



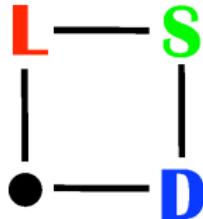
Lattice calculation of composite dark matter form factors

David Schaich (University of Colorado)
for the Lattice Strong Dynamics Collaboration

APS April Meeting, Denver, 13 April 2013

LSD Collaboration, [arXiv:1301.1693](https://arxiv.org/abs/1301.1693) (submitted to *Phys. Rev. D*)

Lattice Strong Dynamics Collaboration



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Exploring the range of possible phenomena
in strongly-coupled gauge theories

Overview: goals, methods and results

Goals

- Constrain models of composite dark matter “baryons” based on a new strong gauge force in the dark sector
- Direct detection depends on electromagnetic form factors, especially dark matter charge radius and magnetic moment

Methods

- Non-perturbative lattice gauge theory calculations
- Explore a couple of SU(3) gauge models, for a range of masses

Results

- From XENON100 data we exclude $M_B \lesssim 10$ TeV for these models
- Cross section dominated by magnetic moment
- Clear directions for future studies

Motivation for composite dark matter

After Planck

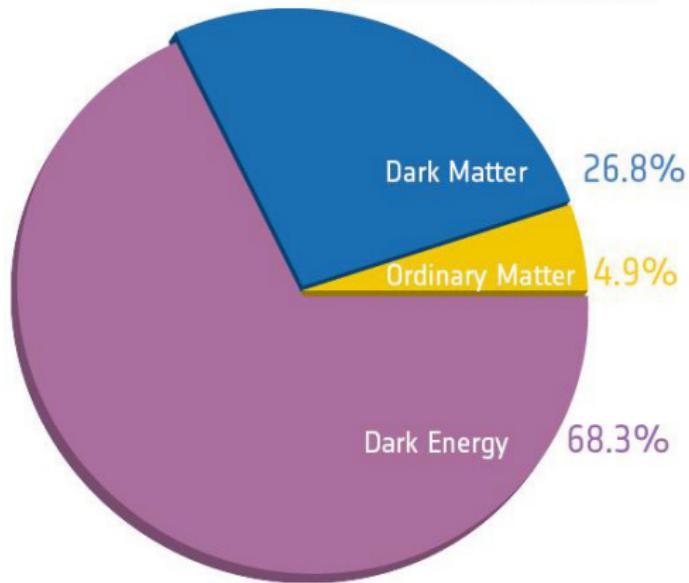
Credit: ESA & Planck

$$\frac{\Omega_{DM}}{\Omega_{SM}} \approx 5$$

May indicate DM–SM coupling

For example:

- Thermal relic DM
(WIMP miracle; WIMPless)
- Asymmetric DM

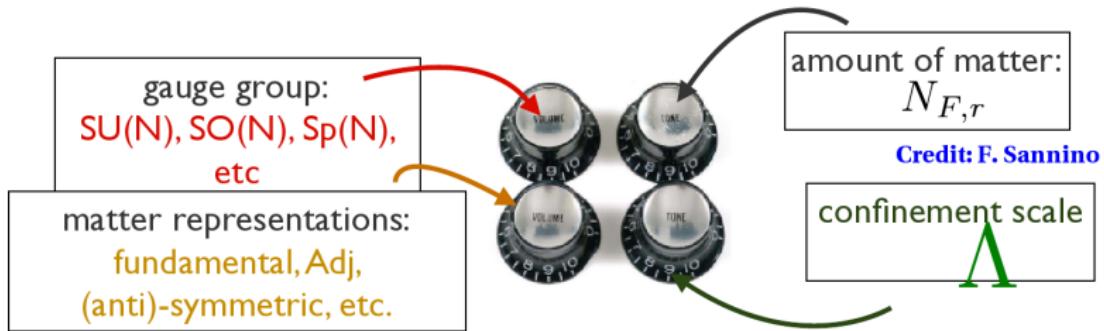


If Ω_{DM} related to coupling between dark matter and standard model,
such interactions must now be suppressed

Composite dark matter models are one way to make this natural

Basic composite dark matter scenario

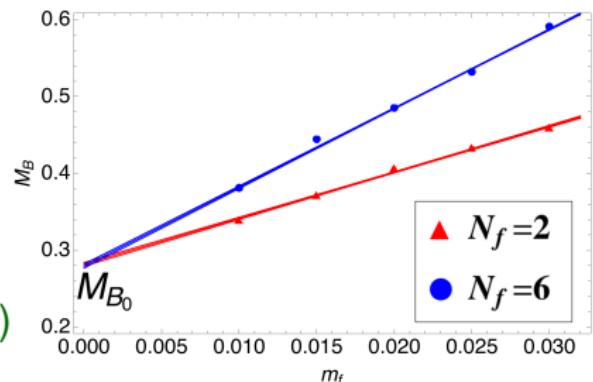
- Hypothesize a new confining gauge force in the dark sector
 - Present-day dark matter: **electroweak-neutral** “baryons” stabilized by analog of baryon number conservation
 - Charged constituents produce active interactions in early universe
 - Electromagnetic form factors allow direct detection now
-
- Strong interactions require non-perturbative analysis \Rightarrow lattice
 - Lattice calculations too expensive to study many potential models





For initial explorations
we re-analyze existing lattices

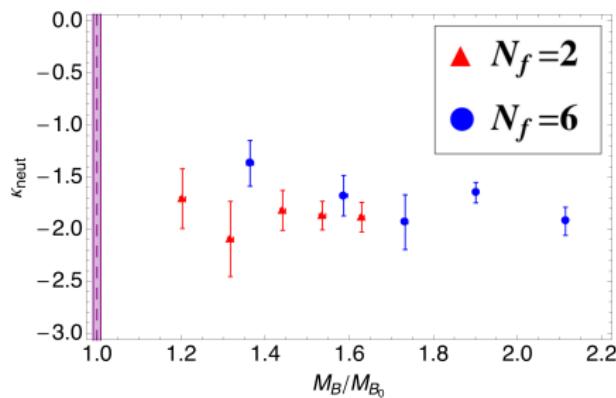
- **Fix** $SU(3)$ gauge group (like QCD)
- **Fix** fermions in fundamental rep.
- **Compare** $N_f = 2$ or 6 degenerate fermions,
with **fixed confinement scale** $\Lambda \sim M_{B_0}$
- **Scan** range of fermion masses m_f
Unlike QCD, fermions are relatively heavy, $0.4 \lesssim M_\pi/M_B \lesssim 0.5$



Form factors for dark matter direct detection

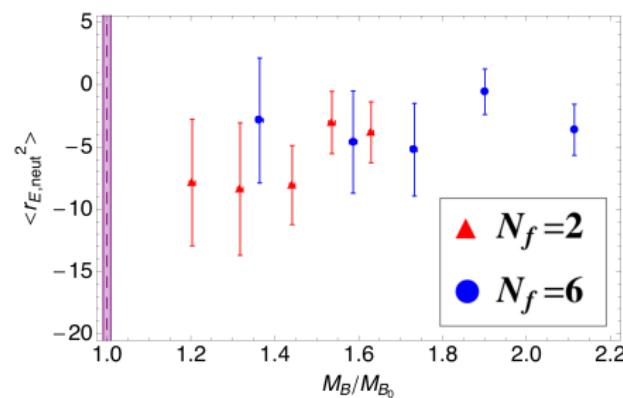
Magnetic moment κ

$$\sim (\bar{\psi} \sigma^{\mu\nu} \psi) F_{\mu\nu} \quad (\text{dim-5})$$



Charge radius $\langle r^2 \rangle$

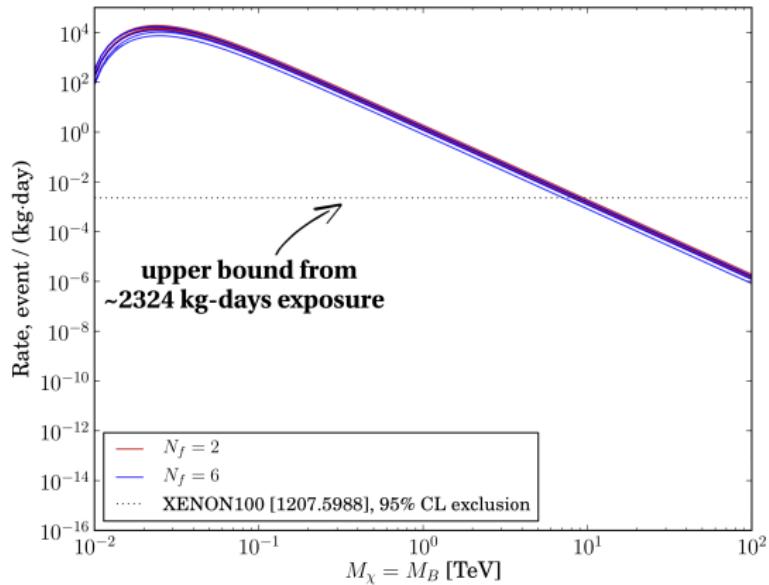
$$\sim (\bar{\psi} \psi) v_\mu \partial_\nu F_{\mu\nu} \quad (\text{dim-6})$$



- Results show little dependence on N_f or on M_B/M_{B_0}
- κ comparable to QCD value $\kappa_{QCD} = -1.91$
- $\langle r^2 \rangle$ different from $\langle r^2 \rangle_{QCD} \approx -38$, due to our larger M_π/M_B

Insert into the usual calculations to predict scattering rates...

Predicted XENON100 event rate

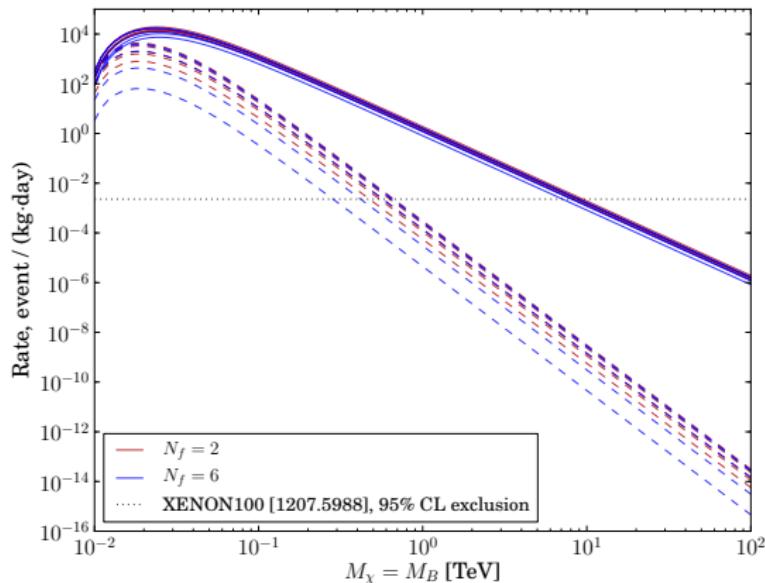


From XENON100 results
(PRL 109:181301, 2012)
we exclude $M_B \lesssim 10$ TeV
for these models

Little dependence
on N_f or on M_B/M_{B_0}
(all ten cases overlaid)

For thermal relic dark matter,
constraint $M_B \gtrsim 10$ TeV nearly saturates unitarity limit $M_B \lesssim 20$ TeV
(Griest & Kamionkowski, 1990)

Magnetic moment dominates for $M_B \gtrsim 25$ GeV



Dashed lines show
charge radius contribution

Suppressed by $1/M_B^2$
relative to mag. moment

No magnetic moment
→ more viable models

Guidance for future directions

- Bosonic “baryons” (e.g., for SU(4)) have no magnetic moment
- Charge radius itself can vanish for symmetric charge assignments
- Dim-7 polarizabilities $\sim (\bar{\psi}\psi) F_{\mu\nu}F^{\mu\nu}$ then leading contribution

Recapitulation

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Methods

- Non-perturbative lattice gauge theory calculations
- Explore SU(3) models with $N_f = 2$ and 6 for a range of masses

Results

- From XENON100 data we exclude $M_B \lesssim 10$ TeV for these models
- Cross section dominated by magnetic moment
- Clear directions for future studies: SU(4) models, polarizabilities

Thank you!

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Collaborators on arXiv:1301.1693

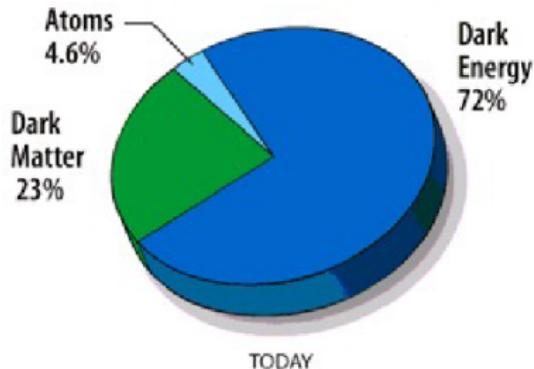
Tom Appelquist, Rich Brower, Mike Buchoff, Michael Cheng,
Saul Cohen, George Fleming, Joe Kiskis, Meifeng Lin, Ethan Neil,
James Osborn, Claudio Rebbi, Chris Schroeder, Sergey Syritsyn,
Gennady Voronov, Pavlos Vranas, Joe Wasem

Funding and computing resources

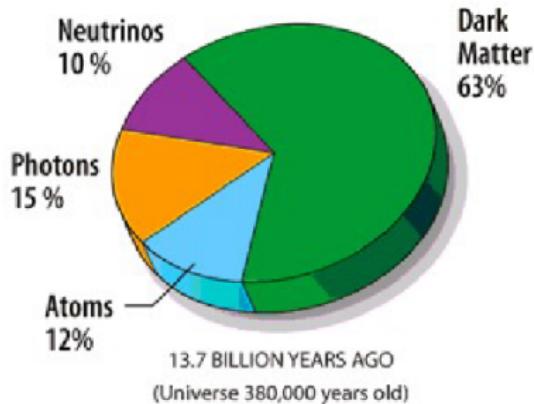


Backup: Dark matter density in cosmological history

$$\frac{\Omega_{DM}}{\Omega_{baryons}} \approx \frac{\Omega_{DM}}{\Omega_{SM}} \approx 5 \text{ now}$$



$$\frac{\Omega_{DM}}{\Omega_{baryons}} \approx 5 \text{ then}$$



Backup: Two roads to natural asymmetric dark matter

$$\begin{aligned}\rho_D &\approx 5\rho_B \\ \implies M_D n_D &\approx 5M_B n_B\end{aligned}$$

- $n_D \sim n_B \implies M_D \sim 5 \text{ GeV}$
High-dimensional interactions relate baryon# and DM# violation
- $M_D \gg M_B \implies n_B \gg n_D \sim \exp[-M_D/T_s]$
Sphaleron transitions above $T_s \sim 200 \text{ GeV}$ distribute asymmetries

Both require coupling between standard model and dark matter

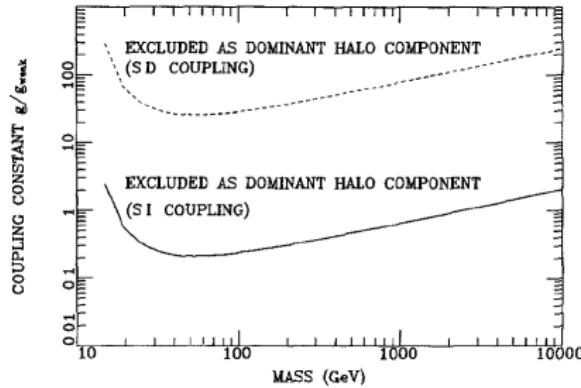
Backup: Electroweak-charged composite dark matter

SU(3) composite dark matter “baryons” are Dirac fermions

Therefore a net electroweak charge

- ⇒ Unsuppressed tree-level Z-exchange interaction with nuclei
- ⇒ Spin-independent cross section $\sigma \sim 10^{-38} \text{ cm}^2$
- ⇒ Ruled out decades ago

(Example: Ahlen et al., 1987)

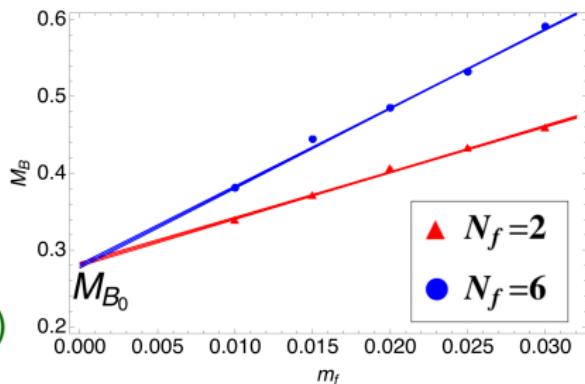


Neutralinos are Majorana fermions, so evade this bound

Backup: More model details

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we re-analyze existing lattices

- **Fix** SU(3) gauge group (like QCD)
- **Fix** fermions in fundamental rep.
- **Compare** $N_f = 2$ or 6 degenerate fermions,
with **fixed confinement scale** $\Lambda \sim M_{B_0}$
- **Scan** range of fermion masses m_f
Unlike QCD, fermions are relatively heavy, $0.4 \lesssim M_\pi/M_B \lesssim 0.5$



Also unlike QCD, fermions are all $SU(2)_L$ singlets

$Q = Y$, assign half of fermions $Q_p = 2/3$, other half $Q_m = -1/3$

Then lightest “baryon” B is electroweak-neutral pmm combination

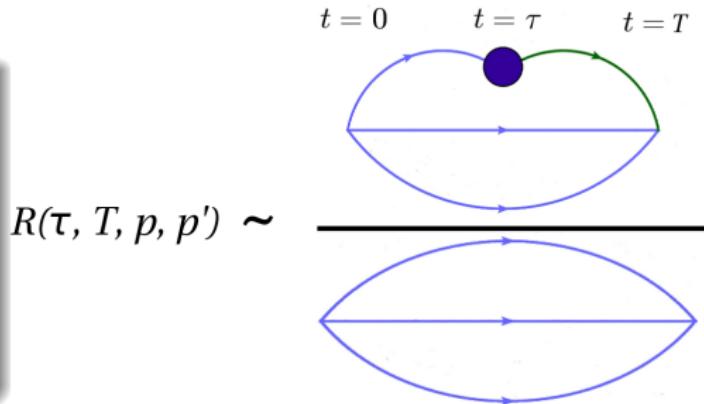
Backup: Calculational details I: Form factors

With $q = p' - p$ and $Q^2 = -q^2 > 0$,

$$\langle B(p') | \bar{\psi} \gamma^\mu \psi | B(p) \rangle = \overline{U}(p') \left[F_1^\psi(Q^2) \gamma^\mu + F_2^\psi(Q^2) \frac{i \sigma^{\mu\nu} q_\nu}{2M_B} \right] U(p)$$

$$\kappa \equiv F_2(0) \quad \langle r^2 \rangle = \int d^3r [r^2 \rho(r)] \equiv -6 \left. \frac{dF_1(Q^2)}{dQ^2} \right|_{Q^2=0} + \frac{3\kappa}{2M_B^2}$$

$$\begin{aligned} R_{\mathcal{O}}(\tau, T, p, p') \\ \longrightarrow \langle B(p') | \mathcal{O} | B(p) \rangle \\ + \mathcal{O}(e^{-\Delta\tau}) + \mathcal{O}(e^{-\Delta T}) \\ + \mathcal{O}(e^{-\Delta(T-\tau)}) \end{aligned}$$



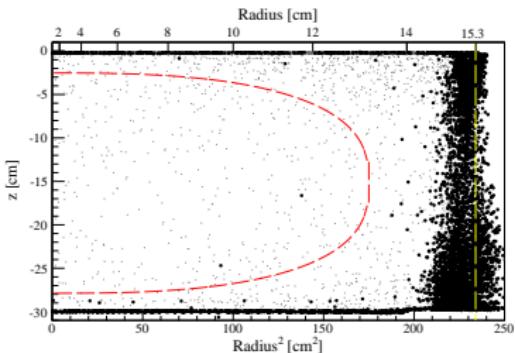
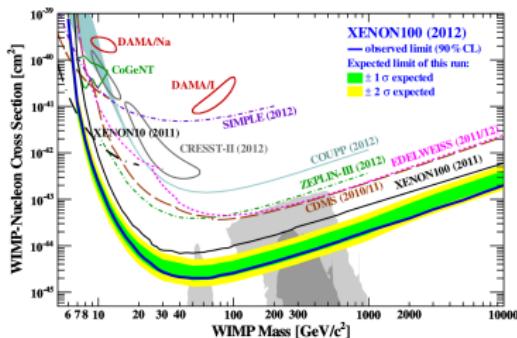
Backup: Calculational details II: Event rate

$$\text{Rate} = \frac{M_{\text{detector}}}{M_T} \frac{\rho_{DM}}{M_B} \int_{E_{\min}}^{E_{\max}} dE_R \mathcal{A}cc(E_R) \left\langle v' \frac{d\sigma}{dE_R} \right\rangle_f$$

$$\frac{d\sigma}{dE_R} = \frac{\overline{|\mathcal{M}_{SI}|^2} + \overline{|\mathcal{M}_{SD}|^2}}{16\pi (M_B + M_T)^2 E_R^{\max}}$$

$$\overline{|\mathcal{M}_{SI}|^2} = e^4 [ZF_c(Q)]^2 \left(\frac{M_T}{M_B} \right)^2 \left[\frac{4}{9} M_B^2 \left\langle r^2 \right\rangle^2 + \left(1 + \frac{M_B}{M_T} \right)^2 \kappa^2 \cot^2 \frac{\theta_{CM}}{2} \right]$$

$$\overline{|\mathcal{M}_{SD}|^2} = e^4 \frac{2}{3} \left(\frac{J+1}{J} \right) \left[\left(A \frac{\mu_T}{\mu_n} \right) F_s(Q) \right]^2 \kappa^2$$



Backup: More on possible future directions

Polarizabilities on the lattice

- Background electric field $\mathcal{E} \propto 2\pi/L^2$ on L^4 lattice
- Extract polarizability α from contribution to energy:

$$E = M + \frac{1}{2}\alpha\mathcal{E}^2 + \mathcal{O}(\mathcal{E}^4)$$

To neglect $\mathcal{O}(\mathcal{E}^4)$, in practice need very large L

Other possibilities

- SU(2) cheaper than SU(4), but pseudoreal representations
 \implies “mesons” and “baryons” interchangeable
- Large fermion masses, so that $M_B \approx N_c m_f$
- Strongly-coupled **scalar** constituents