# Exploring the Origin of Mass with High-Performance Computing

David Schaich

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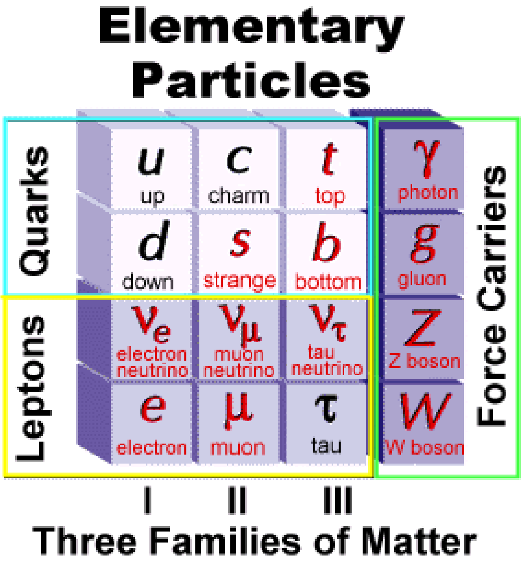
arXiv:1009.5967 (LSD Collaboration)

# Outline

1. Mystery: The Origin of Mass
   - Electroweak symmetry breaking
   - New strong dynamics
   - *S* parameter
2. Methods: High-Performance Computing
3. Results: *S* Parameter on the Lattice




(SLAC)

Interactions described by “gauge symmetries” (invariance under transformations)

**A Mystery**

Why do almost all of these particles possess mass?

# What's mysterious about mass?

## Electroweak symmetry

Unifies quantum electrodynamics and the weak interaction.

### Electromagnetism

- Infinite range
- Massless photon
- Conserves parity

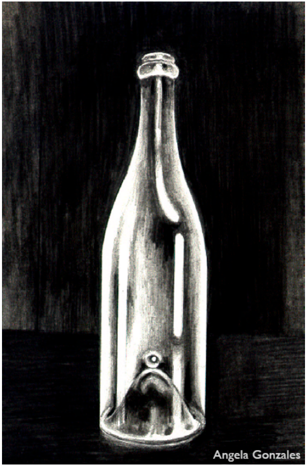
### Weak interaction

- Extremely short range ($\lesssim 10^{-17}$ m)
- Very massive $W^{\pm}$ and $Z$ ($\sim 90 M_{proton} \sim 175,000 m_e$)
- Violates parity

Electroweak unification well-verified experimentally,
but appears to **forbid** elementary particle masses!

# Electroweak symmetry breaking

**"Spontaneous" symmetry breaking**
reconciles electroweak theory with phenomenology




"Symmetry of laws
$\not\Rightarrow$ symmetry of outcomes"

Example: superconductivity

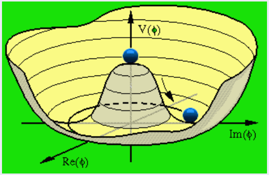
Lagrangian must be gauge invariant
but ground state **hides** symmetry

Must provide longitudinal modes
of massive $W^{\pm}$ and $Z$
$\longrightarrow$ new degrees of freedom

# The standard model

## Simplest solution: generalized Ginzburg–Landau model

- New scalar field $$\Phi = \begin{pmatrix} \phi_1 + i\phi_2 \\ v/\sqrt{2} + h + i\phi_3 \end{pmatrix}$$
- "Winebottle potential" $$V(\Phi) = -\mu^2 \Phi^\dagger \Phi + \lambda \left(\Phi^\dagger \Phi\right)^2$$
  produces spontaneous symmetry breaking
  at the electroweak scale $v = \sqrt{-\mu^2/\lambda} = 246$ GeV

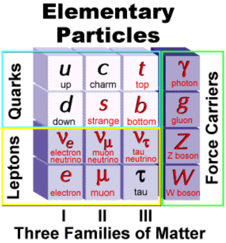



(Imperial)

- $\phi_i$ "eaten" by $W^\pm$ and $Z$ becoming massive
- $h$ remains as massive **Higgs boson**

# Unsatisfied with the standard model

No fundamental scalars observed in nature




**Standard model *can't* be the end of the story**

- High-energy quantum effects make the Higgs decouple!
- Standard model *requires* new physics at high energies

- Higgs mass *extremely sensitive* to physics at high energies
- Properties must be unnaturally "fine-tuned"

Doesn't rule out standard model, but motivates alternatives ⟶ BCS?

# Unsatisfied with the standard model

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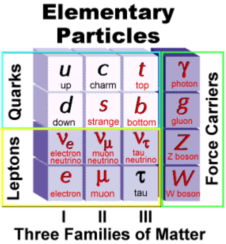


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# Dynamical electroweak symmetry breaking

Instead of BCS, think of QCD (quantum chromodynamics)

## New Strong Dynamics ("Technicolor")

- New "technifermions" $\Psi$ couple through a new **strong** interaction
- Lagrangian decomposes into two parts

$$\mathcal{L}_{TC} = \overline{\Psi}\gamma_\mu \mathcal{D}^\mu \Psi = \overline{\Psi}_L \gamma_\mu \mathcal{D}^\mu \Psi_L + \overline{\Psi}_R \gamma_\mu \mathcal{D}^\mu \Psi_R$$

**Chiral symmetry:** $\Psi_L$ and $\Psi_R$ can transform *independently*

- Strong interactions spontaneously break chiral symmetry, which leads to electroweak symmetry breaking

$$\left\langle \overline{\Psi}\Psi \right\rangle = \left\langle \overline{\Psi}_L \Psi_R + \overline{\Psi}_R \Psi_L \right\rangle \neq 0$$

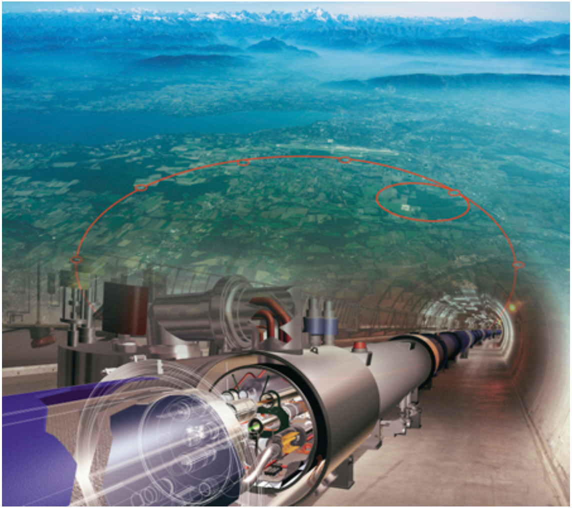
$\left\langle \overline{\Psi}\Psi \right\rangle \sim v^3$

- Instead of Higgs, expect a zoo of "technihadrons" at high energy

**Strong** dynamics $\longrightarrow$ perturbation theory inapplicable

How can we determine which mechanism
of electroweak symmetry breaking
is realized in nature?

# Obvious approach: direct detection



(CERN)

- Experimentally observe and identify Higgs, technihadrons, ...
- “Obvious” does not mean “easy”!

# Outline (reminder)

A less obvious approach is to use
precision measurements of electroweak observables

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# The $S$ parameter

Parameterize effects of physics beyond the standard model
on the neutral gauge bosons


$$= i \; e^2 \; \Pi_{QQ} \; g^{\mu\nu} \; + \cdots$$


$$= i \; \frac{e^2}{cs} \; (\Pi_{3Q} - s^2 \, \Pi_{QQ}) \, g^{\mu\nu} \; + \cdots$$


$$= i \; \frac{e^2}{c^2 s^2} (\Pi_{33} - 2s^2 \, \Pi_{3Q} + s^4 \Pi_{QQ}) \, g^{\mu\nu} + \cdots$$

$$\Pi_{VV} = 2\Pi_{3Q}$$
$$\Pi_{AA} = 4\Pi_{33} - 2\Pi_{3Q}$$

Define the parameter $S = 4\pi \lim_{Q^2 \to 0} \dfrac{d}{dQ^2} \left[ \Pi_{VV}(Q^2) - \Pi_{AA}(Q^2) \right] - \Delta S_{SM}$

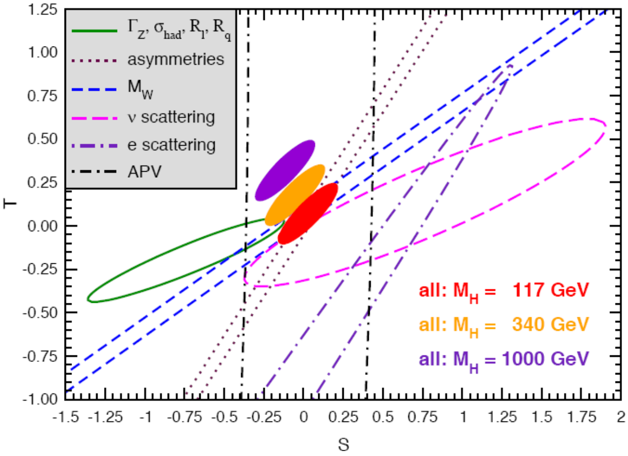
(Peskin and Takeuchi)

$\Delta S_{SM}$ subtracted so that $S = 0$ in the standard model
(assuming a "reference" Higgs boson mass)

# Experimentally, $S \lesssim 0$

Extract $S$ from global fit to experimental data

- $Z$ decay partial widths and asymmetries
- Neutrino scattering cross sections
- $M_W$, $M_Z$
- Atomic parity violation




(PDG)

# What is $S$ for new strong dynamics?

Recall **strong** dynamics $\longrightarrow$ perturbation theory inapplicable

**If** new strong dynamics has exactly the same form as QCD

$SU(3)$ gauge theory with $N_f = 2$ fermions

then we can extract information from low-energy QCD measurements

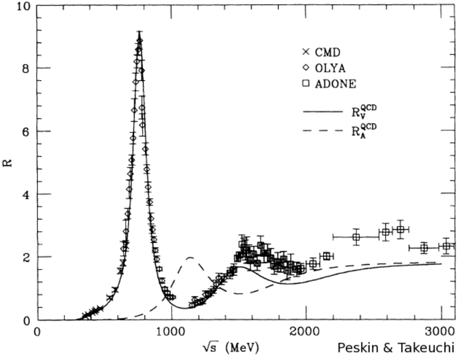
# $S$ from scaling up QCD

Relate polarization functions $\Pi$ to spectral functions $R$

$$R(s) = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

$$\Pi(Q^2) = \Pi(0) + \frac{Q^2}{12\pi^2}\int_0^\infty \frac{ds R(s)}{s+Q^2}$$

$$S = 4\pi\Pi'_{V-A}(0) - \Delta S_{SM}$$



Peskin & Takeuchi

$$S = \frac{1}{3\pi}\int_0^\infty \frac{ds}{s}\left\{R_V - R_A - \frac{1}{4}\left[1 - \left(1 - \frac{M_H^2}{s}\right)^3 \Theta\left(s - M_H^2\right)\right]\right\}$$

Replacing the QCD scale with the electroweak scale, $S = 0.32 \pm 0.03$

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**If** new strong dynamics has exactly the same form as QCD

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Replacing the QCD scale with the electroweak scale, $S = 0.32 \pm 0.03$

$$\text{Guess } S \sim 0.3 \frac{N_f}{2} \frac{N_c}{3} ?$$

This is very far from the experimental $S \approx -0.15 \pm 0.10$,

but does **not** hold for strongly-interacting theories in general

We need a way to perform non-perturbative calculations

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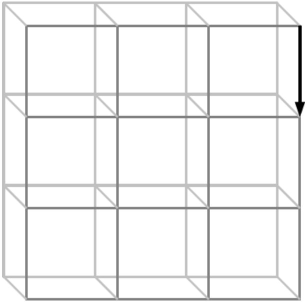
**Motivation**

By working in a discrete euclidean spacetime,
we can perform non-perturbative calculations
of strongly-interacting theories

# Quantum fields on a lattice

A 12-step program for
non-perturbative predictions



(R. Babich)

## Part 1: Formulation

1. Wick rotation $t \to -it$ from Minkowski to euclidean spacetime
2. Replace spacetime with regular lattice of sites connected by links
3. Gauge invariance: fermion fields on sites, gauge fields on links
4. Recover original theory (e.g., Lorentz invariance) in continuum

# Part 2: Simulation

5. Observables $\langle\mathcal{O}\rangle$ defined through path integral

$$\langle\mathcal{O}\rangle = \frac{1}{Z}\int \mathcal{D}U\mathcal{D}\overline{\Psi}\mathcal{D}\Psi\mathcal{O}e^{-S_G(U)-\overline{\Psi}D(U)\Psi}$$

   $D(U)$ is the discrete Dirac operator on the lattice

6. Gaussian integration replaces anti-commuting Grassmann fields

$$\int \mathcal{D}\overline{\Psi}\mathcal{D}\Psi e^{-\overline{\Psi}D\Psi} \propto \det D \propto \int \mathcal{D}\overline{\chi}\mathcal{D}\chi e^{-\overline{\chi}D^{-1}\chi}$$

(Inverting the large sparse matrix $D(U)$ is the main computational cost)

7. With an even number of fermions, we have $\int \mathcal{D}\overline{\chi}\mathcal{D}\chi e^{-\overline{\chi}\left(D^\dagger D\right)^{-1}\chi}$
8. Positive definite action $\longrightarrow$ probability distribution
9. Finite number of degrees of freedom

   $\longrightarrow$ numerical importance sampling (Monte Carlo)

# Part 3: Systematics

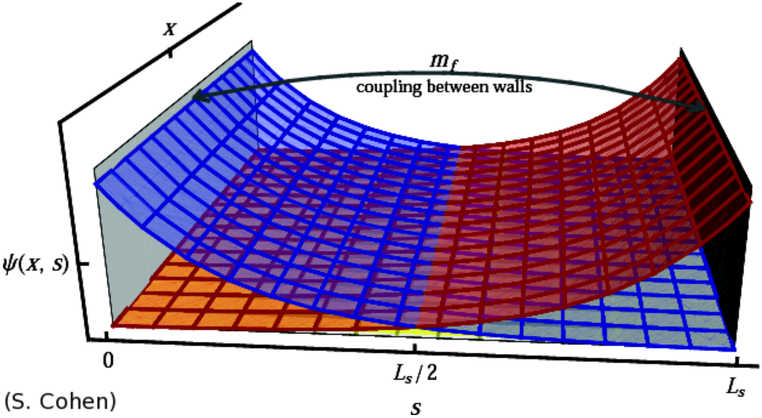
## Must keep in mind systematic effects of working on the lattice

10. **Finite volume**
    Reduce effects by requiring $L \gg \lambda_{max} = \frac{1}{M_P}$
    Need large lattice size $L^3 \times 2L$ or large pseudoscalar mass $M_P$
    (Input is fermion mass $m_f$; $M_P \propto \sqrt{m_f}$ not known *a priori*)

11. **Nonzero "lattice spacing" *a* between sites**
    *Should* repeat calculation at several $a$, extrapolate $a \to 0$
    (Computational cost $\propto 1/a^6$)
    Reduce effects by clever construction of lattice action

12. **Chiral symmetry breaking**
    Explicitly broken by $m_f > 0$ (Computational cost $\propto 1/m_f^{4.5}$)
    Additional explicit breaking from many lattice actions
    (Chiral lattice actions have much larger computational costs)

# Domain wall fermions




- Add fifth dimension of length $L_s$
- Exact chiral symmetry at finite lattice spacing in the limit $L_s \to \infty$
- At finite $L_s$, "residual mass" $m_{res} > 0$; $\quad m = m_f + m_{res}$

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We have a way to perform non-perturbative computations
Let's apply it to calculate the $S$ parameter for new strong dynamics

# Lattice Strong Dynamics Collaboration

L S D

Argonne James Osborn

Boston Ron Babich, Richard Brower, Saul Cohen, Claudio Rebbi, DS

Fermilab Ethan Neil

Harvard Mike Clark

Livermore Mike Buchoff, Michael Cheng, Pavlos Vranas

UC Davis Joseph Kiskis

Yale Thomas Appelquist, George Fleming, Meifeng Lin, Gennady Voronov

Formed in 2007 to pursue non-perturbative studies of strongly interacting theories likely to produce observable signatures at the Large Hadron Collider.

# LSD Philosophy and Simulation Details

- Start from what we know (QCD) and use it as a baseline
  $\longrightarrow$ $SU(3)$ gauge theory with $N_f = 2, \quad 6, \quad 10$
- Work on large lattices so finite-volume effects are small
  $\longrightarrow$ $32^3 \times 64$ with $0.005 \leq m_f \leq 0.030$ gives $M_P L \gtrsim 4$
- Directly compare the different theories
  $\longrightarrow$ Tune parameters to match chiral symmetry breaking scale
  $\longrightarrow$ Plot results versus $M_P^2$ rather than $m = m_f + m_{res}$
- Exploratory calculations
  $\longrightarrow$ $\mathcal{O}(100)$ independent measurements per point
- Studying spontaneous chiral symmetry breaking
  $\longrightarrow$ Domain wall fermions with $L_s = 16$
  $\longrightarrow$ $m_{res} \approx 3\times10^{-5}$ (2f); $\quad 8\times10^{-4}$ (6f); $\quad 2\times10^{-3}$ (10f)

# DWF are expensive, even for exploratory calculations

$\sim$ 300M core-hours on LLNL BGL, USQCD clusters, NSF Teragrid...

# $S$ parameter on the lattice

$$\gamma \sim\bullet\sim \gamma = i\, e^2\, \Pi_{QQ}\, g^{\mu\nu} + \cdots$$

$$Z \sim\bullet\sim \gamma = i\, \frac{e^2}{cs}\, (\Pi_{3Q} - s^2\, \Pi_{QQ})\, g^{\mu\nu} + \cdots$$

$$Z \sim\bullet\sim Z = i\, \frac{e^2}{c^2 s^2}\, (\Pi_{33} - 2s^2\, \Pi_{3Q} + s^4\, \Pi_{QQ})\, g^{\mu\nu} + \cdots$$

$$S = 4\pi N_D \Pi'_{V-A}(0) - \Delta S_{SM}$$

$$\Pi_{VV} = 2\Pi_{3Q}$$

$$\Pi_{AA} = 4\Pi_{33} - 2\Pi_{3Q}$$

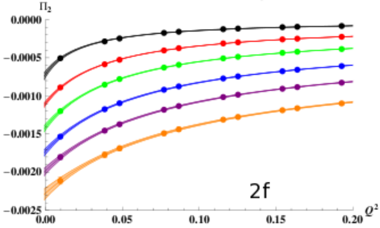
On the lattice, correlators involve a single pair of fermions

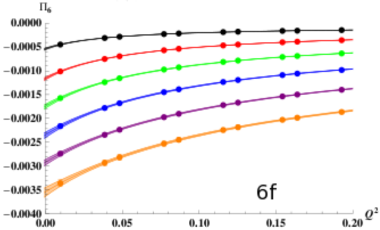
$$\Pi^{\mu\nu}_{V-A}(Q) = Z \sum_x e^{iQ\cdot(x+\widehat{\mu}/2)} \mathrm{Tr}\left[\left\langle \mathcal{V}^{\mu a}(x) V^{\nu b}(0)\right\rangle - \left\langle \mathcal{A}^{\mu a}(x) A^{\nu b}(0)\right\rangle\right]$$

$$\Pi^{\mu\nu}(Q) = \left(\delta^{\mu\nu} - \frac{\widehat{Q}^\mu \widehat{Q}^\nu}{\widehat{Q}^2}\right)\Pi(Q^2) - \frac{\widehat{Q}^\mu \widehat{Q}^\nu}{\widehat{Q}^2}\Pi^L(Q^2) \qquad \widehat{Q} = 2\sin(Q/2)$$

- Conserved currents $\mathcal{V}$ and $\mathcal{A}$ ensure that lattice artifacts cancel
- $\langle \mathcal{V}^{\mu a}(x) \mathcal{V}^{\nu a}(0)\rangle$ and $\langle \mathcal{A}^{\mu a}(x) \mathcal{A}^{\nu a}(0)\rangle$ require $\mathcal{O}(L_s)$ inversions
- Renormalization constant $Z$ computed non-perturbatively
  $Z = 0.85$ (2f); 0.73 (6f); 0.71 (10f)

# Correlator data and fits



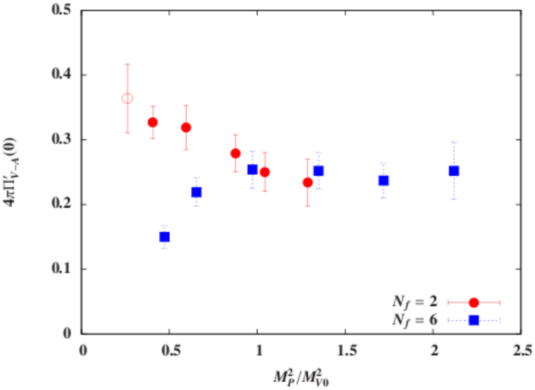




Independently fit $\Pi_{V-A}(Q^2)$ to (1, 2) Padé, $Q^2 < 0.4$

Fits stable with $\chi^2 \ll 1$ as $Q^2$ fit range varies

$$\frac{a_0 + a_1 Q^2}{1 + b_1 Q^2 + b_2 Q^4} = \left[ -F_P^2 + \frac{Q^2 F_V^2}{M_V^2 + Q^2} - \frac{Q^2 F_A^2}{M_A^2 + Q^2} \right]_{F_P^2 = F_V^2 - F_A^2}$$

# Fit results for $\Pi'_{V-A}(0)$, $N_f = 2$ and $N_f = 6$




$$S = 4\pi N_D \Pi'_{V-A}(0) - \Delta S_{SM}$$
$$1 \leq N_D \leq N_f/2$$

Reduction in $\Pi'_{V-A}(0)$ for $M_P^2 < M_{V0}^2 \equiv \lim_{m\to 0} M_V^2$

$\longrightarrow$ naïve scaling $S \sim 0.3 \dfrac{N_f}{2} \dfrac{N_c}{3}$ does **not** hold

(Do expect naïve scaling in heavy-fermion limit $M_P^2 \gg M_{V0}^2$)

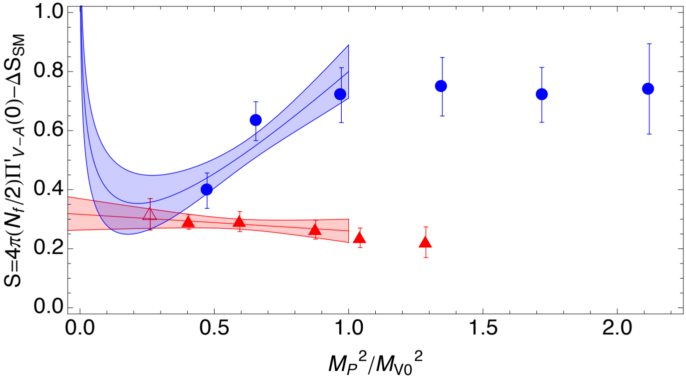
# $\Delta S_{SM}$ with $m_f > 0$

$$S = 4\pi N_D \lim_{Q^2 \to 0} \frac{d}{dQ^2}\left[\Pi_{VV}(Q^2) - \Pi_{AA}(Q^2)\right] - \Delta S_{SM}$$

$$\Delta S_{SM} = \frac{1}{4}\int_{4M_P^2}^{\infty} \frac{ds}{s}\left[1 - \left(1 - \frac{M_{V0}^2}{s}\right)^3 \Theta(s - M_{V0}^2)\right]$$

- $\Delta S_{SM}$ diverges as $s \to 0$ (cancelling out eaten modes)
- With $m_f > 0$, need lower bound $4M_P^2 > 0$ on spectral integral
- For $N_f = 2$, cancellation continues to work as $m_f \to 0$
- For $N_f > 2$, extra $N_f^2 - 4$ uneaten modes must receive masses from other interactions
- Set reference Higgs mass $M_H^{ref} = \lim_{m \to 0} M_V \equiv M_{V0} \sim 1000$ GeV
- Numerically, $\Delta S_{SM} \lesssim 0.04$, only 5–10% reduction

# $S$ parameter, $N_f = 2$ and $N_f = 6$




For $M_P^2 < M_{V0}^2$, fit to form accounting for $N_f^2 - 4$ uneaten modes

$$S = A + BM_P^2 + \frac{1}{12\pi}\left[\frac{N_f^2}{4} - 1\right]\log\left(\frac{M_{V0}^2}{M_P^2}\right)$$

# Conclusion

- Elementary particle masses require electroweak symmetry breaking, which may be due to new strong dynamics
- Strongly-interacting gauge theories need not resemble QCD
- Lattice gauge theory can provide non-perturbative information

For $SU(3)$ gauge theory with $N_f = 6$ compared to $N_f = 2$ we find an $S$ parameter smaller than naïve scaling

Further refinements ongoing:

- Additional data, $m_f = 0.0075$
- Effects of finite volume, topology
- "Twisted" BCs to reduce $Q^2$
- Testing cheaper lattice action
- $N_f = 10$
- OPE for $\Pi_{V-A}$
- ...

# Acknowledgements


## At BU

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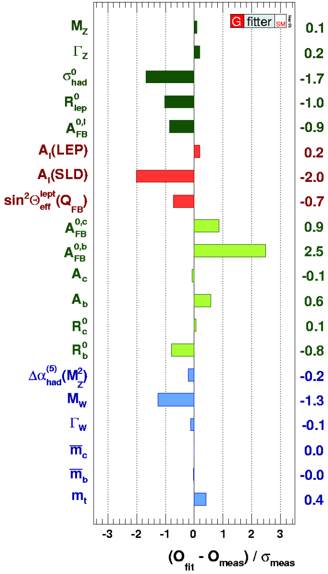
## Elsewhere

Tom Appelquist, Mike Buchoff, Michael Cheng, George Fleming, Fu-Jiun Jiang, Joe Kiskis, Meifeng Lin, Ethan Neil, Pavlos Vranas

## Funding and computing resources

# Bonus slides!

# Experimental confirmation of electroweak theory




(Gfitter Group)

# Gauge invariance example: electromagnetism

Electric and magnetic fields in terms of potentials $\Phi$ and $\mathbf{A}$

$$\mathbf{E} = -\frac{\partial \mathbf{A}}{\partial t} - \nabla\Phi \qquad \mathbf{B} = \nabla \times \mathbf{A}$$

$\mathbf{E}$ and $\mathbf{B}$ are invariant under the gauge transformation

$$\Phi \to \Phi - \frac{\partial \Lambda}{\partial t} \qquad \mathbf{A} \to \mathbf{A} + \nabla\Lambda$$

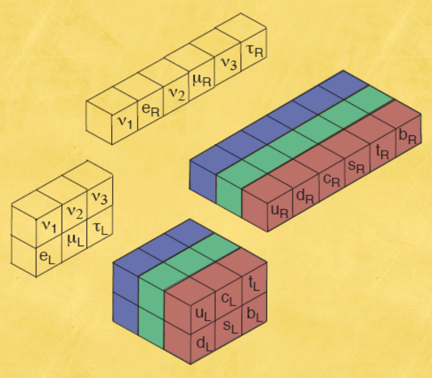
In four-vector notation, $A_\mu = (\Phi, \mathbf{A}) \to A_\mu + \partial_\mu \Lambda$

Photon mass term in lagrangian is

$$\tfrac{1}{2} m_\gamma^2 A_\mu A^\mu = \tfrac{1}{2} m_\gamma^2 \left( \mathbf{A} \cdot \mathbf{A} - \Phi^2 \right)$$

Forbidden by gauge invariance!

# Massless fermions from chiral gauge theory




(Chris Quigg)

Fermion mass term in lagrangian is $m\overline{\psi}\psi = m\left(\overline{\psi}_L\psi_R + \overline{\psi}_R\psi_L\right)$

$$\overline{\psi}_L\psi_R \sim \left(\overline{\psi}_\uparrow \quad \overline{\psi}_\downarrow\right)_L \cdot (\psi)_R$$

Forbidden by gauge invariance!

# Fermion masses in standard model

Need to make a gauge-invariant object involving

$$\overline{\psi}_L \psi_R \sim \left(\overline{\psi}_\uparrow \quad \overline{\psi}_\downarrow\right)_L \cdot (\psi)_R$$

Standard model solution: stick in a Higgs $\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$

$$\lambda_\psi \left(\overline{\psi}_\uparrow \quad \overline{\psi}_\downarrow\right)_L \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} (\psi)_R$$

With vacuum $\langle \Phi \rangle = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}$, identify $m_\psi = \lambda_\psi v/\sqrt{2}$.

All fermion masses and mixing are arbitrary free parameters!

# Gauge boson masses in standard model

$$\Phi = \begin{pmatrix} \phi_1 + i\phi_2 \\ v/\sqrt{2} + h + i\phi_3 \end{pmatrix}$$

$$\mathcal{L}_\Phi = (\mathcal{D}^\mu \Phi)^\dagger (\mathcal{D}_\mu \Phi) + \mu^2 \Phi^\dagger \Phi - \lambda \left(\Phi^\dagger \Phi\right)^2 \Rightarrow v = \sqrt{-\mu^2/\lambda}$$

$$\mathcal{D}_\mu = (\partial_\mu + \tfrac{i}{2} g_1 B_\mu)\,\mathbb{I} + \tfrac{i}{2} g_2 W^a_\mu \sigma^a$$

$W^\pm$ and $Z$ masses pop out of $(\mathcal{D}^\mu \Phi)^\dagger (\mathcal{D}_\mu \Phi)$. Relevant terms:

$$\frac{v^2}{8}(0 \quad 1) \begin{pmatrix} -g_2 W^3_\mu - g_1 B_\mu & g_2(W^1_\mu - iW^2_\mu) \\ g_2(W^1_\mu + iW^2_\mu) & g_2 W^3_\mu - g_1 B_\mu \end{pmatrix}^2 \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\equiv \frac{g_2^2 v^2}{8}(0 \quad 1) \begin{pmatrix} \cdots & \sqrt{2} W^+_\mu \\ \sqrt{2} W^-_\mu & (g_1^2 + g_2^2)^{1/2} Z_\mu / g_2 \end{pmatrix}^2 \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\equiv M_W^2 W^{+\mu} W^-_\mu + \tfrac{1}{2} M_Z^2 Z^\mu Z_\mu + \cdots$$

$$M_W = \tfrac{1}{2} g_2 v = (M_Z/g_2)\sqrt{g_1^2 + g_2^2} \equiv M_Z \cos\theta_W$$

# Gauge boson masses in new strong dynamics

Now we have pions with

$$\mathcal{L}_\chi = F_P^2 \mathrm{Tr}\left[(\mathcal{D}^\mu\Sigma)^\dagger(\mathcal{D}_\mu\Sigma)\right]/4$$

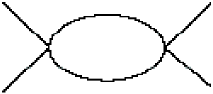
$$\Sigma = \exp(2i\sigma^a\pi^a/F_P) \sim q_L\overline{q}_R$$

$$\mathcal{D}_\mu = \mathbb{I}\partial_\mu - \tfrac{i}{2}g_2\mathcal{W}_\mu^a\sigma^a \qquad\qquad \mathcal{W}_\mu^a = \left(W_\mu^1, W_\mu^2, W_\mu^3 - g_1 B_\mu/g_2\right)$$

$W^\pm$ and $Z$ masses pop out of $F_P^2\mathrm{Tr}|\mathcal{D}_\mu\Sigma|^2/4$. Relevant terms:

$$\begin{aligned}(\partial_\mu\pi^a)^2 - F_P g_2(\partial^\mu\pi^a)\mathcal{W}_\mu^a/2 + F_P^2 g_2^2(\mathcal{W}_\mu^a)^2/16 &= \left[F_P g_2\mathcal{W}_\mu^a/4 - \partial_\mu\pi^a\right]^2\\ &= F_P^2 g_2^2\left[(W_\mu^1)^2 + (W_\mu^2)^2\right]/8 + F_P^2(g_2^2+g_1^2)Z_\mu^2/8\\ &\equiv M_W^2 W^{+\mu}W_\mu^- + \tfrac{1}{2}M_Z^2 Z^\mu Z_\mu \quad\longrightarrow F_P = v\end{aligned}$$

# Triviality of fundamental Higgs

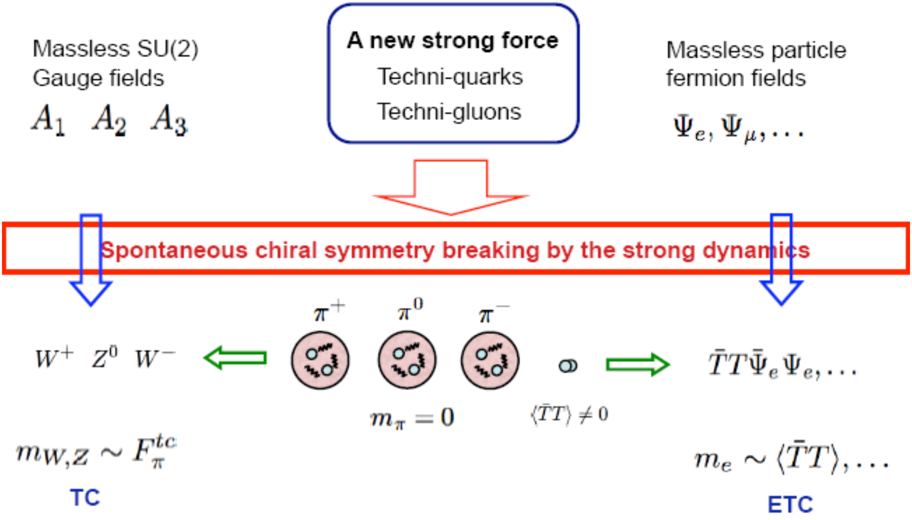
 $\Rightarrow \beta = \dfrac{3\lambda^2}{2\pi^2} > 0$

$$\lambda(\mu) \simeq \frac{1}{[1/\lambda(\Lambda)] + (3/2\pi^2)\log(\Lambda/\mu)} < \frac{2\pi^2}{3\log(\Lambda/\mu)}$$

$$\Lambda \simeq M_H \exp\left(\frac{4\pi^2 v^2}{3M_H^2}\right)$$

$$M_H = 115 \text{ GeV} \longrightarrow \Lambda \sim 10^{28} \text{ GeV}$$
$$M_H = 700 \text{ GeV} \longrightarrow \Lambda \sim 1000 \text{ GeV}$$

# (Extended) technicolor in a picture



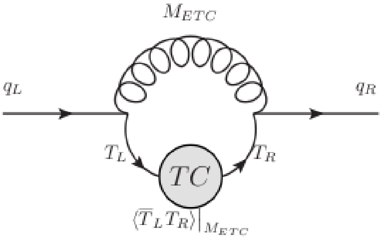

*P. Vranas, LLNL*

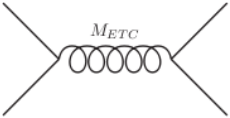
# Fermion masses in extended technicolor

Integrating out ETC gauge bosons produces four-fermion operators that provide both SM fermion masses and FCNCs

Masses: $\frac{(\overline{T}T)(\overline{q}q)}{M_{ETC}^2}$ FCNCs: $\frac{(\overline{q}q)(\overline{q}q)}{M_{ETC}^2}$

FCNCs required by CKM mixing, limit obtainable SM fermion masses.

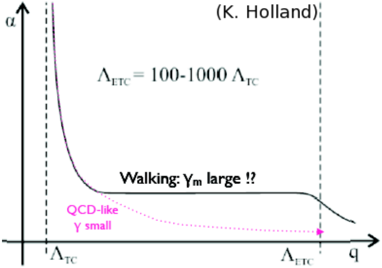


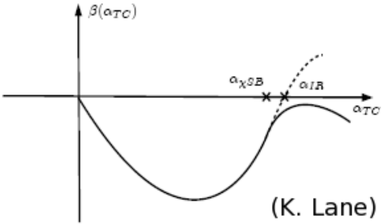



# "Walking" Technicolor

$$\langle \overline{T}T \rangle|_{M_{ETC}} = \langle \overline{T}T \rangle|_{\Lambda_{TC}} \exp\left( \int_{\Lambda_{TC}}^{M_{ETC}} \frac{d\mu}{\mu} \gamma(\mu) \right) \approx \langle \overline{T}T \rangle|_{\Lambda_{TC}} \left( \frac{M_{ETC}}{\Lambda_{TC}} \right)^{\gamma}$$

- $\gamma(\mu) \sim 1$ for $\Lambda_{TC} \lesssim \mu \lesssim M_{ETC}$ enhances fermion masses
- Implies large, slowly-running ("walking") coupling, small $\beta$ function



# Perturbative Yang–Mills $\beta$ function

For $SU(N_c)$ Yang–Mills theory with $N_f$ fermions in representation $r$

$$\beta(g) = \mu\frac{\partial g}{\partial\mu} = \beta_0 g^3 + \beta_1 g^5 + \beta_2 g^7 + \cdots$$

$$\beta_0 = -\frac{1}{(4\pi)^2}\left(\frac{11}{3}N_c - \frac{4}{3}N_f C(r)\right)$$

$$\beta_1 = -\frac{1}{(4\pi)^4}\left[\frac{34}{3}N_c^2 - \left(\frac{13}{3}N_c - \frac{1}{N_c}\right)N_f C(r)\right]$$

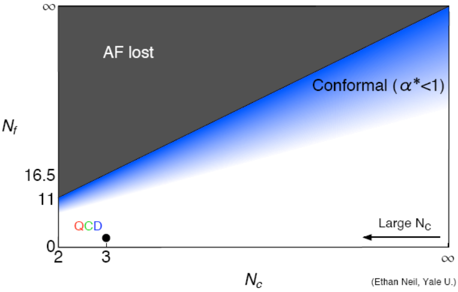
Higher-order $\beta_i$ depend on choice of renormalization scheme

$$C(N) = \frac{1}{2} \qquad C(Adj) = N_c \qquad C_2(N) = \frac{d(Adj)}{d(N)}C(N) = \frac{N_c^2 - 1}{2N_c}$$

# Conformal window

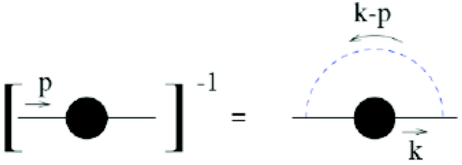
- Strongly-coupled gauge theories can look very different than QCD
- With many fermions, theory has perturbative IR fixed point;
  it is in a conformal phase with no spontaneous $\chi$SB
- The **conformal window** ranges from loss of asymptotic freedom
  to some (unknown) critical $N_f^c < N_f^{AF}$
- With $N_f \lesssim N_f^c$, may be approximately conformal (walking!)
  for some range of scales

Visualization of conformal window for $SU(N_c)$ fermions in fundamental rep:



# Anomalous dimension

From "rainbow approximation" to "gap" (Schwinger–Dyson) equation



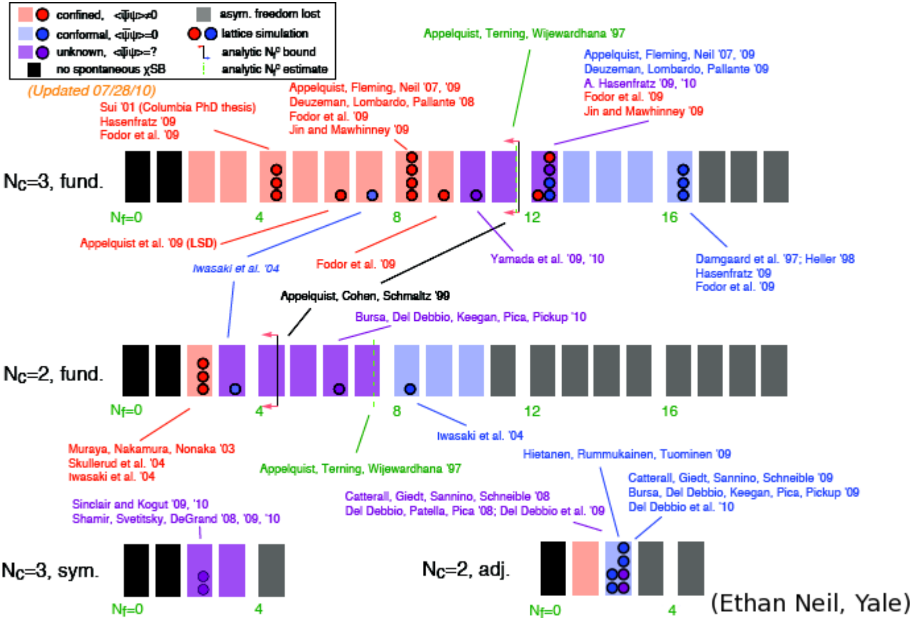

$$\gamma(\mu) = 1 - \sqrt{1 - 3C_2(r)\alpha(\mu)/\pi} \leq 1$$

Assume spontaneous chiral symmetry breaking when

$$\alpha(\mu) \geq \frac{\pi}{3C_2(r)} \equiv \alpha_{\chi SB}$$

When $\alpha(\mu) = \alpha_{\chi SB}$, this gives $\gamma(\mu) = 1$

# Searching for conformal windows




(Ethan Neil, Yale)

# NLO$\chi$PT for general $N_f$

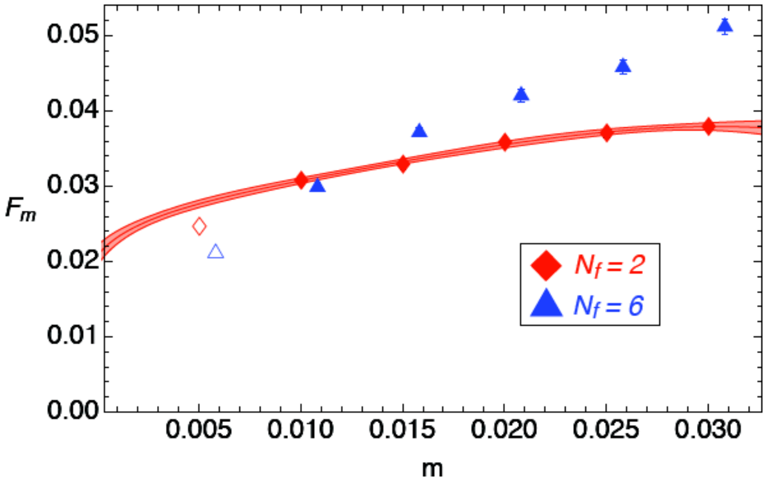
$$\frac{M_P^2}{2m} = B\left\{1 + \frac{2mB}{(4\pi F)^2}\left[\alpha_m + \frac{1}{N_f}\log\left(\frac{2mB}{(4\pi F)^2}\right)\right]\right\}$$

$$F_P = F\left\{1 + \frac{2mB}{(4\pi F)^2}\left[\alpha_F - \frac{N_f}{2}\log\left(\frac{2mB}{(4\pi F)^2}\right)\right]\right\}$$

$$\langle\overline{\psi}\psi\rangle = F^2 B\left\{1 + \frac{2mB}{(4\pi F)^2}\left[\alpha_C - \frac{N_f^2 - 1}{N_f}\log\left(\frac{2mB}{(4\pi F)^2}\right)\right]\right\}$$

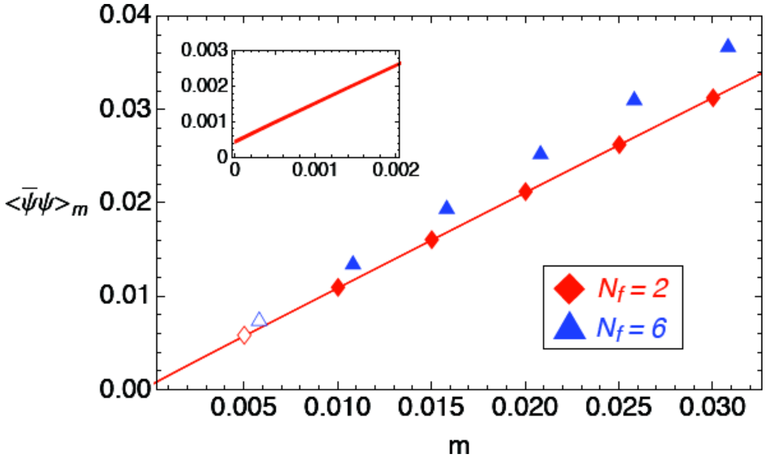
- $\alpha_C$ includes "contact term" $m\Lambda^2 \sim ma^{-2}$
- NNLO $M_P^2$ coefficients enhanced by $N_f^2$ (Bijnens & Lu, 2009)

# Goldstone decay constant



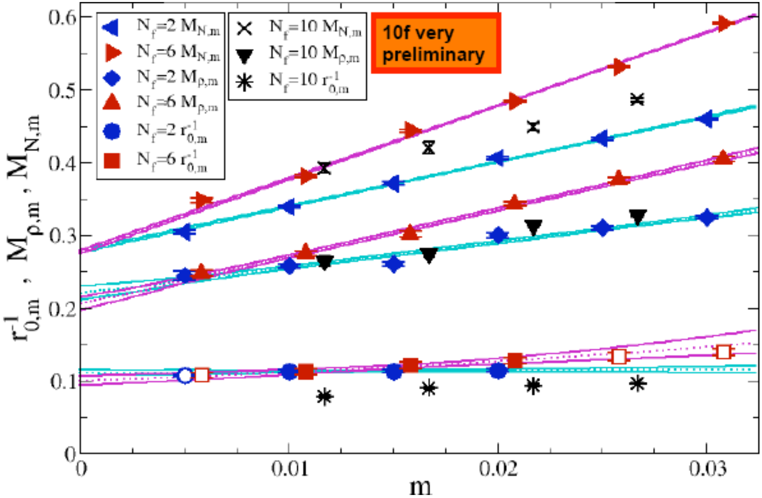

Joint NNLO$\chi$PT fit to $N_f = 2$ $F_P$, $M_P^2$, $\langle\overline{\psi}\psi\rangle$

# Chiral condensate




Joint NNLO$\chi$PT fit to $N_f = 2$ $F_P$, $M_P^2$, $\langle\bar{\psi}\psi\rangle$
Linear term clearly dominant

# “Sommer scale”, vector and nucleon masses




$N_f = 2$ and $N_f = 6$ all match at 10% level

# Chiral condensate enhancement: preliminaries

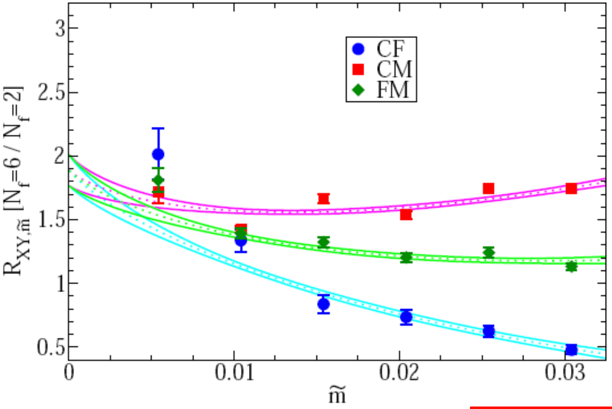
- Search for enhancement through $\langle\overline{\psi}\psi\rangle / F^3$
- Not RG invariant: keep cutoff fixed in physical units
- Focus on the ratio $R$ of $\langle\overline{\psi}\psi\rangle / F^3$ between $N_f = 6$ and $N_f = 2$

$$R = \frac{(\langle\overline{\psi}\psi\rangle / F^3)_{6f}}{(\langle\overline{\psi}\psi\rangle / F^3)_{2f}} = \frac{\exp\left(\int_{M_\rho}^{5M_\rho} \left.\frac{\gamma(\mu)}{\mu}\right|_{6f} d\mu\right)}{\exp\left(\int_{M_\rho}^{5M_\rho} \left.\frac{\gamma(\mu)}{\mu}\right|_{2f} d\mu\right)}$$

$\overline{MS}$ perturbation theory & perturbative conversion to lattice scheme predicts $R = 1.27(7)$

# Enhancement of $\langle \overline{\psi}\psi \rangle / F^3$, $N_f = 2$ to $N_f = 6$

Find significant enhancement compared with perturbative $R = 1.27(7)$

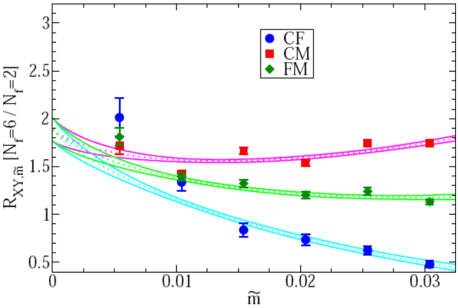


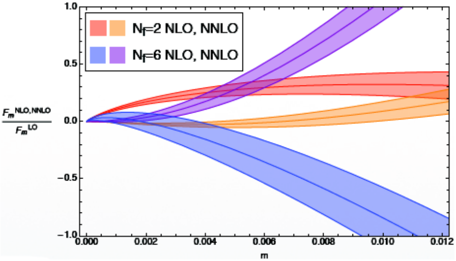

$$FM = M_\pi^2 / 2mF_\pi$$

$$CF = < \bar{\Psi}\Psi > / F_\pi^3$$

$$CM = \frac{< \bar{\Psi}\Psi >}{[\frac{\sqrt{2m\bar{\Psi}\Psi}}{m_\pi}]^3}$$

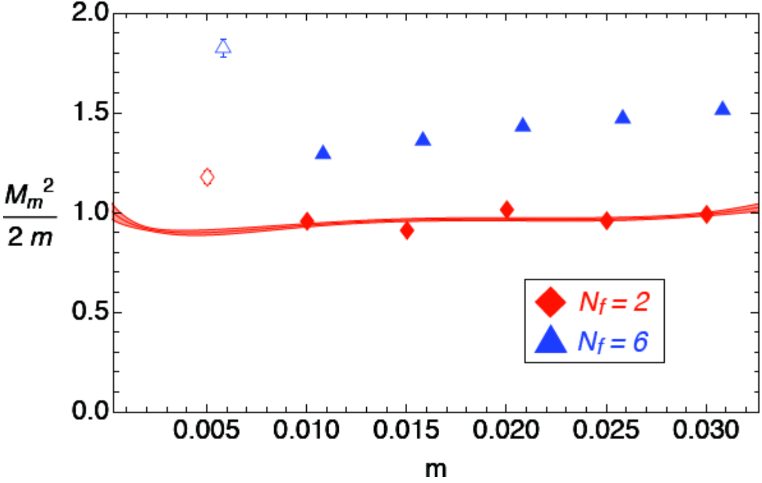
# NLO $\chi$PT fits, $N_f = 2$ and $N_f = 6$



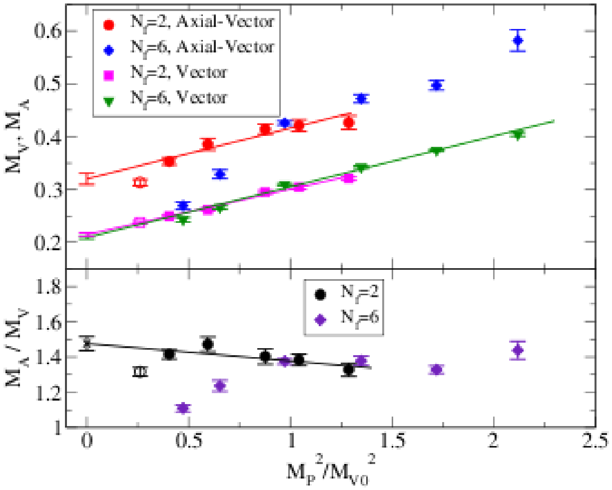

- NLO$\chi$PT fits work for $N_f = 2$ but not $N_f = 6$ (lighter $m_f$ required)
- GMOR $\Rightarrow \frac{\langle\overline{\psi}\psi\rangle}{F_\pi^3} = \frac{M_\pi^3}{\sqrt{(2m)^3\langle\overline{\psi}\psi\rangle}} = \frac{M_\pi^2}{2mF_\pi} \equiv \mathcal{R}$ as $m \to 0$
- Fit ratios to $\mathcal{R}\left[1 + \widetilde{m}(\alpha_{XY10} + \alpha_{11}\log\widetilde{m})\right]$ where $\widetilde{m} \equiv \sqrt{m_2 m_6}$

# Pseudo Nambu–Goldstone boson mass




- Slope of $M_P^2$ with $m$ significantly larger for $N_f = 6$
- Plot against $M_P^2$, to provide more physical comparison

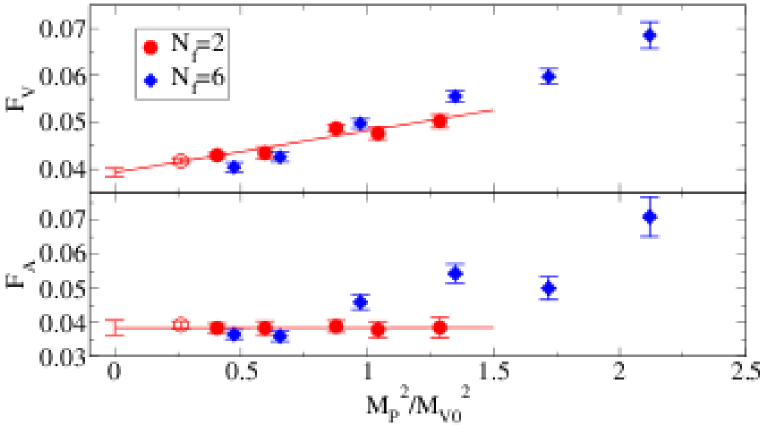
# Vector and axial spectrum




Signs of $N_f = 6$ parity-doubling as $M_P^2$ decreases

$\Rightarrow$ implications for $S$ parameter?

# Vector and axial decay constants




Need $F_V \approx F_A$ for parity-doubling to produce vanishing $S$ parameter

# Wilson gauge action

$$
\begin{aligned}
U_{x,\mu} &= \exp\left[iagA_\mu(x+\widehat{\mu}/2)\right] \quad \text{(directed from } x+\widehat{\mu} \text{ to } x\text{)} \\
P_{x,\mu\nu} &= \mathrm{Tr}\left[U_{x,\mu}U_{x+\widehat{\mu},\nu}U^\dagger_{x+\widehat{\nu},\mu}U^\dagger_{x,\nu}\right] \\
S_G &= \frac{1}{g^2}\sum_x\sum_{\mu\neq\nu}\left(3 - P_{x,\mu\nu} - P^\dagger_{x,\mu\nu}\right) \\
&\to \int \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \mathcal{O}(a^2) \quad \text{as } a \to 0
\end{aligned}
$$

# Domain wall Dirac operator

$$D^W_{x,y}(M_5) = (4 - M_5)\delta_{x,y} - \frac{1}{2}\Big[(1+\gamma^\mu)U^\dagger_{x,\mu}\delta_{x,y+\mu} + (1-\gamma^\mu)U_{x,\mu}\delta_{x+\mu,y}\Big]$$

$$D_{s,s'}(m) = \left[D^W(M_5) + 1\right]\delta_{s,s'} + P_L\left[(1+m)\delta_{s,L_s-1}\delta_{s',0} - \delta_{s+1,s'}\right] + P_R\left[(1+m)\delta_{s,0}\delta_{s',L_s-1} - \delta_{s,s'+1}\right]$$

$$D(m) = \begin{pmatrix} D^W+1 & -P_L & 0 & \cdots & mP_R \\ -P_R & D^W+1 & -P_L & \cdots & 0 \\ 0 & -P_R & D^W+1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ mP_L & 0 & 0 & \cdots & D^W+1 \end{pmatrix}$$

$P_L = \frac{1}{2}(1-\gamma_5)$, $P_R = \frac{1}{2}(1+\gamma_5)$; $M_5 < 2$ is height of domain wall

# Conserved and local domain wall currents

Conserved currents:

$$\mathcal{V}^{\mu a}(x) = \sum_{s=0}^{L_s-1} j^{\mu a}(x,s) \qquad \mathcal{A}^{\mu a}(x) = \sum_{s=0}^{L_s-1} \mathrm{sign}\left(s - \frac{L_s-1}{2}\right) j^{\mu a}(x,s)$$

$$\begin{aligned} j^{\mu a}(x,s) = {} & \overline{\Psi}(x+\widehat{\mu},s)\frac{1+\gamma^\mu}{2}\tau^a U^\dagger_{x,\mu}\Psi(x,s) \\ & -\overline{\Psi}(x,s)\frac{1-\gamma^\mu}{2}\tau^a U_{x,\mu}\Psi(x+\widehat{\mu},s) \end{aligned}$$

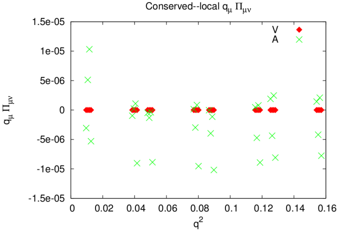
Local currents:

$$V^\mu(x) = \overline{q}(x)\gamma^\mu\tau^a q(x) \qquad A^\mu(x) = \overline{q}(x)\gamma^\mu\gamma^5\tau^a q(x)$$

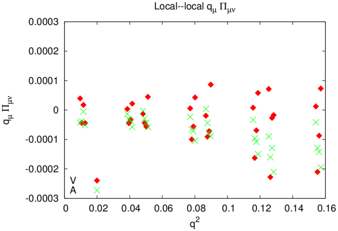
$$q(x) = P_L\Psi(x,0) + P_R\Psi(x,L_s-1)$$

# Ward identities and violations

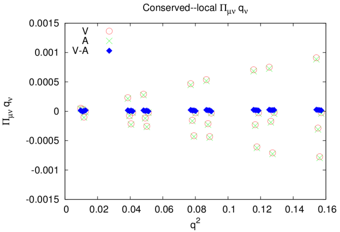
$$\widehat{Q}_\mu \left[\textstyle\sum_x e^{iQ\cdot(x+\widehat{\mu}/2)} \left\langle \mathcal{V}^a_\mu(x) V^a_\nu(0)\right\rangle\right] = 0$$



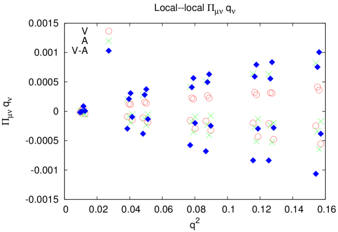

$$\widehat{Q}_\mu \left[\textstyle\sum_x e^{iQ\cdot x} \left\langle V^a_\mu(x) V^a_\nu(0)\right\rangle\right] \neq 0$$




$$\left[\textstyle\sum_x e^{iQ\cdot(x+\widehat{\mu}/2)} \left(\left\langle \mathcal{V}^a_\mu V^a_\nu\right\rangle - \left\langle \mathcal{A}^a_\mu A^a_\nu\right\rangle\right)\right] \widehat{Q}_\nu \approx 0$$



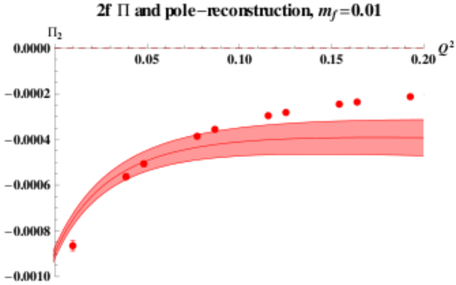

$$\left[\textstyle\sum_x e^{iQ\cdot x} \left(\left\langle V^a_\mu V^a_\nu\right\rangle - \left\langle A^a_\mu A^a_\nu\right\rangle\right)\right] \widehat{Q}_\nu \neq 0$$

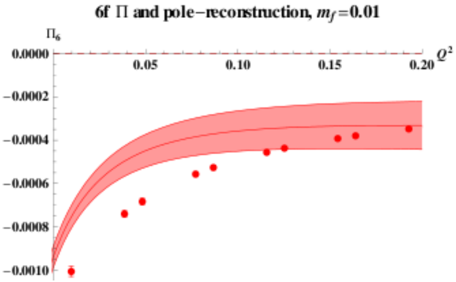


# Single-pole approximations to $\Pi_{V-A}$

$$R_V(s) = 12\pi^2 F_V^2 \delta(s - M_V^2) \qquad R_A(s) = 12\pi^2 F_A^2 \delta(s - M_A^2)$$

$$\Pi_{V-A}(Q^2) = -F_P^2 + \frac{Q^2 F_V^2}{M_V^2 + Q^2} - \frac{Q^2 F_A^2}{M_A^2 + Q^2}$$



# $S$ in $\chi$PT, for $N_f = 2$

$$S = \frac{1}{12\pi}\left(\bar{\ell}_5 + \log\left[\frac{M_P^2 \frac{v^2}{F_P^2}}{M_H^2}\right] - \frac{1}{6}\right)$$

$\bar{\ell}_5$ is extracted from (Gasser and Leutwyler)

$$\Pi_{V-A}(Q^2) = -F_P^2 + Q^2\left[\frac{1}{24\pi^2}\left(\bar{\ell}_5 - \frac{1}{3}\right) + \frac{2}{3}(1+x)\bar{J}(x)\right]$$

$$\bar{J}(x) = \frac{1}{16\pi^2}\left(\sqrt{1+x}\log\left[\frac{\sqrt{1+x}-1}{\sqrt{1+x}+1}\right] + 2\right), \quad x \equiv 4M_P^2/Q^2$$

- Our $N_f \geq 6$ simulations have $M_P$ too large to apply $\chi$PT
- General-$N_f$ corrections for $\bar{\ell}_5$ not yet known
- Must take only two flavors to the chiral limit, any others remain massive