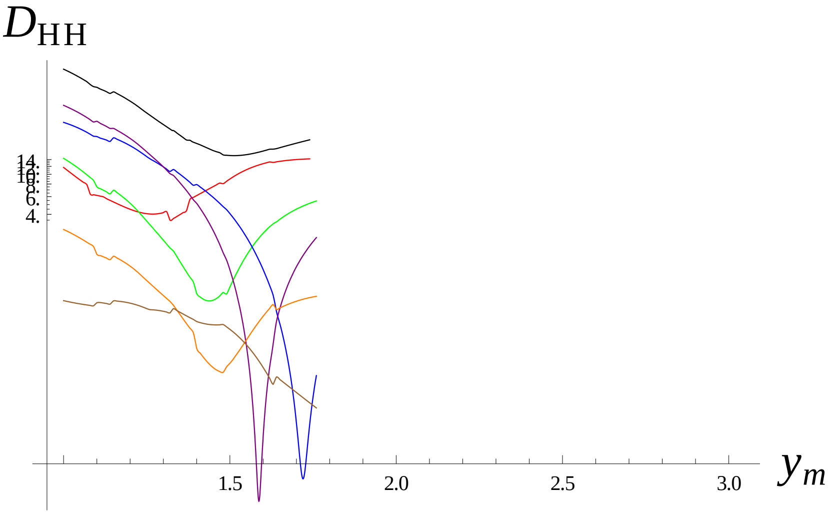
$N_f = 6$



$N_f = 10$



Backup: Finite-volume scaling not yet viable



$y_m = \gamma_m + 1$, $16^3 \times 32$ and $32^3 \times 64$ volumes
Quality functions (like χ^2/dof) for M_P, M_V, M_A, F_P, F_V, F_A, and sum