Exploring Electroweak Symmetry Breaking on the Lattice

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Outline

1. Electroweak symmetry breaking
2. New Strong Dynamics
3. Lattice Strong Dynamics

Avertissement

- Aiming to be “accessible to a non-specialized audience”.
- Tension between covering background
  and discussing work in progress.
- Some flexibility (and question time) remains.

Electroweak symmetry breaking

How elementary particles acquire mass

- The standard model unifies EM and the weak interaction in a spontaneously-broken electroweak gauge theory.

$$SU(2)_L \times U(1)_Y \to U(1)_{em}$$

- Electroweak symmetry breaking (EWSB) is well-established experimentally.
- The mechanism of EWSB remains unknown, one of the greatest mysteries of the standard model.

Minimal EWSB mechanism

Standard Model Higgs Mechanism

- Simplest mechanism introduces a single scalar field $\phi(x)$.
- $\phi(x)$ is a complex $SU(2)$ doublet, with a potential V designed to produce spontaneous symmetry breaking.
- $V(\phi) = \frac{1}{4}\lambda\left(\phi^\dagger\phi - \frac{1}{2}v^2\right)^2$ minimized when $|\phi|^2 = \frac{1}{2}v^2$.
- Parameterize ϕ in terms of a gauge transformation U

$$\phi(x) = U(x)\frac{1}{\sqrt{2}}\left(\begin{array}{c} v + h(x) \\ 0 \end{array}\right) \qquad \langle h(x)\rangle = 0$$

- $v = 2^{-1/4}G_F^{-1/2} = 246$ GeV, but λ and $m_h = \frac{v}{2}\sqrt{\lambda}$ unknown.

How to eat a Higgs

- Performing a gauge transformation to eliminate $U(x)$, massive vector bosons pop out of $(D_\mu\phi)^\dagger D^\mu\phi$.
- $D_\mu = (\partial_\mu + \frac{i}{2}g_1 B_\mu)\,\mathbb{I} - \frac{i}{2}g_2 W_\mu^a \sigma^a$ is the gauge covariant derivative.
- Relevant terms in $|D_\mu\phi|^2$:

$$\begin{aligned}
&\frac{v^2}{8}(1 \quad 0)\begin{pmatrix} g_2 W_\mu^3 - g_1 B_\mu & g_2(W_\mu^1 - iW_\mu^2) \\ g_2(W_\mu^1 + iW_\mu^2) & -g_2 W_\mu^3 - g_1 B_\mu \end{pmatrix}^2 \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\
&\equiv \frac{g_2^2 v^2}{8}(1 \quad 0)\begin{pmatrix} (g_1^2 + g_2^2)^{1/2} Z_\mu / g_2 & \sqrt{2} W_\mu^+ \\ \sqrt{2} W_\mu^- & \cdots \end{pmatrix}^2 \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\
&\equiv M_W^2 W^{+\mu} W_\mu^- + \tfrac{1}{2} M_Z^2 Z^\mu Z_\mu
\end{aligned}$$

Also get fermion masses from $\lambda_e\,(\overline{e}_L \quad \overline{\nu}_L)\begin{pmatrix} v \\ 0 \end{pmatrix} e_R$, etc.

Shortcomings of the minimal model

- **Ad hoc**: New type of particle with particular potential.
  All fermion masses and mixings arbitrary free parameters.
- **Trivial**: $\beta > 0$
  Must be new physics, or else Higgs decouples.
  Also holds nonperturbatively.
- **Unnatural**: Quadratically sensitive to highest energies.
  Fine-tuning required to maintain hierarchy.

$$\lambda(\mu) \simeq \frac{\lambda(\Lambda)}{1 + (24/16\pi^2)\lambda(\Lambda)\log(\Lambda/\mu)} \Longrightarrow \Lambda \simeq m_h \exp\left(\frac{4\pi^2 v^2}{3m_h^2}\right)$$

$$m_h = 115 \text{ GeV} \Longrightarrow \Lambda \sim 10^{28} \text{ GeV}$$

$$m_h = 700 \text{ GeV} \Longrightarrow \Lambda \sim 1000 \text{ GeV}.$$

An alternative: how to eat a pion

- Pions: $f^2\, Tr\left[(D_\mu\Sigma)^\dagger\, D^\mu\Sigma\right]/4$ where

$$\Sigma = \exp\left(2i\sigma^a\pi^a/f\right) \sim q_L\overline{q}_R$$

$$D_\mu = \mathbb{I}\partial_\mu - \tfrac{i}{2}g_2\mathcal{W}^a_\mu\sigma^a \qquad \mathcal{W}^a_\mu = \left(W^1_\mu, W^2_\mu, W^3_\mu - g_1 B_\mu/g_2\right)$$

- Relevant terms in $f^2\, Tr|D_\mu\Sigma|^2/4$:

Dynamical Electroweak Symmetry Breaking (Technicolor)

$$\begin{aligned}(\partial_\mu\pi^a)^2 - fg_2(\partial^\mu\pi^a)\mathcal{W}^a_\mu/2 + f^2g_2^2(\mathcal{W}^a_\mu)^2/16 &= \left[fg_2\mathcal{W}^a_\mu/4 - \partial_\mu\pi^a\right]^2 \\ &= f^2g_2^2\left[(W^1_\mu)^2 + (W^2_\mu)^2\right]/8 + f^2(g_2^2+g_1^2)Z^2_\mu/8 \\ &\equiv M_W^2 W^{+\mu}W^-_\mu + \tfrac{1}{2}M_Z^2 Z^\mu Z_\mu\end{aligned}$$

- With $f = v$, same masses and fields as before.
- No fermion masses yet,

  but can “extend” technicolor to produce them.

Extended technicolor

Flavor changing neutral currents (FCNCs)

- Introduce additional gauge interactions involving both SM fermions q and technifermions Q
- These interactions generate both masses and FCNCs,

$$\text{Masses:} \quad \frac{(\overline{Q}Q)(\overline{q}q)}{\Lambda_{ETC}^2} \qquad\qquad \text{FCNCs:} \quad \frac{(\overline{q}q)(\overline{q}q)}{\Lambda_{ETC}^2}$$

- Strongest FCNC constraints require $\Lambda_{ETC} \gtrsim 1000$ TeV from K_L–K_S mass difference, K–$\overline{K}$ mixing
- Scaled-up QCD produces m_s about fifty times too small.

Top quark causes special problems

Top quark mass too close to electroweak (technicolor) scale.

Precision electroweak observables

S parameter related to splitting of the vector and axial spectra

$$\begin{aligned} S &= \frac{1}{3\pi}\int_0^\infty \frac{ds}{s}\left[\rho_V(s) - \rho_A(s)\right] \\ &= -4\pi \frac{d}{dq^2}\left[\Pi_{VV}(q^2) - \Pi_{AA}(q^2)\right]_{q^2=0} \\ \Pi_{VV}^{\mu\nu}(q) &= ig^{\mu\nu}\Pi_{VV}(q^2) + (q^\mu q^\nu \text{ terms}) \\ &= \int d^4x e^{-iq\cdot x}\left\langle \overline{\psi}(x)\gamma^\mu\psi(x)\overline{\psi}(0)\gamma^\nu\psi(0)\right\rangle \end{aligned}$$

Example: vector meson dominance,

$$\rho_{V/A}(s) = f_{V/A}^2\delta(s - m_{V/A}^2) \Rightarrow S \propto f_V^2/m_V^2 - f_A^2/m_A^2.$$

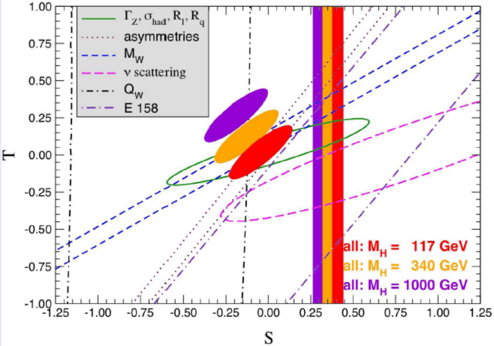
Also appears in chiral perturbation theory and elsewhere.

T parameter related to custodial symmetry, $M_W^2 = M_Z^2\cos^2\theta_W$.

Precision electroweak observables

Prediction for S from scaled-up QCD

is in roughly 2.5σ disagreement with experiment.



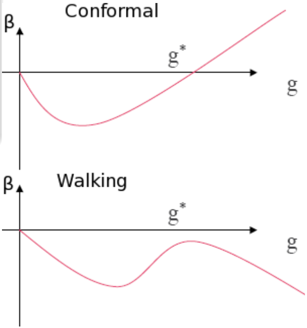
$$S \simeq N_{TF} N_{TC}/(12\pi)$$

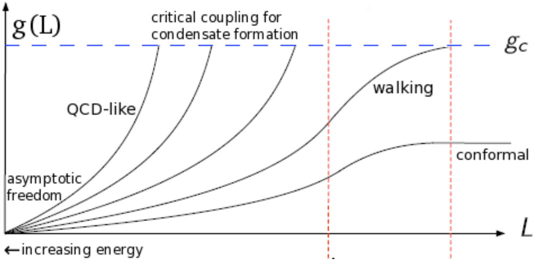
Need to subtract Higgs effects to isolate new physics.

Technicolor must go beyond QCD

- Scaled-up QCD ruled out.
- Technicolor can look completely different.
- Approximate conformality ("walking") might solve many problems (not top).
- Walking may result from simply increasing the number of quark flavors.
- Little known beyond QCD: lattice can help

$$\beta = \mu\frac{\partial g}{\partial \mu}$$





Lattice gauge theory

Nonperturbative calculations from first principles

- Regularize quantum field theories by formulating them on a discrete spacetime lattice.
- "Only" millions of degrees of freedom, simulated stochastically.
- Typical approach: "hybrid Monte Carlo" hamiltonian evolution followed by Metropolis step.

Need unphysically large quark masses

- Bound state Compton wavelengths need to fit in "box" to reduce effects of finite lattice size.
- Critical slowing down in chiral limit, $m_q \to 0$.

Chiral fermions on the lattice

A longstanding and difficult problem

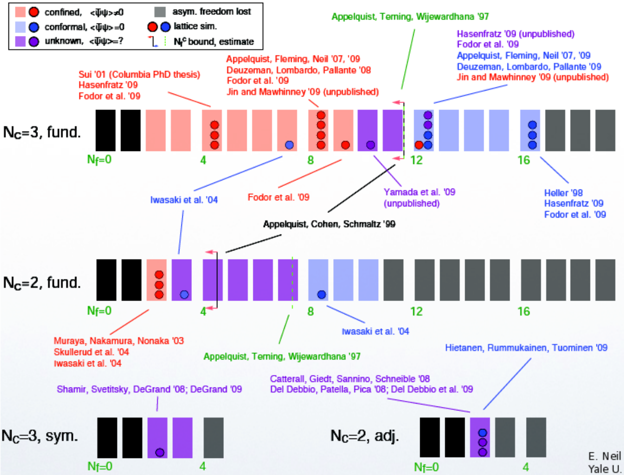
- Quark masses break chiral symmetry, as do most lattice fermion discretizations.
- Interested in *spontaneous* chiral symmetry breaking, need to minimize *explicit* chiral symmetry breaking.

Domain wall fermions

- Chiral symmetry breaking exponentially suppressed, encapsulated in “residual mass” m_{res}.
- Need to introduce fifth dimension of length L_s, $m_{res} \to 0$ as $L_s \to \infty$.
- Typically $L_s = 16$, still significant computational cost.

Lattice explorations beyond QCD

Many possible gauge groups, colors, flavors, representations.



Computational cost increases $\propto N_f^{3/2}, N_c^3, d(R)^3$.

Lattice Strong Dynamics Collaboration

Argonne	James Osborn
Boston	Adam Avakian, Ron Babich, Rich Brower, Saul Cohen, Claudio Rebbi, DS
Harvard	Mike Clark
Livermore	Michael Cheng, Ron Soltz, Pavlos Vranas
UC Davis	Joe Kiskis
Yale	Tom Appelquist, George Fleming, Ethan Neil, Gennady Voronov

Lattice Strong Dynamics Collaboration

Lattice Strong Dynamics program

- Carried out step-scaling studies with $N_f = 8, 12$ flavors, finding $N_f = 12$ conformal, $N_f = 8$ not.
- Finishing studies of $N_f = 6$ spectrum and condensate compared to $N_f = 2$ (QCD).
- Similar studies for $N_f = 9$ getting underway.
- Work in progress: calculation of S parameter, comparing $N_f = 6$ to $N_f = 2{+}1$ and $N_f = 2$.
- Drawing board: $N_f = 10$ step scaling; studies with $N_c = 2$.

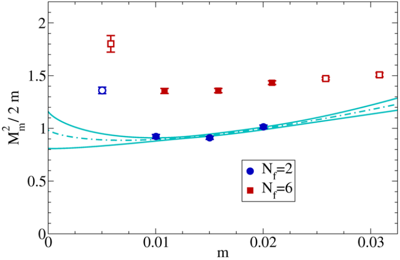
Pushing the limits
of current computational resources and capabilities.

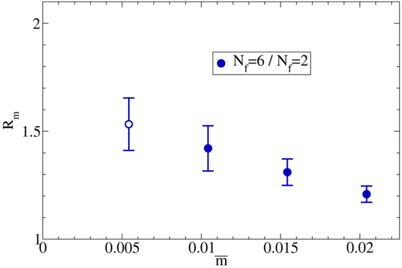
Latest LSD result: condensate enhancement

- Recall that in extended technicolor,

  Masses: $(\overline{Q}Q)(\overline{q}q)/\Lambda^2_{ETC}$ FCNCs: $(\overline{q}q)(\overline{q}q)/\Lambda^2_{ETC}$

- Larger $\left\langle\overline{Q}Q\right\rangle$ would weaken FCNC constraints.
- 0910.2224: $N_f = 6$ produces $\gtrsim$ 50% enhancement in $\left\langle\overline{\psi}\psi\right\rangle/F^3 = M^2/(2mF)$.





S parameter on the lattice

- Recall QCD-like S too large compared to experiment.
- Can argue qualitatively that walking reduces S: expect degenerate vector and axial spectra.
- Can we get quantitative, nonperturbative measurements of S on the lattice?

Other groups are also working on this

- 0806.4222: JLQCD Collaboration measures $S = 0.38(4)$ for $N_f = 2$ overlap fermions.
- 0909.4931: Edinburgh group measures $S = 0.42(7)$ for $N_f = 2{+}1$ domain wall fermions. Also find good agreement with vector meson dominance.

Stiff competition

- 0806.4222: JLQCD Collaboration measures $S = 0.38(4)$ for $N_f = 2$ overlap fermions.
- 0909.4931: Edinburgh group measures $S = 0.42(7)$ for $N_f = 2{+}1$ domain wall fermions.

- Both extract S by fitting to chiral perturbation theory.
- LSD Collaboration has found that χPT more difficult for larger N_f.
- Good news: Edinburgh group uses different lattices and methods than we do.
- Bad news: what they use is likely superior.

Future work

Lots to do

- Check S for $N_f = 2{+}1$ against others' results
  and vector meson dominance.
- See how well we can determine S for $N_f = 6$,
  through chiral perturbation theory or other approach(es).
- Issues with topology in LSD lattices.
- Improved computational methods:
  Möbius fermions and GPUs.
- Longer term: domain wall multigrid?