Quantum simulation for quantum field theories

David Schaich (U. Liverpool)

Northern Quantum Meeting VIII — York — 12 June 2023

arXiv:2112.07651 arXiv:2301.02230

and more to come with Chris Culver

Quantum computing & simulation for lattice quantum field theory

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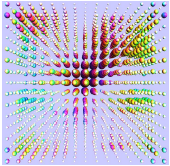
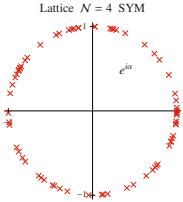
Overview and plan

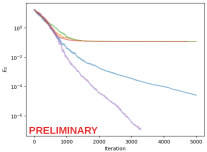


Lattice quantum field theory (QFT) is a promising domain for quantum computing application & co-design



Context: Lattice QFT

Connections to quantum computing / simulation

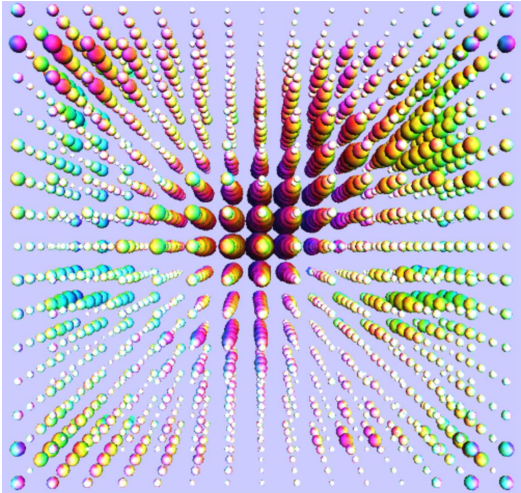


Application to spontaneous supersymmetry breaking

Context: QFT

Quantum mechanics
+ special relativity
= quantum field theory

Picture relativistic quantum fields
filling $(d+1)$-dimensional space-time



(Image credit: Claudio Rebbi)

The QFT / StatMech Correspondence

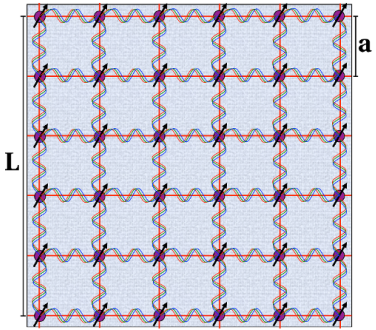
Generating functional (Feynman path integral)	Partition function
$\mathcal{Z} = \int \mathcal{D}\Phi \;\; e^{iS[\Phi]\,/\,\hbar}$	$\int \mathcal{D}q\,\mathcal{D}p \;\; e^{-H(q,p)\,/\,k_B T}$
Action $S[\Phi] = \int d\vec{x}dt\;\mathcal{L}[\Phi(\vec{x},t)]$	Hamiltonian H
$\hbar\,(=1) \longleftrightarrow$ quantum fluctuations	$k_B T \longleftrightarrow$ thermal fluctuations

Lattice QFT in a nutshell

Formally $\langle \mathcal{O} \rangle = \frac{1}{\mathcal{Z}} \int \mathcal{D}\Phi \;\; \mathcal{O}(\Phi) \;\; e^{iS[\Phi]}$

Regularize by formulating theory in finite, discrete space-time

↖ Non-perturbative, $(d+1)$-dimensional, gauge-invariant



P. Vranas LLNL

Spacing between lattice sites ("a")

$\longrightarrow$ UV cutoff scale $1/a$

Remove cutoff: $a \to 0 \quad (L/a \to \infty)$

Discrete $\longrightarrow$ continuous symmetries ✓

Classical computing and lattice QFT

$$\langle \mathcal{O} \rangle = \frac{1}{\mathcal{Z}} \int \mathcal{D}\Phi \;\; \mathcal{O}(\Phi) \;\; e^{iS[\Phi]}$$

Wick-rotate $t \longrightarrow i\tau$

$e^{iS} \longrightarrow e^{-S}$

$S > 0 \longrightarrow$ importance sampling

Long history using & co-designing supercomputers

$\longrightarrow$ numerical predictions for standard model & beyond

[particle masses, decays, structure, scattering, thermodynamics, $(g-2)$, composite Higgs, dark matter, holographic duality, ...]

Classical computing limitations

Wick rotation only allows access to equilibrium physics

Complex e^{iS} unavoidable in real-time dynamics

Action S itself can have complex phase or sign fluctuations

[net fermion number; supersymmetric QFTs; ...]

Arranging $\langle \mathcal{O} \rangle = \frac{1}{\mathcal{Z}} \int \mathcal{D}\Phi \;\; \mathcal{O} \;\; e^{iS} = \frac{\int \mathcal{D}\Phi \;\; \mathcal{O} \; \frac{e^{iS}}{e^{-|S|}} \;\; e^{-|S|}}{\int \mathcal{D}\Phi \; \frac{e^{iS}}{e^{-|S|}} \;\; e^{-|S|}} = \frac{\left\langle \mathcal{O} \; \frac{e^{iS}}{e^{-|S|}} \right\rangle_{||}}{\left\langle \frac{e^{iS}}{e^{-|S|}} \right\rangle_{||}}$

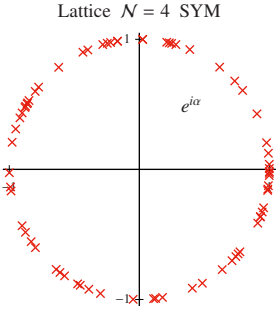
Sign problem: $\langle \mathrm{sign} \rangle_{||} = \left\langle \frac{e^{iS}}{e^{-|S|}} \right\rangle_{||} \to 0$ exponentially quickly

Sign problem examples

$$\frac{e^{iS}}{e^{-|S|}} \text{ is pure phase}$$

for lattice supersymmetric Yang–Mills theory

Clearly averages to zero



Spontaneous supersymmetry breaking

Requires vanishing **Witten index**

$$\mathcal{W} = \mathrm{Tr}\left[(-1)^F e^{-iHt}\right] = \mathrm{Tr}_B\left[e^{-iHt}\right] - \mathrm{Tr}_F\left[e^{-iHt}\right] \propto \langle \mathrm{sign} \rangle_{||}$$

$\langle \mathrm{sign} \rangle_{||} = 0 \longrightarrow$ maximally bad sign problem

Turn to quantum computing

In principle evade exponential classical computing costs

Change perspective

Path integral $\longrightarrow$ continuous-time hamiltonian H on spatial lattice

Generic targets:

Find ground state $|\Omega\rangle \longrightarrow$ test spontaneous symmetry breaking

Real-time evolution $|\Psi(t)\rangle = e^{-iHt}\,|\Psi(0)\rangle \sim \left(\exp\left[-iH\delta_T\right]\right)^{N_T} |\Psi(0)\rangle$

(1+1)-dimensional Wess–Zumino model

arXiv:2301.02230

Supersymmetric $H \longleftrightarrow$ matched boson / fermion d.o.f. at each lattice site:

$$H = \sum_n \left[\frac{p_n^2}{2} + \frac{1}{2}\left(\frac{\phi_{n+1} - \phi_{n-1}}{2}\right)^2 + \frac{1}{2}[W(\phi_n)]^2 + W(\phi_n)\frac{\phi_{n+1} - \phi_{n-1}}{2} \right.$$
$$\left. + (-1)^n W'(\phi_n)\left(\chi_n^\dagger \chi_n - \frac{1}{2}\right) + \frac{1}{2}\left(\chi_n^\dagger \chi_{n+1} + \chi_{n+1}^\dagger \chi_n\right)\right]$$

Prepotential $W(\phi)$ ensures supersymmetric interactions

$W = \phi \longrightarrow$ free theory $\longrightarrow$ expect supersymmetric $|\Omega\rangle$

$W = \phi^2 \longrightarrow$ expect dynamical supersymmetry breaking

Wess–Zumino set up for quantum computing

Lattice $\longrightarrow$ finite number of d.o.f.

Need to map bosons and fermions to finite number of qubits

Fermions — usual Jordan–Wigner transformation $\longrightarrow$ one qubit per site

Bosons — retain lowest $\Lambda = 2^B$ harmonic oscillator modes

binary encoding $\longrightarrow$ B qubits per site

Different treatment breaks supersymmetry, recovered as $\Lambda \to \infty$

Current focus on exploratory development & testing

$\longrightarrow$ Qiskit simulator for rapid turnaround

[github.com/chrisculver/WessZumino]

Variational quantum eigensolver (VQE)

Well-known 'hybrid' quantum–classical algorithm

Quantum circuit implements wave-function ansatz $|\Psi(\theta_i)\rangle$ with tunable params

Loss function measurements $\longrightarrow$ classical optimizer adjusts θ_i

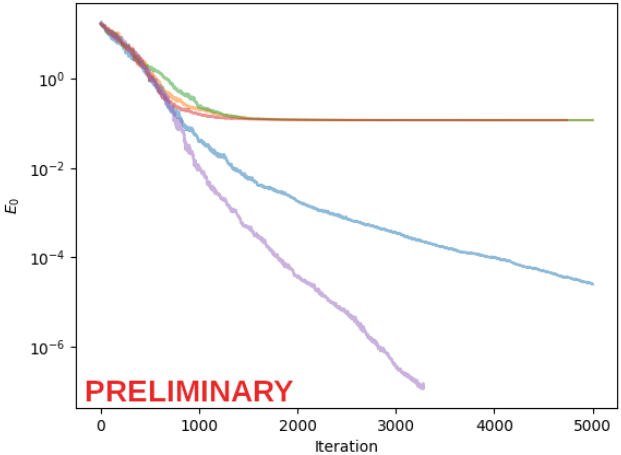
Energy as loss function $\longrightarrow$ approximate ground state

Apply to Wess–Zumino model

Spontaneous supersymmetry breaking $\longleftrightarrow$ non-zero ground-state energy

Wess–Zumino VQE

$W = \phi$ with $\Lambda = 16$ and 2 lattice sites



Even free theory
doesn't always converge

For interacting cases,
easier to converge to non-zero E
harder to decide whether $E \to 0$

Variational quantum deflation (VQD)

Supersymmetry
$\longrightarrow$ all nonzero-energy states paired

Idea: Find three smallest energies, check pairing

Use VQD algorithm
[Higgott–Wang–Brierley, arXiv:1805.08138]

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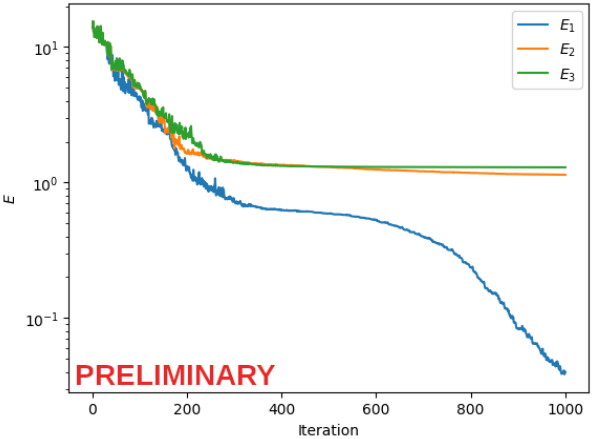
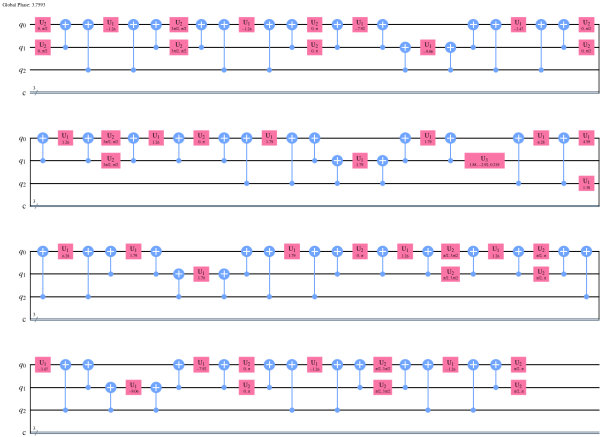


Real-time evolution

For now, need far too many gates for reasonable $\Lambda \gtrsim \mathcal{O}(10)$



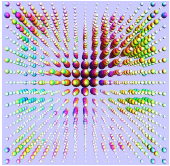
Here $\Lambda = 4$ on single site

$\longrightarrow$ supersymmetric quantum mech.

[arXiv:2112.07651]

Smarter Trotterization & transpilation should help

Recap: Lattice QFT and quantum computing

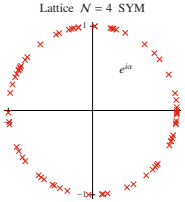


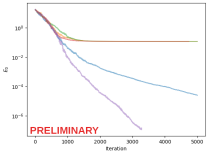
Lattice QFT is a promising domain
for quantum computing application & codesign

Lattice QFT at forefront of scientific computing

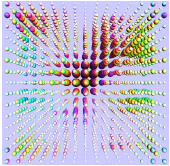
Classically intractable problems motivate quantum computing

Work in progress on spontaneous supersymmetry breaking





Recap: Lattice QFT and quantum computing



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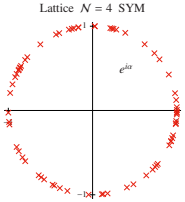
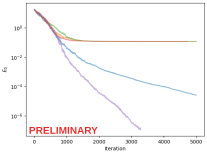


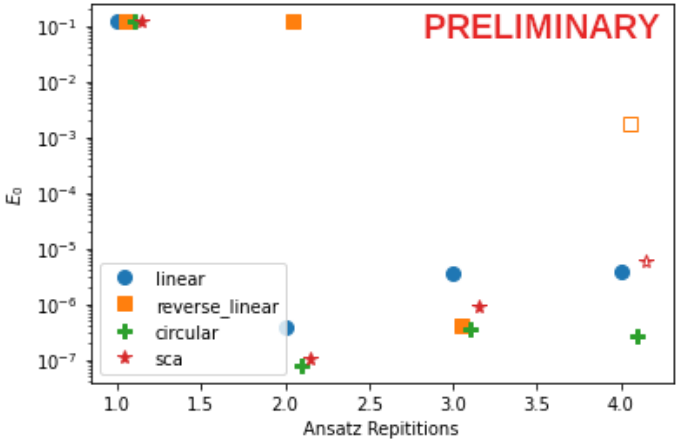
Thanks for your attention! Any questions?



Chris Culver

UK Research and Innovation

Backup: Ansatz testing for Wess–Zumino VQE



More repetitions

$\longrightarrow$ more params to optimize

In principle more expressive

In practice harder to converge

Backup: Supersymmetric quantum mechanics

arXiv:2112.07651

Reduce to single spatial site:

$$H_{\mathrm{SQM}} = \frac{1}{2}\left[\hat{p}^2 + [W(\hat{q})]^2 - W'(\hat{q})\left(\hat{b}^\dagger\hat{b} - \hat{b}\hat{b}^\dagger\right)\right].$$

Spontaneous supersymmetry breaking no longer dynamical

$\longrightarrow$ determined by prepotential

Harmonic oscillator $W_{\mathrm{HO}} = m\hat{q} \longrightarrow$ expect (free) supersymmetric $|\Omega\rangle$

Anharmonic oscillator $W_{\mathrm{AHO}} = m\hat{q} + g\hat{q}^3 \longrightarrow$ expect supersymmetric $|\Omega\rangle$

Double-well $W_{\mathrm{DW}} = m\hat{q} + g\left(\hat{q}^2 + \mu^2\right) \longrightarrow$ expect spont. susy breaking

Backup: Supersymmetric quantum mechanics

arXiv:2112.07651

Ground-state energies from VQE

Free theory clearly converges to zero energy

More params $\longrightarrow$ harder to converge, especially for anharmonic oscillator W_{AHO}

Clear non-zero energy (spont. susy breaking) for double-well W_{DW}

Λ	HO	VQE	AHO	VQE	DW	VQE
2	0	5.34e-10	9.38e-01	9.38e-01	1.08e+00	1.08e+00
4	0	1.07e-09	1.27e-01	1.27e-01	9.15e-01	9.15e-01
8	0	4.06e-09	2.93e-02	2.93e-02	8.93e-01	8.93e-01
16	0	1.13e-08	1.83e-03	6.02e-02	8.92e-01	8.94e-01
32	0	4.81e-08	1.83e-05	6.63e-01	8.92e-01	8.95e-01

Backup: Supersymmetric quantum mechanics

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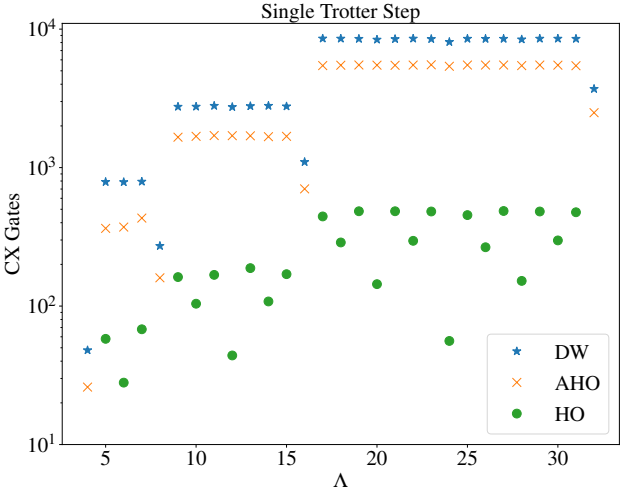

Count number of entangling gates
for single Trotter step
(default Qiskit transpilation)

Big improvements when $\Lambda = 2^B$
(note log scale)