

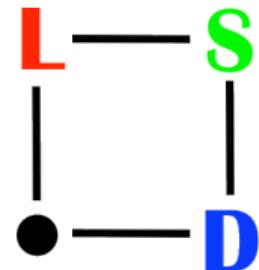
Composite dark matter and the role of lattice field theory

David Schaich (U. Liverpool)

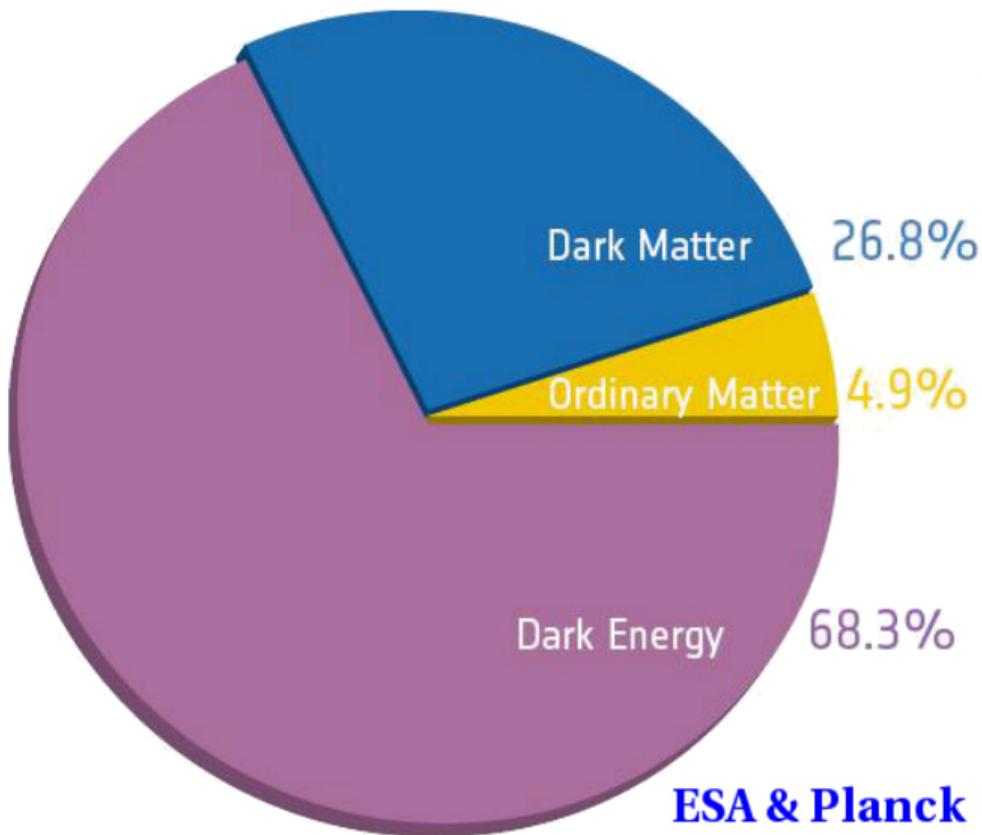
Theoretical Physics Seminar
Dublin Institute for Advanced Studies
17 November 2021



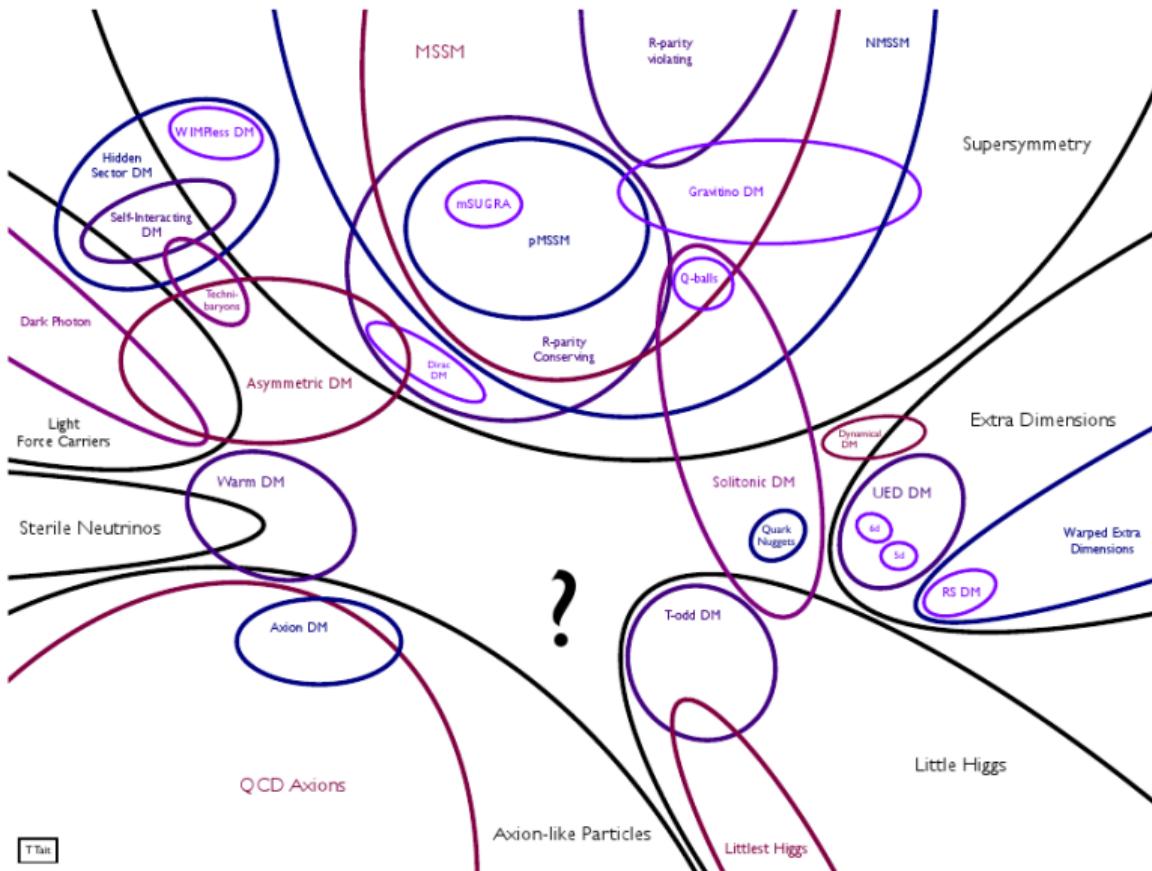
[arXiv:2006.16429](https://arxiv.org/abs/2006.16429) and more to come
with the Lattice Strong Dynamics Collaboration



Dark matter — we observe it...



...we don't yet know what it is



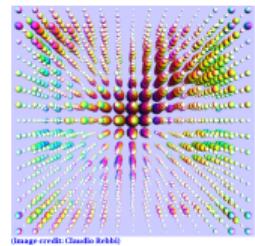
TTar

Overview and plan

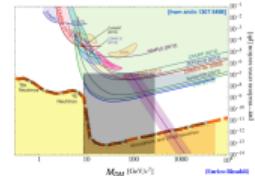
Composite dark matter is an attractive possibility



Lattice field theory is needed
to test models against experimental results



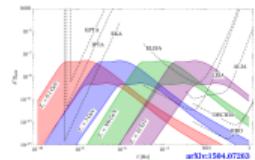
Why: Composite dark matter



How: Lattice field theory

What: Recent, ongoing & planned work

Direct detection experiments



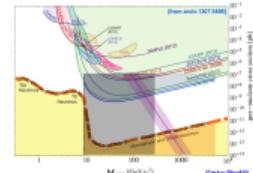
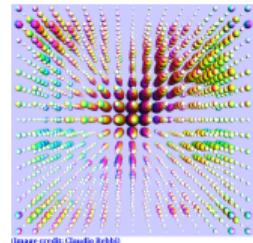
Gravitational-wave observatories

Collider experiments, galactic sub-structure, ...

Overview and plan

Composite dark matter is an attractive possibility

Lattice field theory is needed
to test models against experimental results



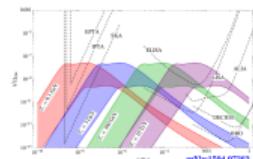
Why: Composite dark matter

How: Lattice field theory

What: Recent, ongoing & planned work

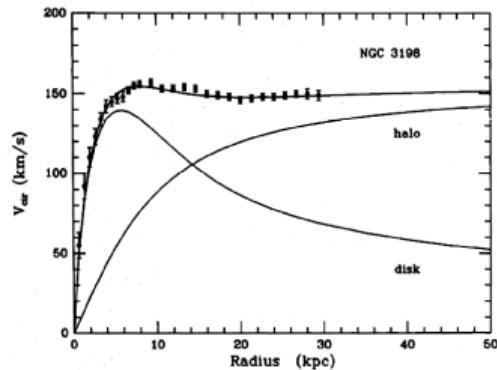
These slides: davidschaich.net/talks/2111Dublin.pdf

Interaction encouraged — complete coverage unnecessary

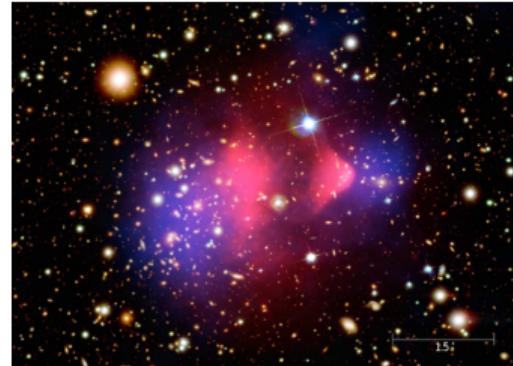


Gravitational evidence for dark matter

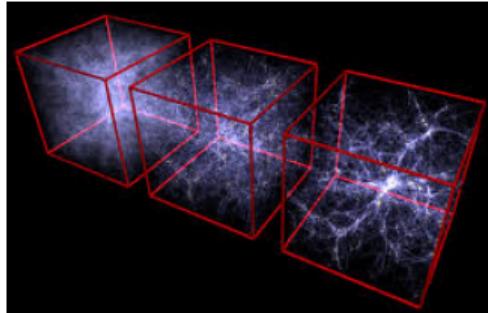
Rotation $\sim 10^3\text{--}10^6$ light-years



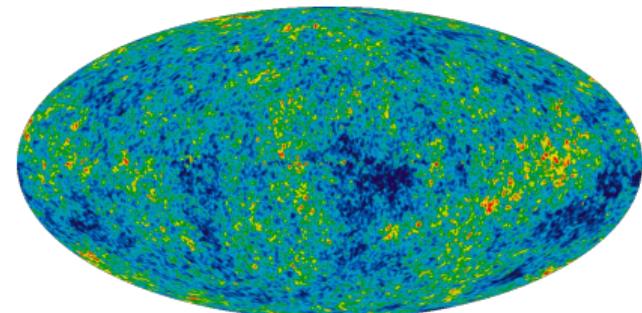
Lensing $\sim 10^6$ light-years



Structure $\sim 10^9$ light-years



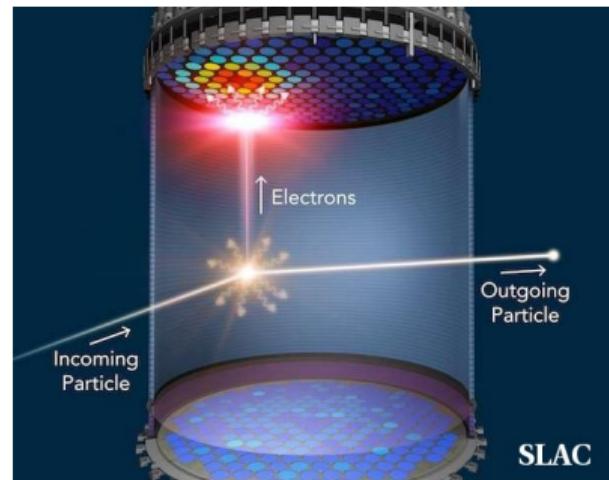
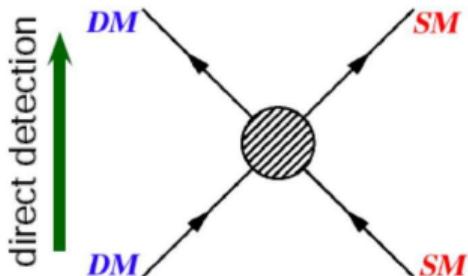
Cosmic background $\sim 10^{10}$ ly



Non-gravitational dark matter interactions

Three search strategies

Direct scattering in underground detectors

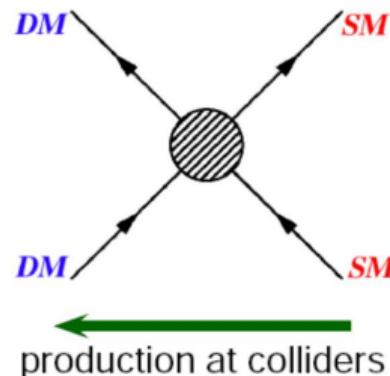


Non-gravitational dark matter interactions

Three search strategies

Direct scattering in underground detectors

Collider production at high energies



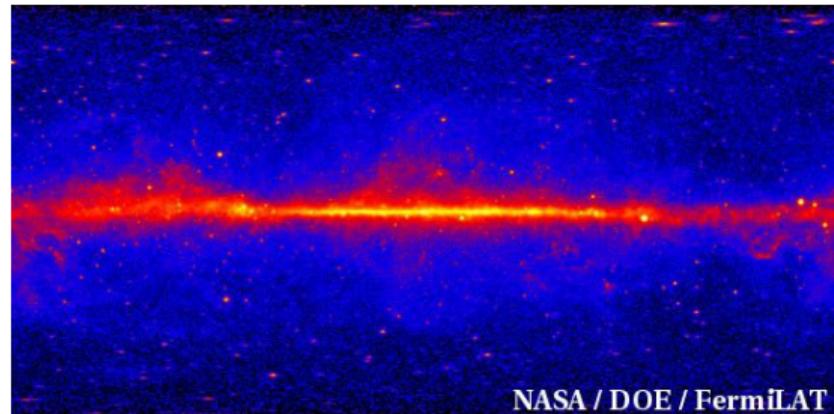
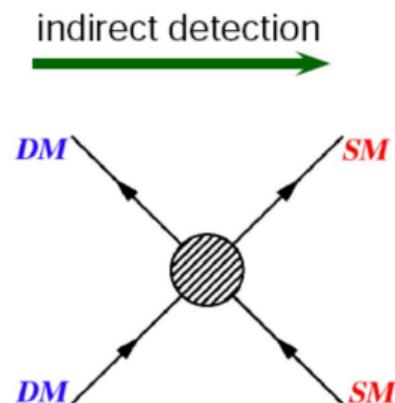
Non-gravitational dark matter interactions

Three search strategies

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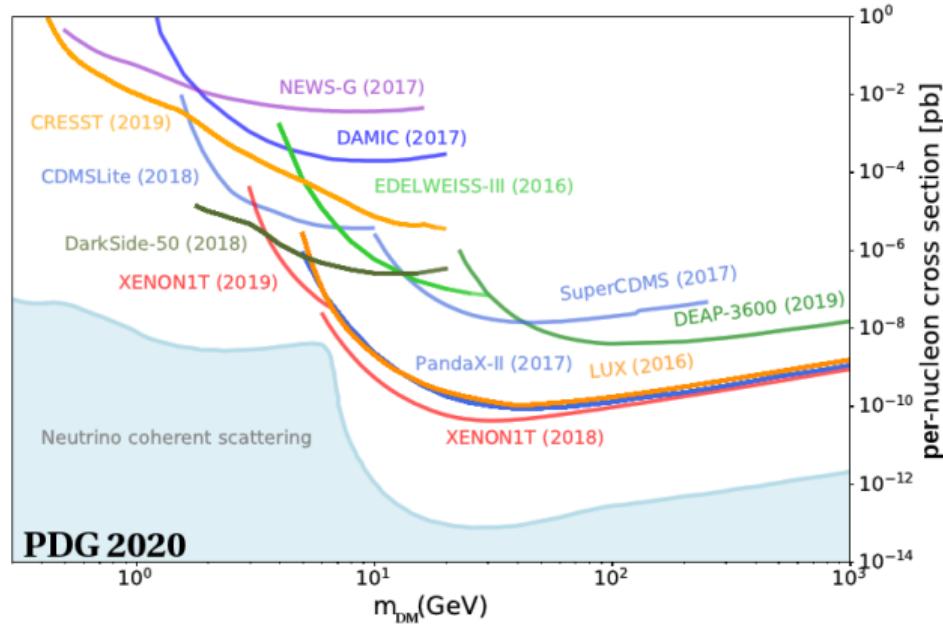
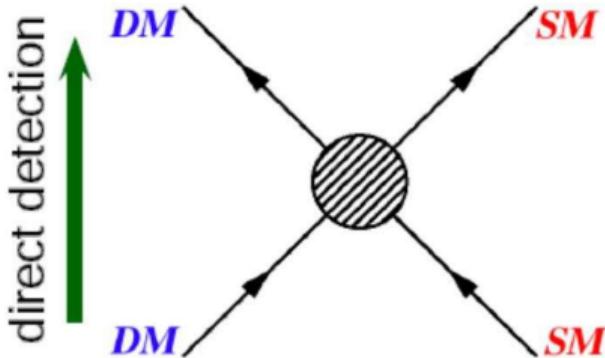
Indirect annihilation into cosmic rays



NASA / DOE / FermiLAT

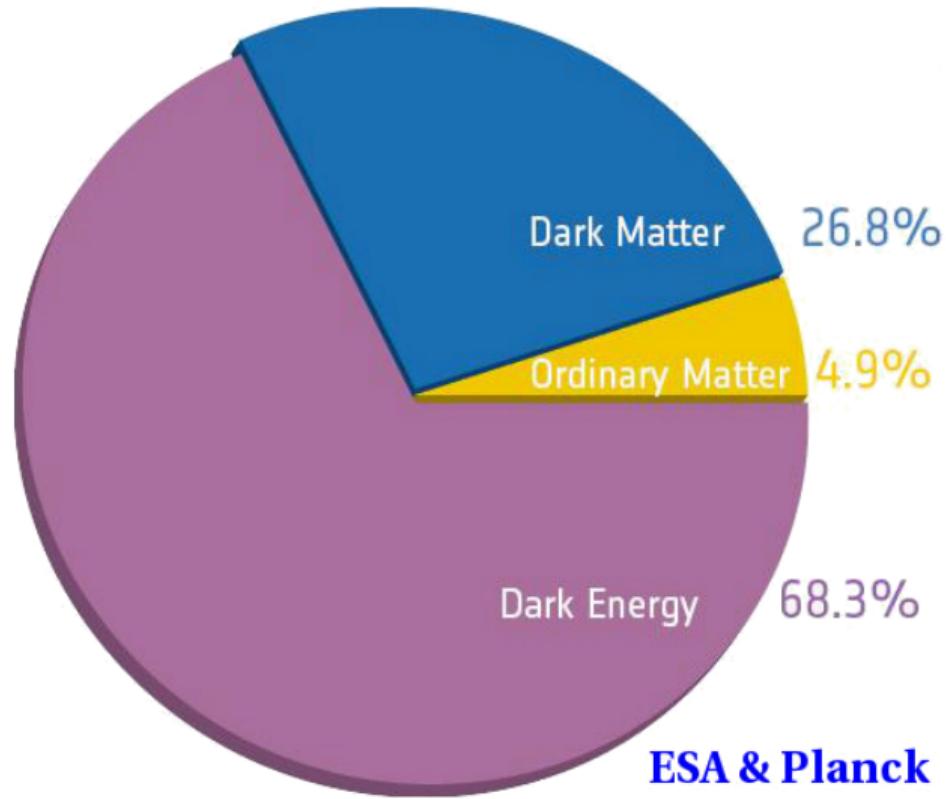
Non-gravitational dark matter interactions

No clear signals so far



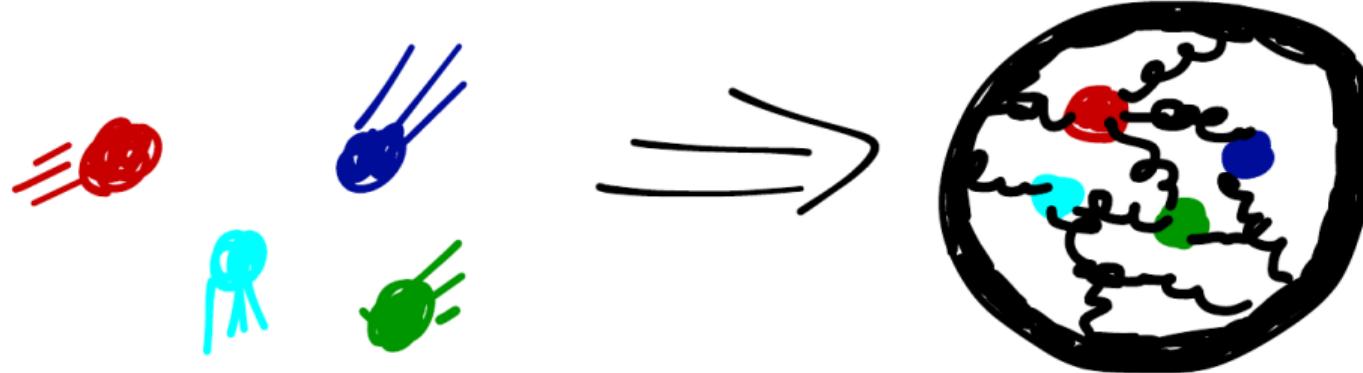
Why we expect non-gravitational interactions

$$\frac{\Omega_{\text{dark}}}{\Omega_{\text{ordinary}}} \approx 5 \quad \dots \text{not } 10^5 \text{ or } 10^{-5}$$



Explained by non-gravitational
interactions in the early universe

Composite dark matter



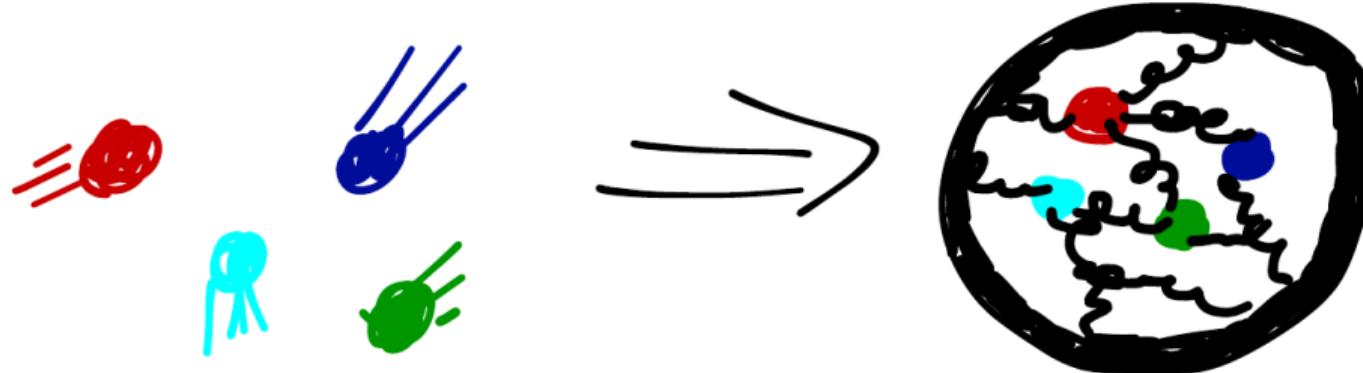
Early universe

Deconfined charged fermions → explain relic density

Present day

Confined neutral ‘dark baryons’ → no experimental detections

Composite dark matter



Present day

Confined neutral ‘dark baryons’ → no experimental detections

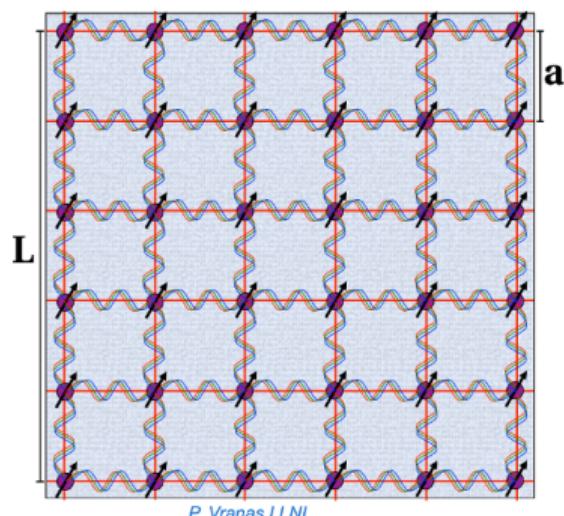
Interact via charged constituents

→ need **lattice calculations** for quantitative predictions

Lattice field theory in a nutshell

Formally $\langle \mathcal{O} \rangle = \frac{1}{\mathcal{Z}} \int \mathcal{D}\Phi \ \mathcal{O}(\Phi) \ e^{-S[\Phi]}$

Regularize by formulating theory in finite, discrete, euclidean space-time
↗ Gauge invariant, non-perturbative, 4-dimensional



Spacing between lattice sites (" a ")
→ UV cutoff scale $1/a$

Remove cutoff: $a \rightarrow 0$ ($L/a \rightarrow \infty$)

Hypercubic → Poincaré symmetries ✓

Numerical lattice field theory calculations

High-performance computing → evaluate up to \sim billion-dimensional integrals
(Dirac operator as $\sim 10^9 \times 10^9$ matrix)

Results to be shown, and work in progress, require state-of-the-art resources

Many thanks to national labs, USQCD–DOE, and computing centres!



Lassen @Livermore



USQCD @Fermilab



Barkla @Liverpool

Numerical lattice field theory calculations



Lassen @Livermore



USQCD @Fermilab



Barkla @Liverpool

Importance sampling Monte Carlo

Algorithms sample field configurations with probability $\frac{1}{\mathcal{Z}} e^{-S[\Phi]}$

$$\langle \mathcal{O} \rangle = \frac{1}{\mathcal{Z}} \int \mathcal{D}\Phi \ \mathcal{O}(\Phi) \ e^{-S[\Phi]} \rightarrow \frac{1}{N} \sum_{i=1}^N \mathcal{O}(\Phi_i) \text{ with stat. uncertainty } \propto \frac{1}{\sqrt{N}}$$

Numerical lattice field theory calculations

Importance sampling Monte Carlo

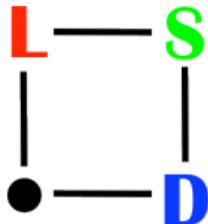
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Lattice calculation requires specific theory \longleftrightarrow lattice action $S[\Phi]$

Our strategy aims to gain generic insights into composite dark matter

Lattice Strong Dynamics Collaboration



- Argonne Xiao-Yong Jin, James Osborn
- Bern Andy Gasbarro
- Boston Venkitesh Ayyar, Rich Brower, Evan Owen, Claudio Rebbi
- Colorado Anna Hasenfratz, Ethan Neil, Curtis Peterson
- UC Davis Joseph Kiskis
- Livermore Dean Howarth, Pavlos Vranas
- Liverpool Chris Culver, DS
- Michigan Enrico Rinaldi
- Nvidia Evan Weinberg
- Oregon Graham Kribs
- Siegen Oliver Witzel
- Trieste James Ingoldby
- Yale Thomas Appelquist, Kimmy Cushman, George Fleming

Exploring the range of possible phenomena in strongly coupled field theories

Direct detection of composite dark matter

Charged constituents \rightarrow **form factors** \rightarrow experimental signals

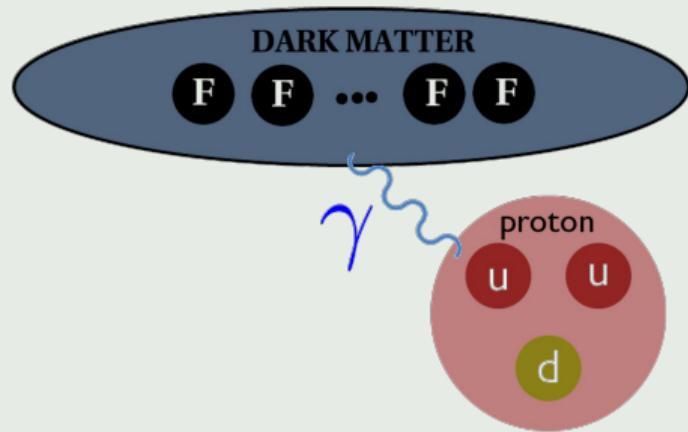
Photon exchange from electromagnetic form factors

Effective interactions suppressed by powers of dark matter mass

$$\text{Magnetic moment} \sim \frac{1}{M_{DM}}$$

$$\text{Charge radius} \sim \frac{1}{M_{DM}^2}$$

$$\text{Polarizability} \sim \frac{1}{M_{DM}^3}$$

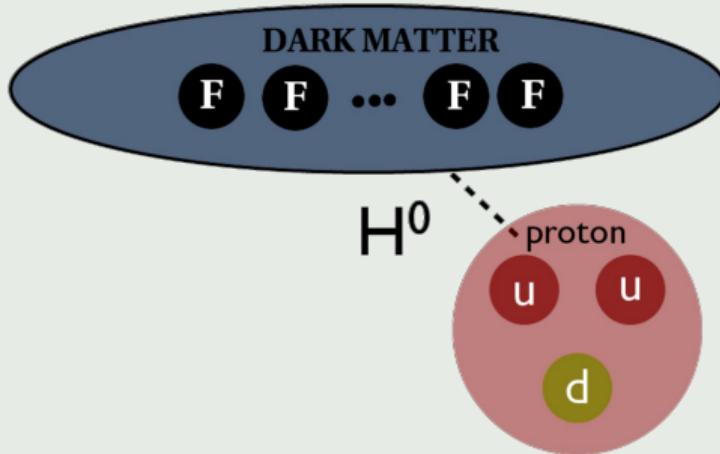


Direct detection of composite dark matter

Charged constituents \rightarrow **form factors** \rightarrow experimental signals

Higgs exchange from scalar form factor

Can dominate cross section... if F mass comes from Higgs

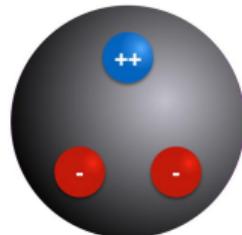


Direct detection of composite dark matter

Charged constituents → **form factors** → experimental signals

Simple first case: Dark matter as a “more-neutral neutron”

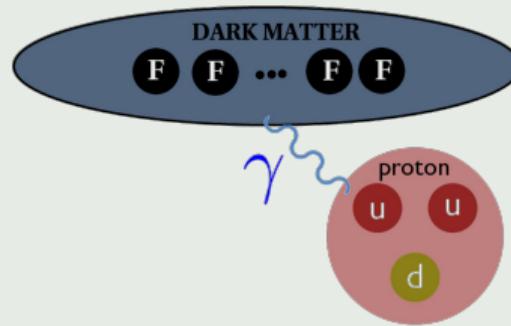
SU(3) with weak singlets → no Higgs-exchange interaction



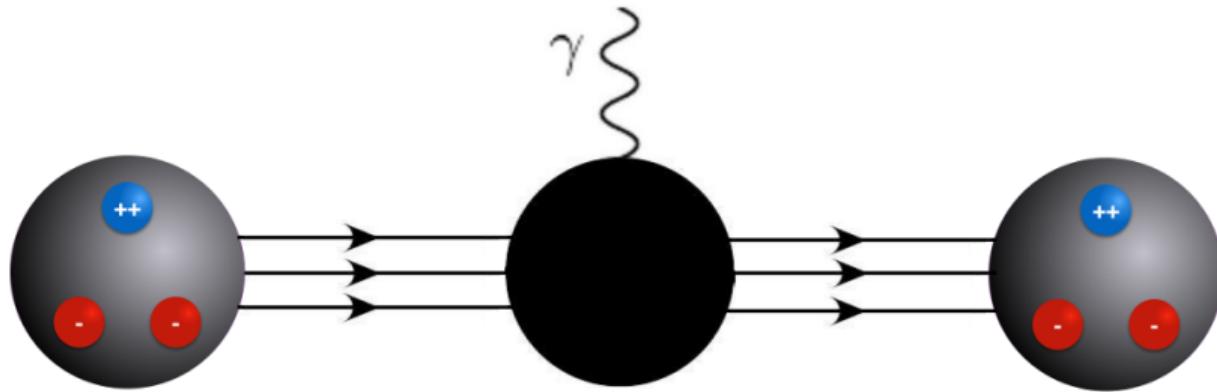
Investigate leading photon-exchange contributions

$$\text{Magnetic moment} \sim \frac{1}{M_{DM}}$$

$$\text{Charge radius} \sim \frac{1}{M_{DM}^2}$$



Magnetic moment and charge radius



$$\langle DM(p') | \Gamma_\mu(q^2) | DM(p) \rangle \sim F_1(q^2) \gamma_\mu + F_2(q^2) \frac{i\sigma_{\mu\nu}q^\nu}{2M_{DM}}, \quad q = p' - p$$

Electric charge: $F_1(0) = 0$

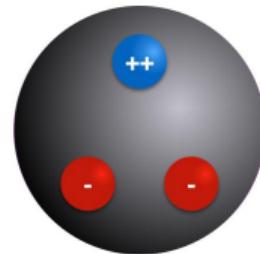
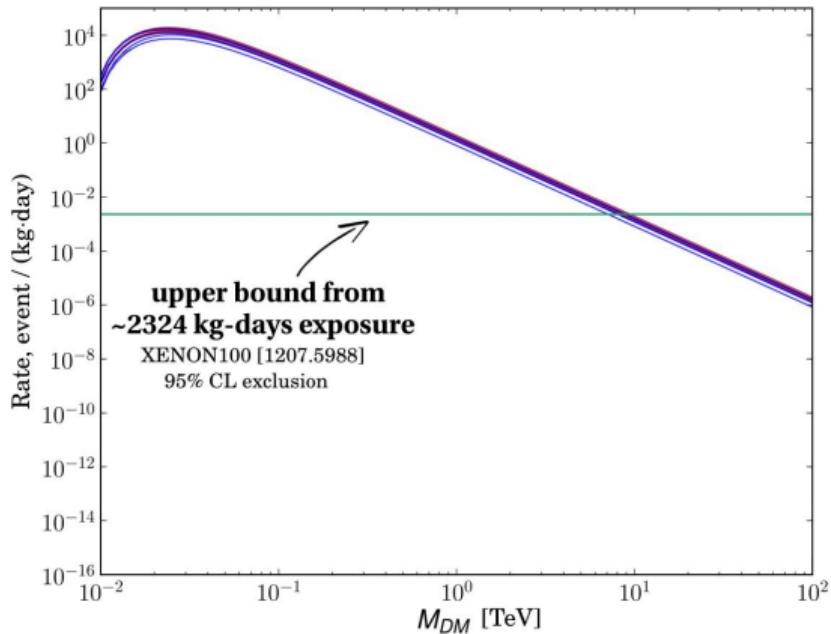
Magnetic moment: $F_2(0)$

Charge radius: $\langle r_E^2 \rangle = -6 \frac{dF_1(q^2)}{dq^2} \Big|_{q^2=0} + \frac{3F_2(0)}{2M_{DM}^2}$

Resulting direct detection constraints

Lattice calculations of magnetic moment and charge radius

→ event rate vs. dark matter mass



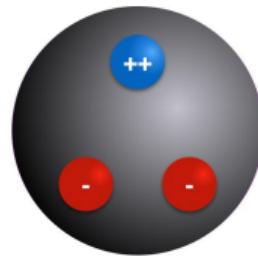
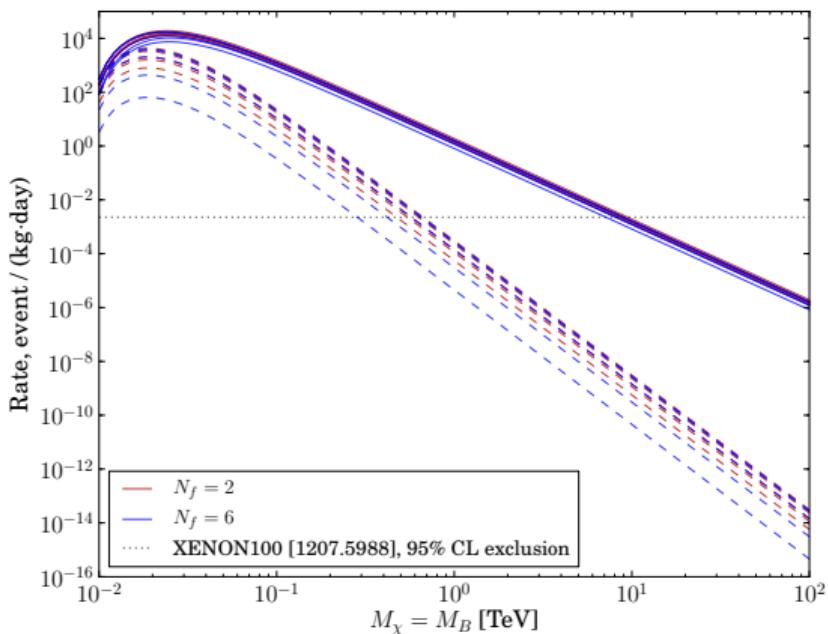
XENON100 → $M_B \gtrsim 10$ TeV

XENON1T → $M_B \gtrsim 30$ TeV [1805.12562]

Little effect from varying model params

Magnetic moment dominates event rate

Dashed charge radius contributions suppressed $\sim 1/M_{DM}^2$



Can change symmetries to forbid both magnetic moment and charge radius

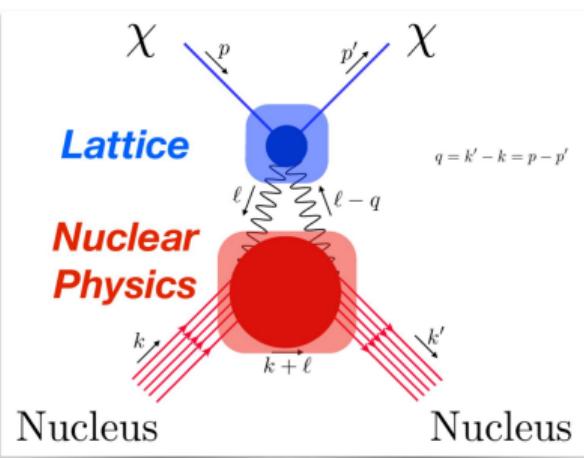
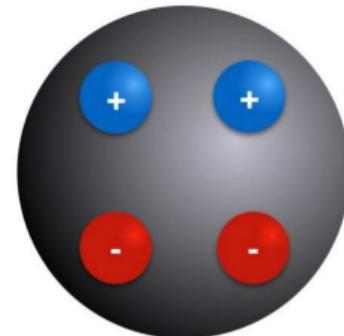
→ More interesting second case:
‘Stealth Dark Matter’

SU(4) Stealth Dark Matter

Fermions now include weak doublet & singlets

Scalar ‘baryon’ → no magnetic moment ✓

+/- charge symmetry → no charge radius ✓



(Tiny) Coupling to Higgs needed for nucleosynthesis

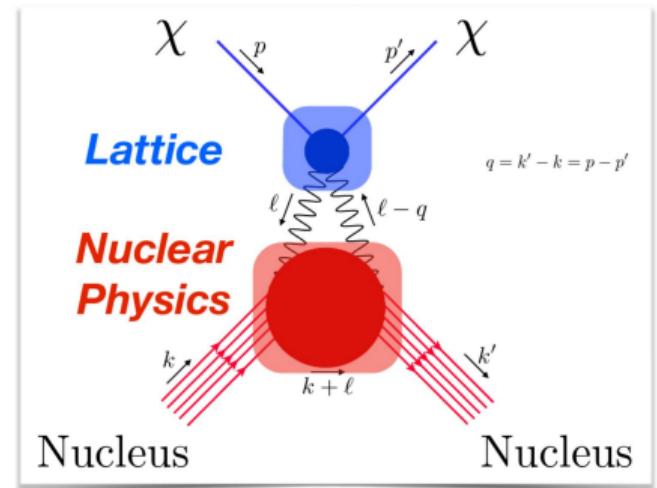
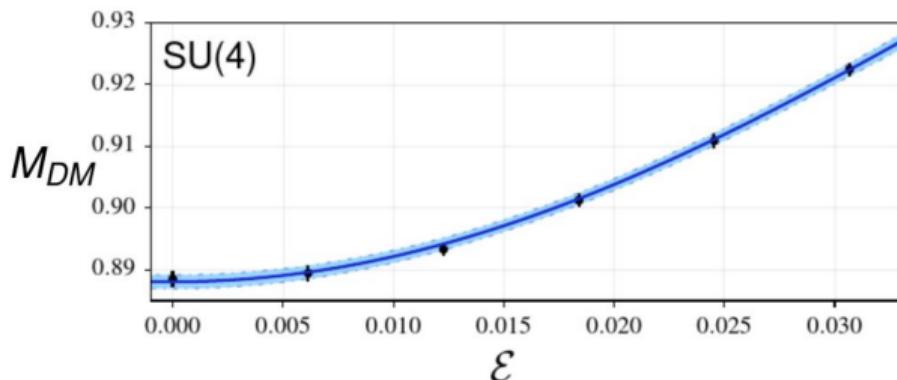
Polarizability $\sim 1/M_{DM}^3$ dominates direct detection

→ Unavoidable lower bound
on broad set of composite dark matter models

Polarizability of Stealth Dark Matter

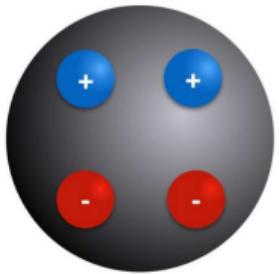
Unavoidable lower bound
on broad set of composite dark matter models

Nuclear physics very complicated
with large uncertainties



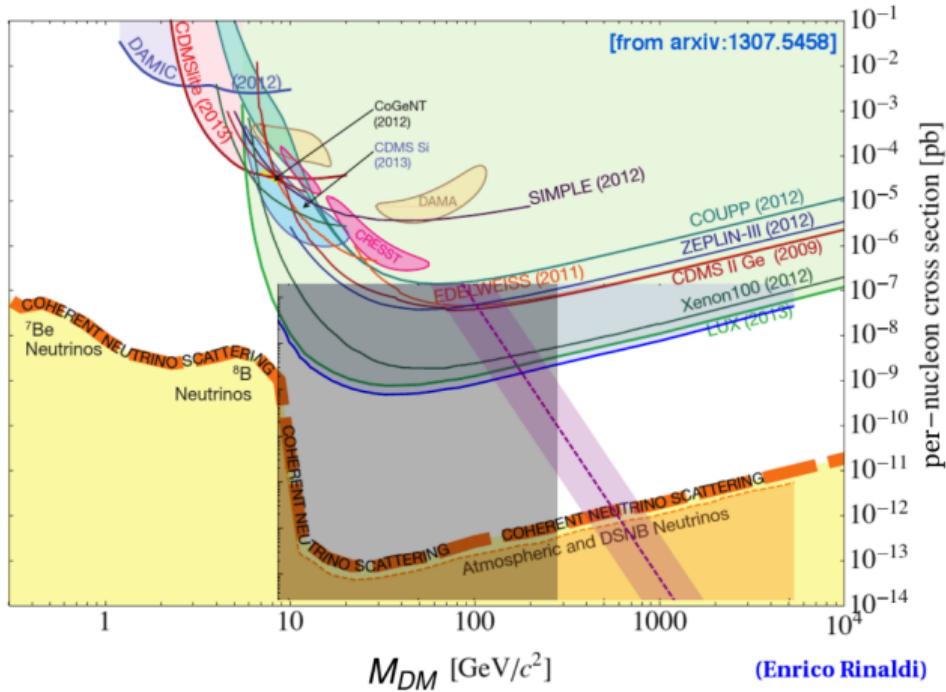
Polarizability is dependence
of lattice M_{DM} on external field \mathcal{E}

Lower bound on direct detection



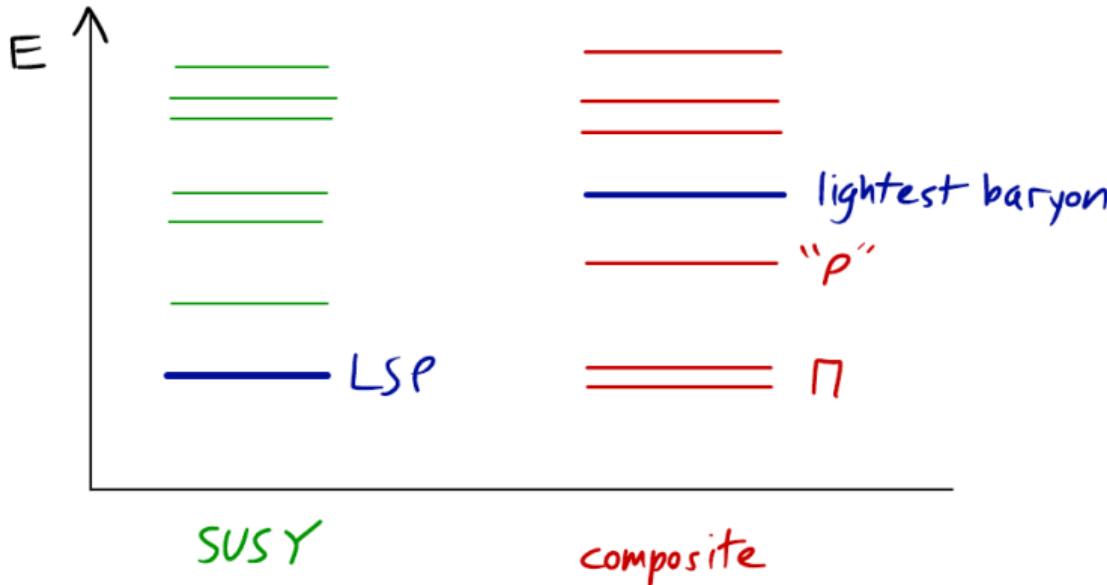
Results specific
to Xenon detectors

Uncertainty dominated
by Xenon nuclear physics



Shaded region is complementary constraint from particle colliders

Collider constraints



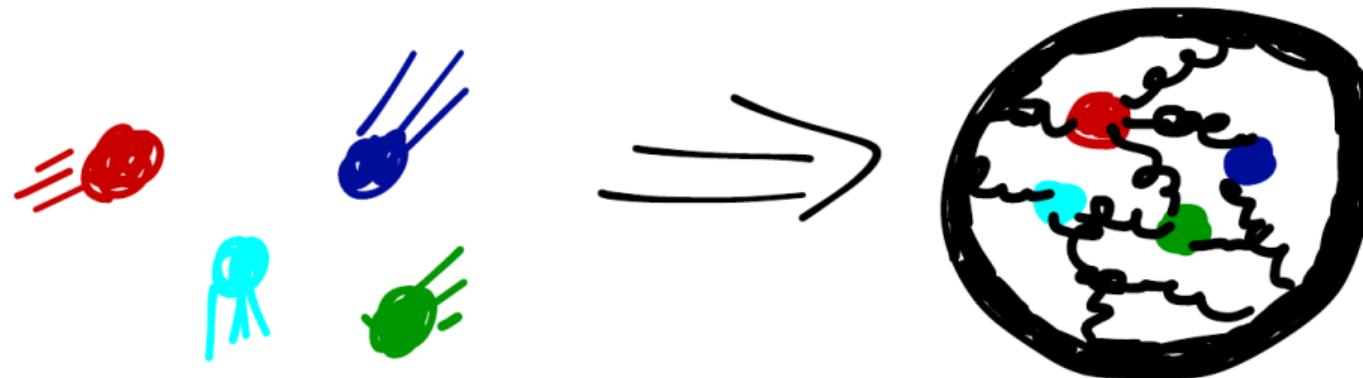
Dark baryon not lightest
composite particle

'Missing energy' searches
inefficient

Collider constraints from lighter **charged** ' Π ' plus lattice calculation of M_{DM}/M_Π

Gravitational waves

Gravitational-wave observatories opening new window on cosmology



First-order confinement transition \longrightarrow stochastic background of grav. waves
 \implies Lattice studies of Stealth Dark Matter phase transition

Stealth Dark Matter phase diagram

arXiv:2006.16429

Pure-gauge transition is first order

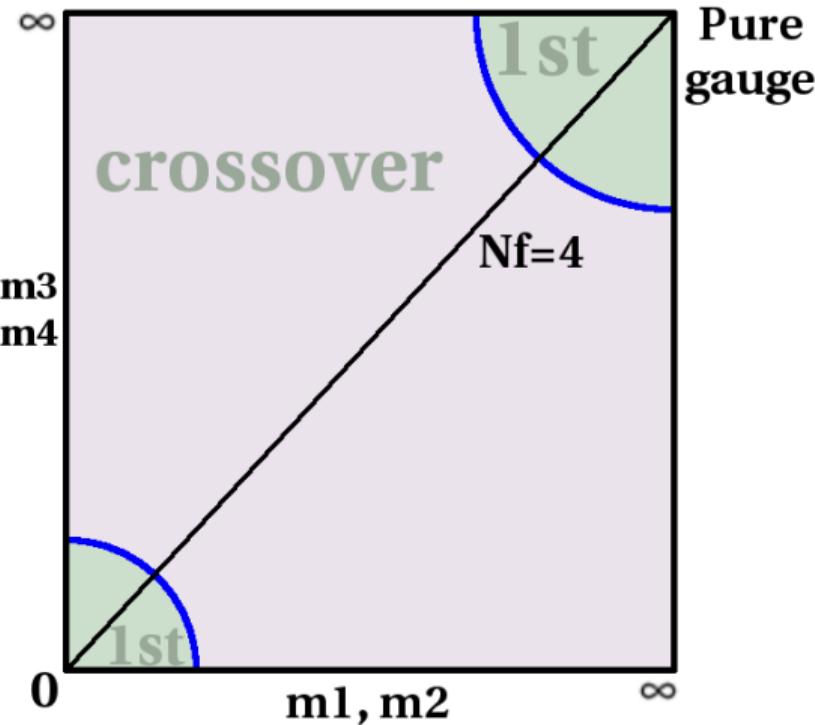
Becomes stronger as N increases

First-order transition persists
for sufficiently heavy fermions

$$\rightarrow M_P/M_V \gtrsim 0.9$$

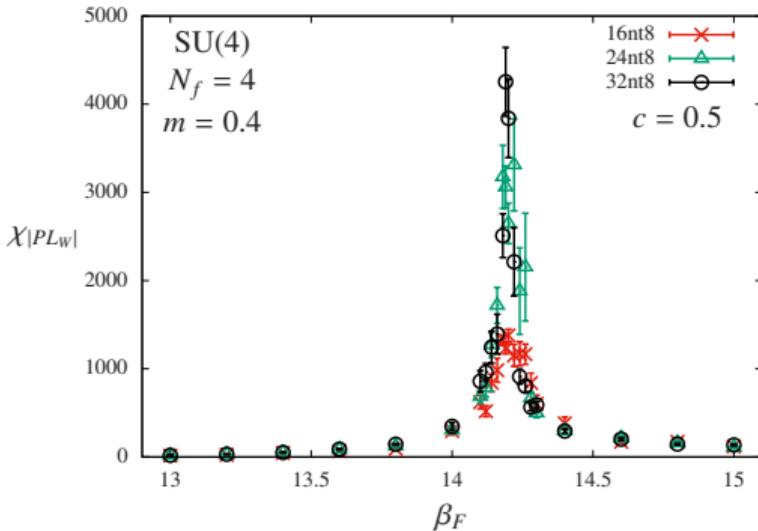
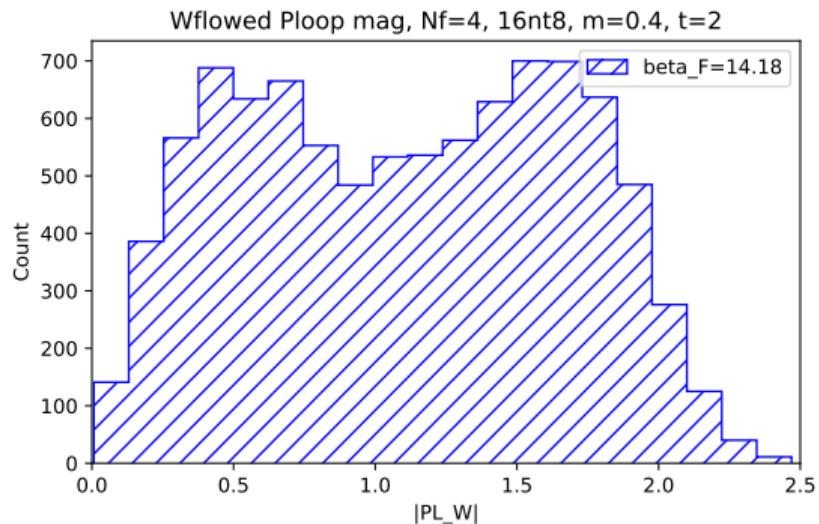
Form factor calculations considered

$$0.55 \leq M_P/M_V \leq 0.77$$



Determining order of thermal transition

arXiv:2006.16429



Left: Phase coexistence in Polyakov loop magnitude histogram

Right: Volume scaling of Polyakov loop susceptibility

From first-order transition to gravitational wave signal

First-order transition \rightarrow gravitational wave background will be produced

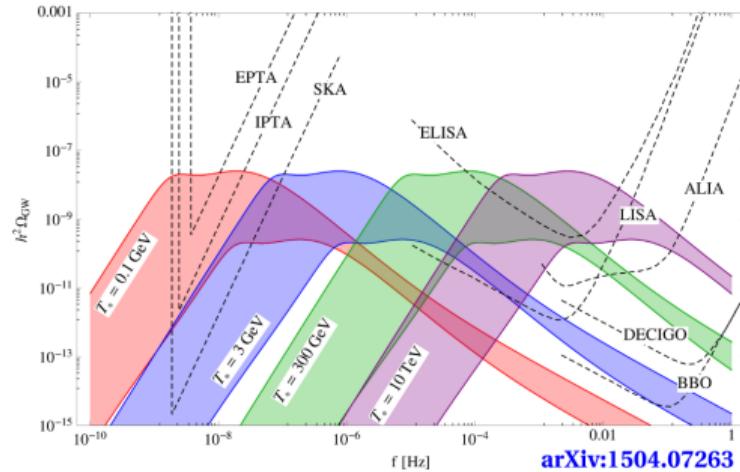
Four key parameters

Transition temperature $T_* \lesssim T_c$

Vacuum energy fraction from **latent heat**

Bubble nucleation rate (transition duration)

Bubble wall speed



Low frequencies require space-based observatories or pulsar timing arrays

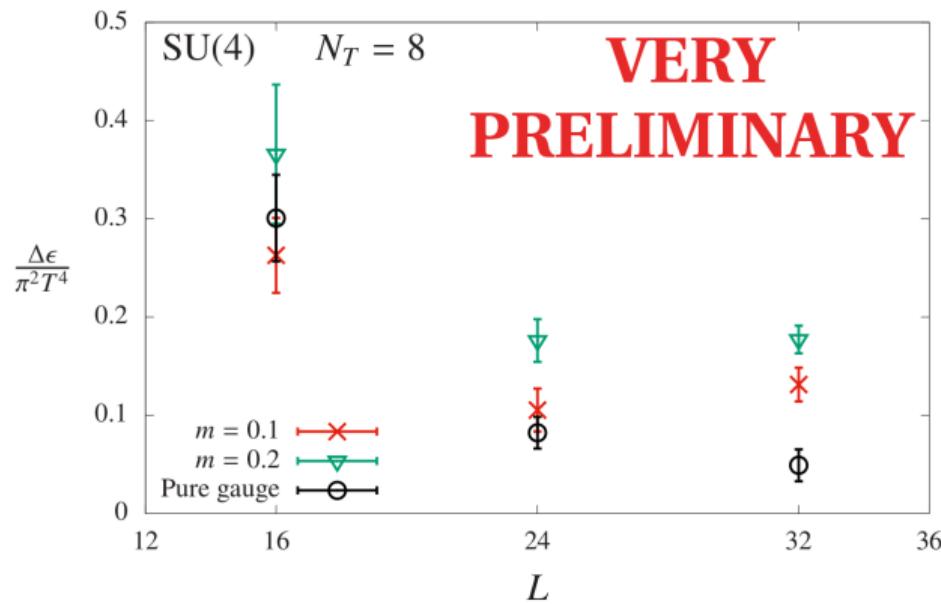
Work in progress: Latent heat $\Delta\epsilon$

First-order transition \rightarrow gravitational wave background will be produced

Vacuum energy fraction

$$\alpha \approx \frac{30}{4N(N^2 - 1)} \frac{\Delta\epsilon}{\pi^2 T_*^4}$$

Latent heat $\Delta\epsilon$
is change in energy density
at transition



Work in progress: Density of states

Markov-chain importance sampling can struggle at first-order transition:
difficult to tunnel between coexisting phases

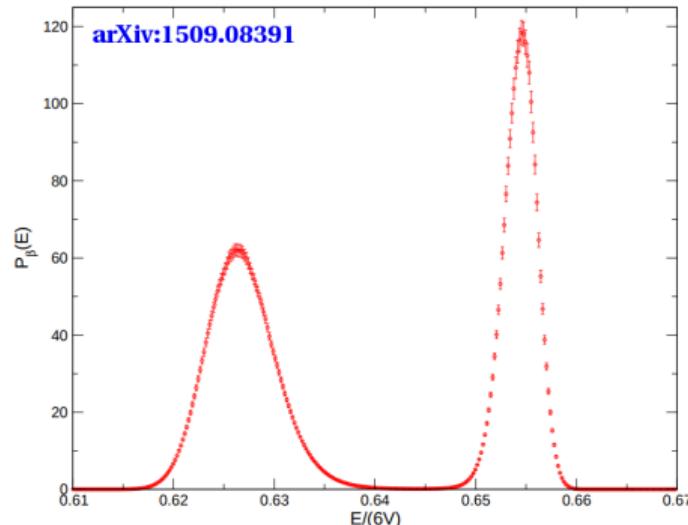
'LLR' generalization of Landau–Wang algorithm

→ continuous density of states $\rho(E)$ with exponential error suppression

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int \mathcal{D}\Phi \ \mathcal{O}(\Phi) \ e^{-S[\Phi]}$$
$$\rightarrow \frac{1}{Z} \int dE \ \mathcal{O}(E) \ \rho(E) \ e^{-E}$$

Work by Felix Springer

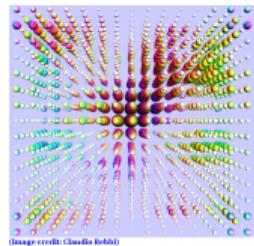
SU(4) code developed, analyses underway



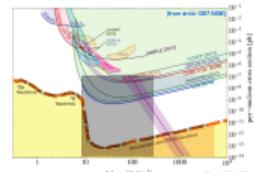
Recapitulation and outlook

Composite dark matter is an attractive possibility

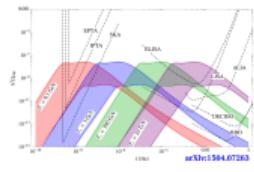
Lattice field theory is needed
to test models against experimental results



Form factors for direct detection
→ Stealth Dark Matter setting lower bound



First-order early-universe transition
→ gravitational waves depending on latent heat etc.



And more: Collider experiments; galactic sub-structure;
indirect detection; relic abundance; ...

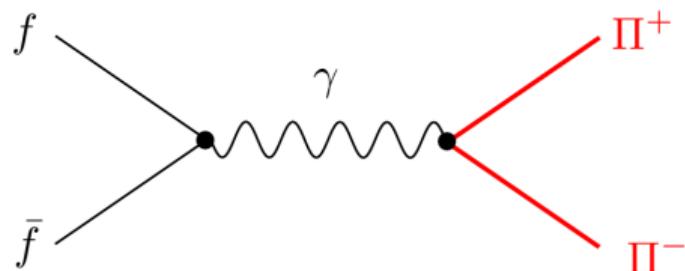
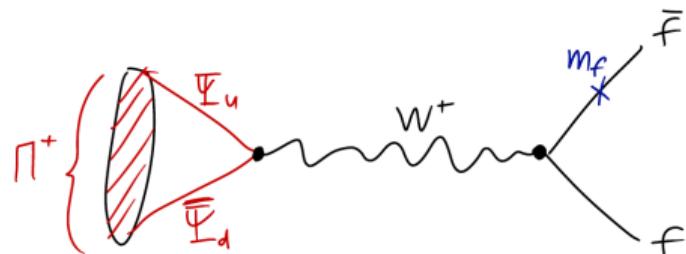
Thank you!

Lattice Strong Dynamics Collaboration & Felix Springer

Funding and computing resources

UK Research
and Innovation



Production**Decay**

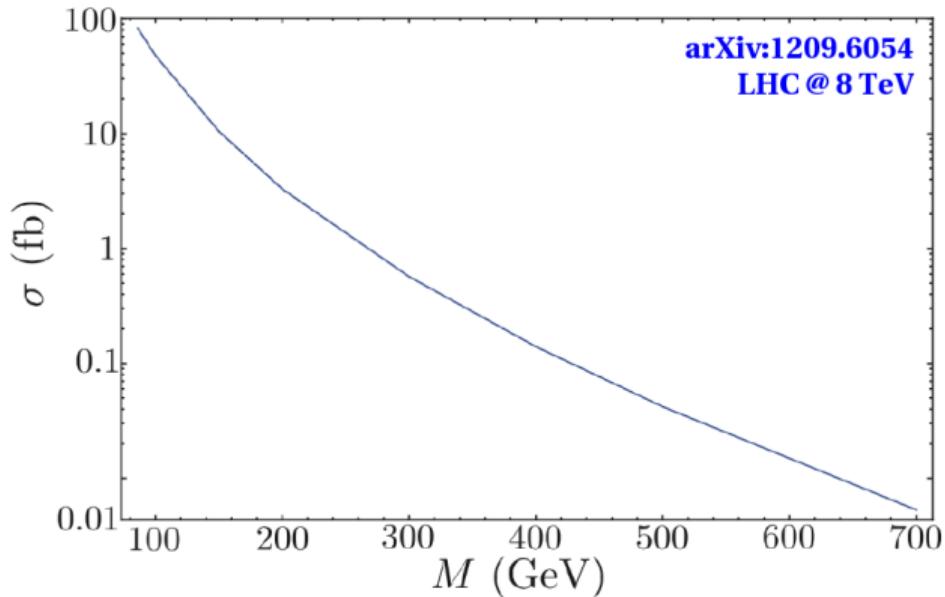
“Particularly tricky” at the LHC

Published bounds $M_\Pi \gtrsim 130$ GeV similar to $M_\Pi \gtrsim 100$ GeV from LEP

[ATLAS-CONF-2020-051 reports $M_\Pi \gtrsim 340$ GeV for lifetimes ~ 0.1 ns]

More form factors to compute: $F_1(4M_\Pi^2)$ for Π and decay constant F_Π

Form factors for collider searches



Π pair production cross section
Integrate over proton parton dist.,
here setting $F_1(4M_\Pi^2) = 1$

For $M_\Pi \gtrsim 200$ GeV, LHC can search for $\Pi^+\Pi^- \rightarrow t\bar{b} + \bar{t}b$

in addition to $\tau^+\tau^- + E_T$

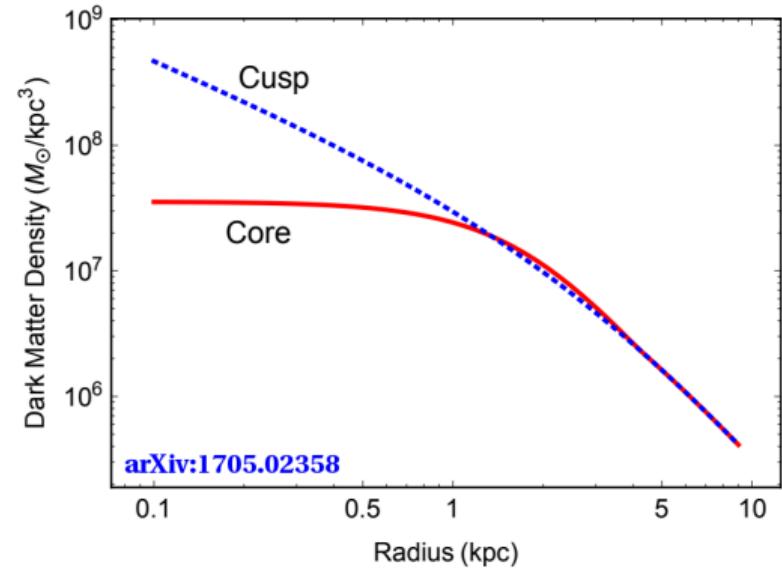
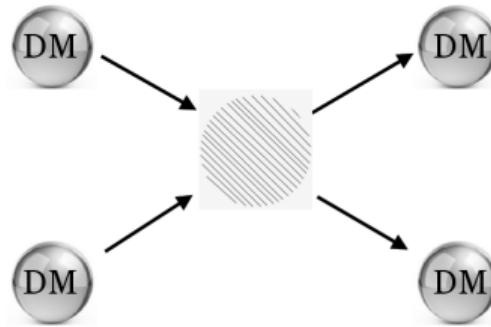
Supplement: Self-interactions and ‘small-scale’ structure

Astrophysical observations vs. collisionless dark matter

Persistent discrepancies on galactic scales

[“core vs. cusp”; “too big to fail”; “missing satellites”; “diversity” — Review: [arXiv:1705.02358](https://arxiv.org/abs/1705.02358)]

Can be addressed by
dark matter self-interactions

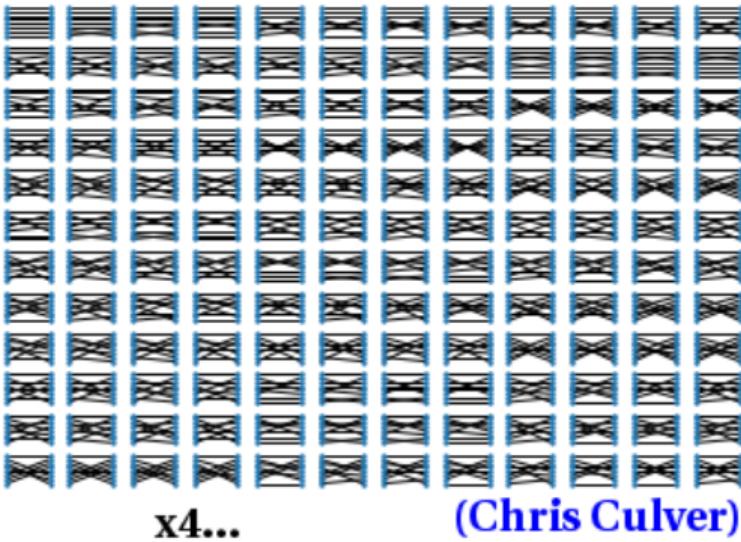
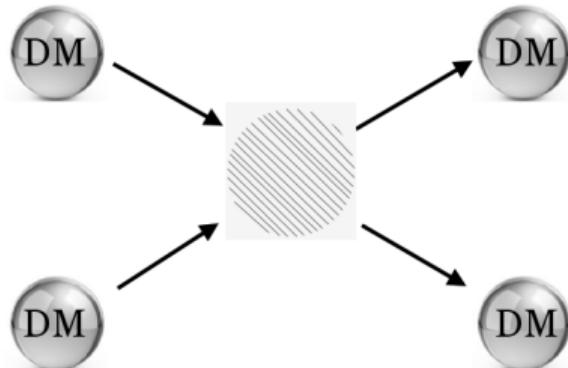


Baryon–baryon scattering work in progress

$2 \times 4\text{fermions} \times \text{SU}(4)$ gauge group \longrightarrow proliferation of contractions

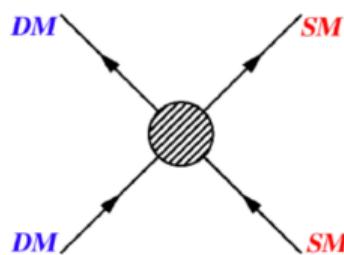
[comparable to QCD triton or He nucleus]

Work in progress to apply state-of-the-art stochastic LapH methods



Backup: Thermal freeze-out for relic density

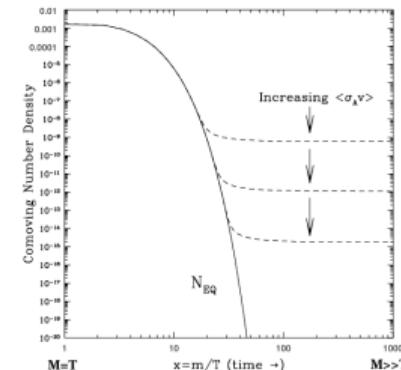
Requires non-gravitational interactions with known particles



$\text{DM} \longleftrightarrow \text{SM}$ for $T \gtrsim M_{DM}$

$\text{DM} \rightarrow \text{SM}$ for $T \lesssim M_{DM}$
 \Rightarrow rapid depletion of Ω_{DM}

Hubble expansion
 \Rightarrow dilution \rightarrow freeze-out



$2 \rightarrow 2$ scattering relates coupling and mass, $200\alpha \sim \frac{M_{DM}}{100 \text{ GeV}}$

Strong $\alpha \sim 16 \rightarrow$ ‘natural’ mass scale $M_{DM} \sim 300 \text{ TeV}$

Smaller $M_{DM} \gtrsim 1 \text{ TeV}$ possible from $2 \rightarrow n$ scattering or asymmetry

Backup: Two roads to natural asymmetric dark matter

Idea: Dark matter relic density related to baryon asymmetry

$$\begin{aligned}\Omega_D &\approx 5\Omega_B \\ \implies M_D n_D &\approx 5M_B n_B\end{aligned}$$

$$n_D \sim n_B \implies M_D \sim 5M_B \approx 5 \text{ GeV}$$

High-dim. interactions relate baryon# and DM# violation

$$M_D \gg M_B \implies n_B \gg n_D \sim \exp[-M_D/T_s] \quad T_s \sim 200 \text{ GeV}$$

Electroweak sphaleron processes above T_s distribute asymmetries

Both require non-gravitational interactions with known particles

Backup: More details about form factors

Photon exchange via electromagnetic form factors

Interactions suppressed by powers of confinement scale $\Lambda \sim M_{DM}$

Dimension 5: Magnetic moment $\rightarrow (\bar{X} \sigma_{\mu\nu} X) F^{\mu\nu}/\Lambda$

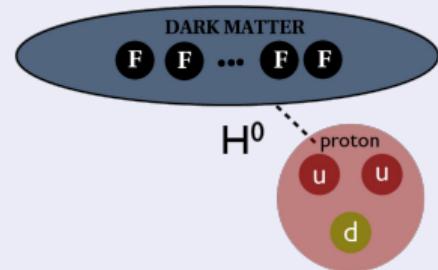
Dimension 6: Charge radius $\rightarrow (\bar{X} X) v_\mu \partial_\nu F^{\mu\nu}/\Lambda^2$

Dimension 7: Polarizability $\rightarrow (\bar{X} X) v_\mu v_\nu F^{\mu\alpha} F_\alpha^\nu/\Lambda^3$

Higgs exchange via scalar form factors

Higgs couples through σ terms $\langle B | m_\psi \bar{\psi} \psi | B \rangle$

Produces rapid charged ' Π ' decay
needed for Big Bang nucleosynthesis

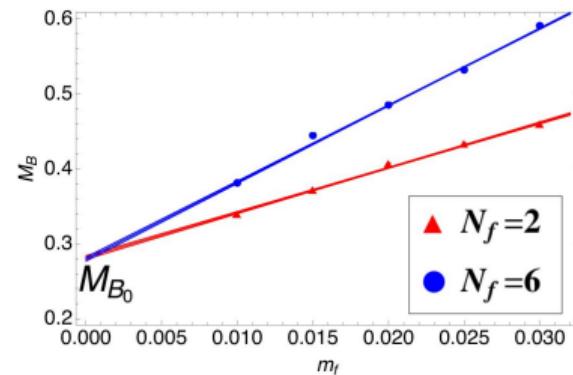


Backup: More details about SU(3) composite dark matter model

Same SU(3) gauge group as QCD

Re-analyze existing data sets:

$32^3 \times 64$ lattices, domain wall fermions



Scan relatively heavy fermion masses $m_F \rightarrow 0.55 \lesssim M_\Pi/M_V \lesssim 0.75$

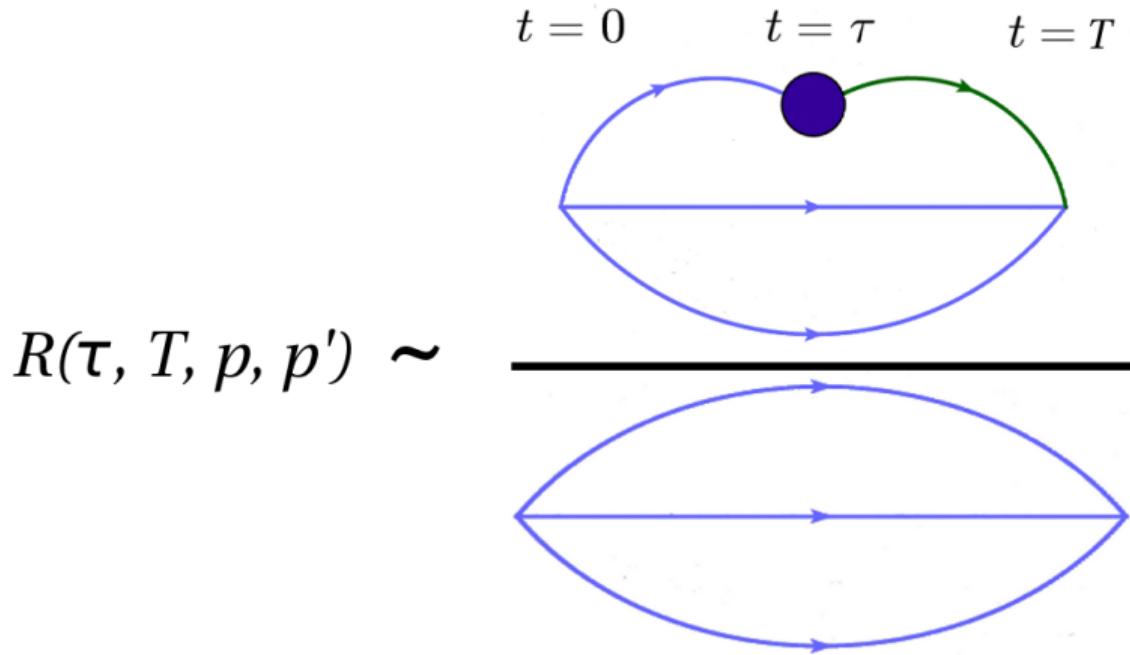
Compare $N_F = 2$ or 6 degenerate flavors with same $M_{B_0} \equiv \lim_{m_F \rightarrow 0} M_B$

Unlike QCD, fermions are all $SU(2)_L$ singlets $\rightarrow Q = Y$

Setting $Q_P = 2/3$ and $Q_M = -1/3$,

dark matter candidate is singlet “dark baryon” $B = PMM$

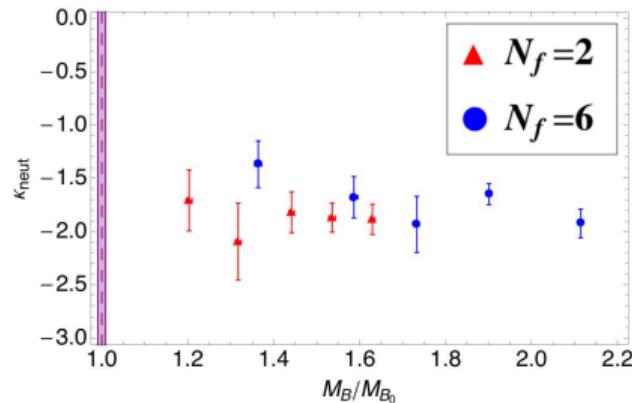
Backup: Form factor calculations on the lattice



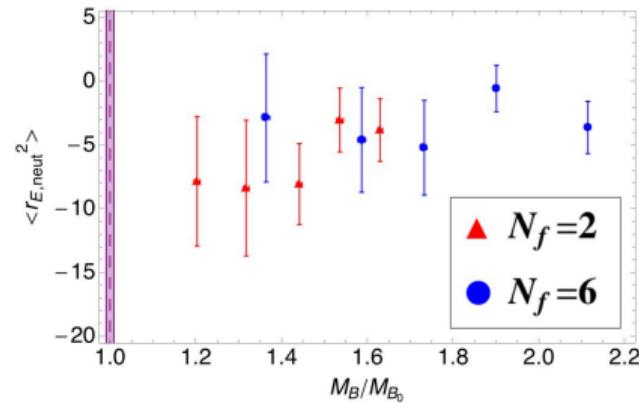
$$R_\Gamma(\tau, T, p, p') \longrightarrow \langle DM(p') | \Gamma_\mu(q^2) | DM(p) \rangle + \mathcal{O}(e^{-\Delta\tau}, e^{-\Delta T}, e^{-\Delta(T-\tau)})$$

Backup: Electromagnetic form factor results

Magnetic moment κ



Charge radius $\langle r^2 \rangle$



Little dependence on N_F or on $m_F \sim M_B/M_{B_0}$

κ comparable to neutron's $\kappa_N = -1.91$

$\langle r^2 \rangle$ smaller than neutron's $\langle r^2 \rangle_N \approx -38$ (related to larger M_Π/M_V)

Insert into standard event rate formulas...

Backup: Event rate formulas and lattice input

$$\text{Rate} = \frac{M_{\text{detector}}}{M_T} \frac{\rho_{DM}}{M_{DM}} \int_{E_{\min}}^{E_{\max}} dE_R \, \mathcal{A}cc(E_R) \left\langle v_{DM} \frac{d\sigma}{dE_R} \right\rangle_f$$

$$\frac{d\sigma}{dE_R} = \frac{|\mathcal{M}_{SI}|^2 + |\mathcal{M}_{SD}|^2}{16\pi (M_{DM} + M_T)^2 E_R^{\max}} \quad E_R^{\max} = \frac{2M_{DM}^2 M_T v_{col}^2}{(M_{DM} + M_T)^2}$$

From **magnetic moment** κ and **charge radius** $\langle r^2 \rangle$

$$\frac{|\mathcal{M}_{SI}|^2}{e^4 [ZF_c(Q)]^2} = \left(\frac{M_T}{M_{DM}} \right)^2 \left[\frac{4}{9} M_{DM}^4 \langle r^2 \rangle^2 + \frac{\kappa^2 (M_T + M_{DM})^2 (E_R^{\max} - E_R)}{M_T^2 E_R} \right]$$

$$|\mathcal{M}_{SD}|^2 = e^4 \frac{2}{3} \left(\frac{J+1}{J} \right) \left[\left(A \frac{\mu_T}{\mu_n} \right) F_s(Q) \right]^2 \kappa^2$$

Backup: Event rate formulas and lattice input

$$\text{Rate} = \frac{M_{\text{detector}}}{M_T} \frac{\rho_{DM}}{M_{DM}} \int_{E_{\min}}^{E_{\max}} dE_R \text{Acc}(E_R) \left\langle v_{DM} \frac{d\sigma}{dE_R} \right\rangle_f$$

$$\frac{d\sigma}{dE_R} = \frac{\overline{|\mathcal{M}_{SI}|^2} + \overline{|\mathcal{M}_{SD}|^2}}{16\pi (M_{DM} + M_T)^2 E_R^{\max}}$$

$$E_R^{\max} = \frac{2M_{DM}^2 M_T v_{col}^2}{(M_{DM} + M_T)^2}$$

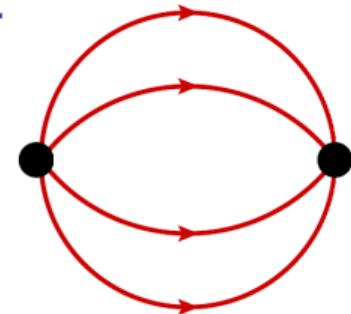
From **polarizability** C_F

$$\sigma_{SI} = \frac{Z^4}{A^2} \frac{144\pi\alpha_{em}^4 \tilde{M}_{n,DM}^2}{M_{DM}^6 R^2} C_F^2 \propto \frac{Z^4}{A^2} \quad \text{per nucleon}$$

Backup: More details about SU(4) Stealth Dark Matter

Quenched SU(4) lattice ensembles

Lattice volumes up to $64^3 \times 128$,
several lattice spacings to check systematic effects



Flavor combinations

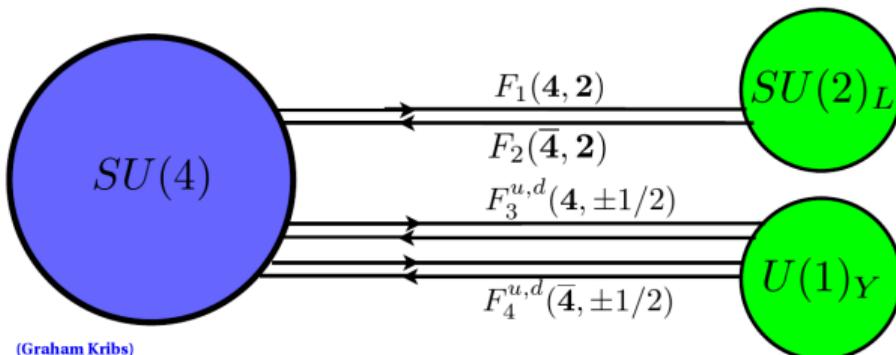
$$\square \otimes \square \otimes \square \otimes \square = \begin{array}{c} \square \\ \square \\ \square \\ \square \end{array} \oplus \begin{array}{c} \square & \square \\ \square & \square \\ \square & \end{array} \oplus \begin{array}{c} \square & \square \\ \square & \square \\ \square & \end{array} \oplus \begin{array}{c} \square & \square & \square \\ \square & \square & \end{array} \oplus \begin{array}{c} \square & \square & \square & \square \end{array}$$

S=0 **S=1** **S=2**

Dark matter candidate is spin-zero baryon \rightarrow no magnetic moment

Need at least two flavors to anti-symmetrize \rightarrow no charge radius

Backup: Even more details about SU(4) Stealth Dark Matter



Field	$SU(N_D)$	$(SU(2)_L, Y)$	Q
$F_1 = \begin{pmatrix} F_1^u \\ F_1^d \end{pmatrix}$	\mathbf{N}	$(\mathbf{2}, 0)$	$(+1/2, -1/2)$
$F_2 = \begin{pmatrix} F_2^u \\ F_2^d \end{pmatrix}$	$\bar{\mathbf{N}}$	$(\mathbf{2}, 0)$	$(+1/2, -1/2)$
F_3^u	\mathbf{N}	$(\mathbf{1}, +1/2)$	$+1/2$
F_3^d	\mathbf{N}	$(\mathbf{1}, -1/2)$	$-1/2$
F_4^u	$\bar{\mathbf{N}}$	$(\mathbf{1}, +1/2)$	$+1/2$
F_4^d	$\bar{\mathbf{N}}$	$(\mathbf{1}, -1/2)$	$-1/2$

Mass terms $m_V (F_1 F_2 + F_3 F_4) + y (F_1 \cdot H F_4 + F_2 \cdot H^\dagger F_3) + \text{h.c.}$

Vector-like masses evade Higgs-exchange direct detection bounds

Higgs couplings \rightarrow charged meson decay before Big Bang nucleosynthesis
Both required \rightarrow four flavors

Backup: ‘Stealth’ composites from conspicuous constituents

Direct detection cross section (pb)

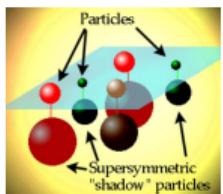


Neutrino
 $\sigma \sim 10^{-2}$

Radar cross section (m^2)



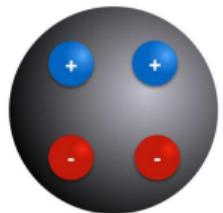
747
 $\sigma \sim 10^2$



SUSY neutralino
 $10^{-6} \lesssim \sigma \lesssim 10^{-5}$



Falcon
 $\sigma \sim 10^{-2}$



Stealth Dark Matter
 $\sigma \sim \left(\frac{200 \text{ GeV}}{M_{DM}}\right)^6 \times 10^{-9}$



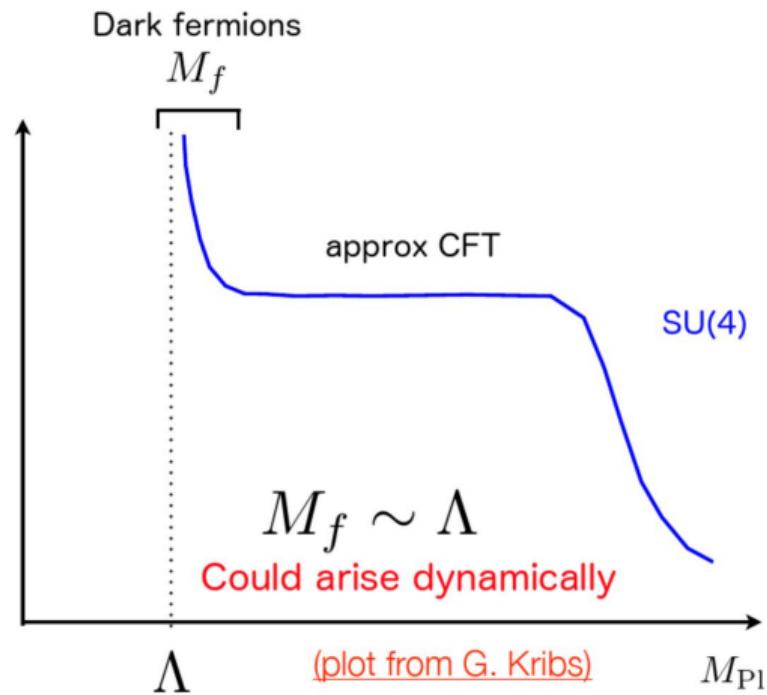
Stealth F-22
 $\sigma < 10^{-3}$

Backup: Stealth Dark Matter mass scales

Lattice studies focus on $m_\psi \simeq \Lambda_{DM}$ where effective theories least reliable

$m_\psi \simeq \Lambda_{DM}$ could arise dynamically

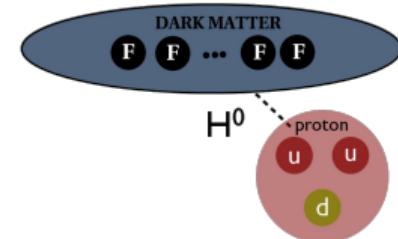
Collider constraints on M_{DM}
become stronger as m_ψ decreases



Backup: Effective Higgs interaction

$M_H = 125 \text{ GeV} \longrightarrow$ Higgs exchange can dominate direct detection

$$\sigma_H^{(SI)} \propto \left| \frac{\tilde{M}_{DM,N}}{M_H^2} \ y_\psi \langle DM | \bar{\psi}\psi | DM \rangle \ y_q \langle N | \bar{q}q | N \rangle \right|^2$$



Quark $y_q = \frac{m_q}{v}$

Dark $y_\psi = \alpha \frac{m_\psi}{v}$ suppressed by $\alpha \equiv \left. \frac{v}{m_\psi} \frac{\partial m_\psi(h)}{\partial h} \right|_{h=v} = \frac{yv}{yv + m_\psi}$

Determine using Feynman–Hellmann theorem $\langle DM | \bar{\psi}\psi | DM \rangle = \frac{\partial M_{DM}}{\partial m_\psi}$

Backup: Feynman–Hellmann theorem

$m_\psi \bar{\psi} \psi$ is the only term in the hamiltonian that depends on m_ψ

$$\implies \left\langle B \left| \frac{\partial \hat{H}}{\partial m_\psi} \right| B \right\rangle = \langle B | \bar{\psi} \psi | B \rangle$$

Since $\hat{H} |B\rangle = M_B |B\rangle$ and $\langle B| \hat{H} = \langle B| M_B$ we have

$$\begin{aligned} \frac{\partial}{\partial m_\psi} M_B &= \frac{\partial}{\partial m_\psi} \left\langle B \left| \hat{H} \right| B \right\rangle = \left\langle \frac{\partial B}{\partial m_\psi} \left| \hat{H} \right| B \right\rangle + \left\langle B \left| \hat{H} \right| \frac{\partial B}{\partial m_\psi} \right\rangle + \left\langle B \left| \frac{\partial \hat{H}}{\partial m_\psi} \right| B \right\rangle \\ &= M_B \langle \frac{\partial B}{\partial m_\psi} | B \rangle + M_B \langle B | \frac{\partial B}{\partial m_\psi} \rangle + \langle B | \bar{\psi} \psi | B \rangle \\ &= M_B \frac{\partial}{\partial m_\psi} \langle B | B \rangle + \langle B | \bar{\psi} \psi | B \rangle = \langle B | \bar{\psi} \psi | B \rangle \quad \square \end{aligned}$$

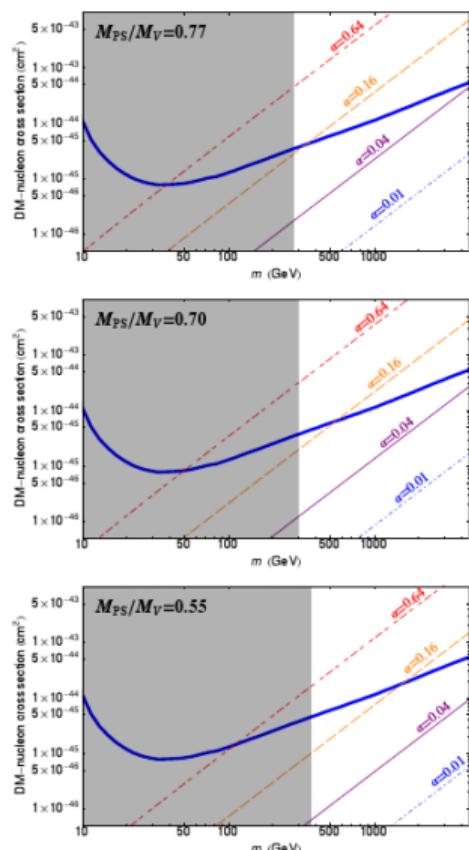
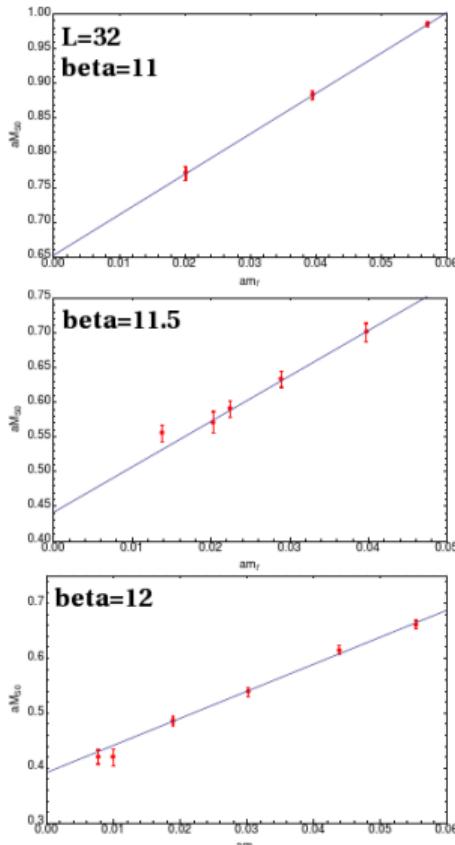
Backup: Lattice results for Higgs exchange constrain α

$$\sigma_H^{(SI)} \propto |y_\psi \langle DM |\bar{\psi}\psi| DM \rangle|^2$$

Matrix element $\propto \frac{\partial M_{DM}}{\partial m_\psi}$
 (Feynman–Hellmann)

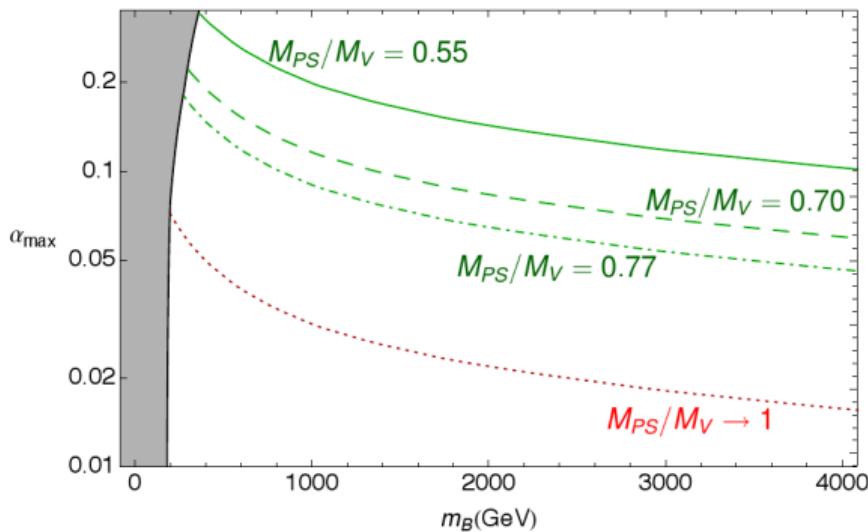
Stealth Dark Matter:
 $0.15 \lesssim \frac{m_\psi}{M_{DM}} \frac{\partial M_{DM}}{\partial m_\psi} \lesssim 0.34$

Larger than QCD
 $0.04 \lesssim \frac{m_q}{M_N} \frac{\partial M_N}{\partial m_q} \lesssim 0.08$



Backup: Bounds on effective Higgs coupling

Higgs-exchange cross section \rightarrow maximum α allowed by LUX [1310.8214]



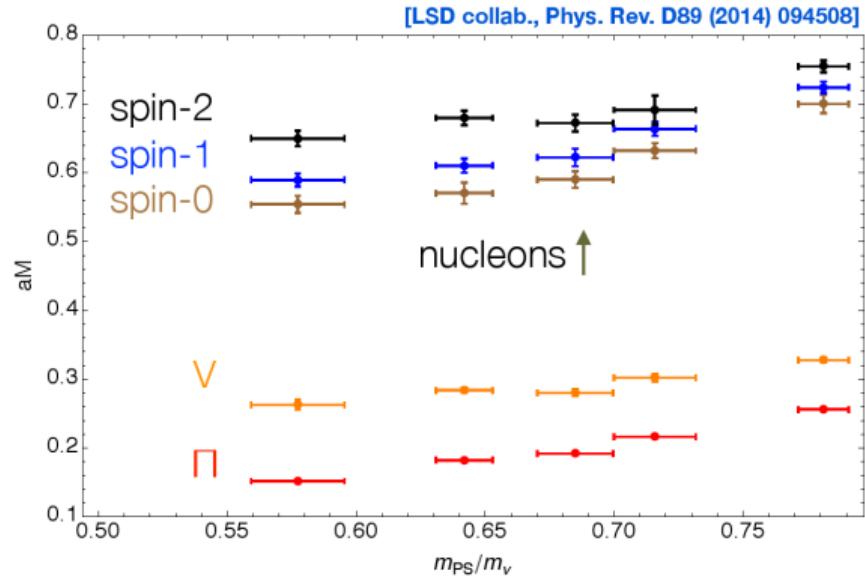
Maximum α depends on M_{Π}/M_V
and dark matter mass

Smaller $M_{\Pi}/M_V \longleftrightarrow m_F$
 \rightarrow stronger constraints from colliders

Effective Higgs interaction tightly constrained

$\alpha \lesssim 0.3$ for $M_{\Pi}/M_V \gtrsim 0.55 \rightarrow$ fermion masses must be mainly vector-like

Backup: Indirect detection



Lattice results for composite spectrum
Predict γ -rays from splitting between
baryons with spin $S = 0, 1$ and 2

Much more challenging future work

DM–DM annihilation into (many) lighter Δ that then decay

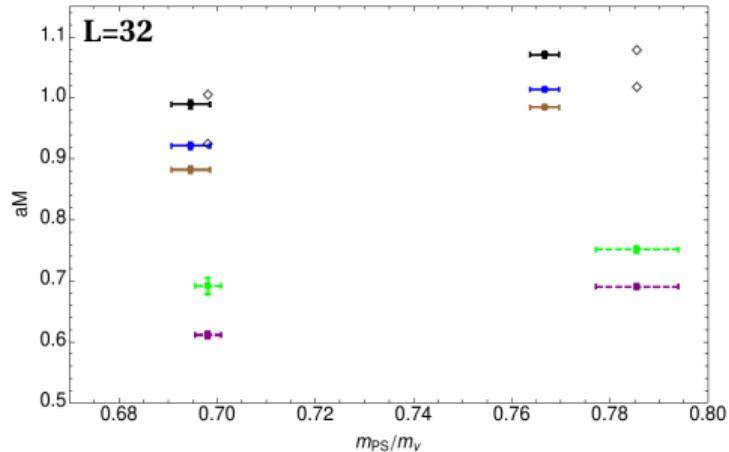
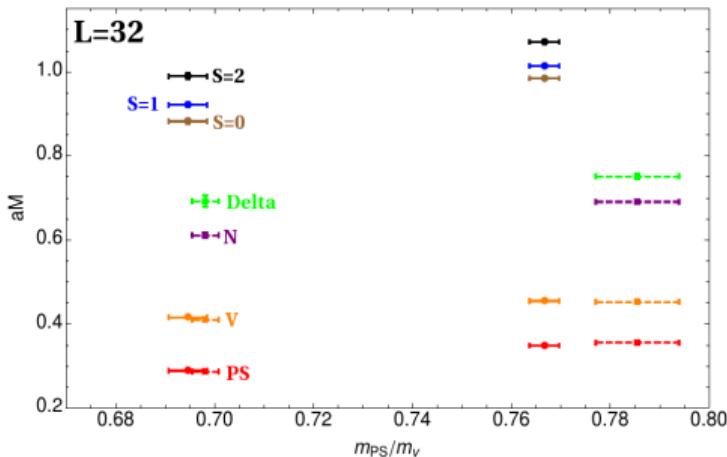
Backup: Large- N predictions for SU(4) baryons

Tune (β, m_F) to match SU(3) M_Π and M_V (dashed)

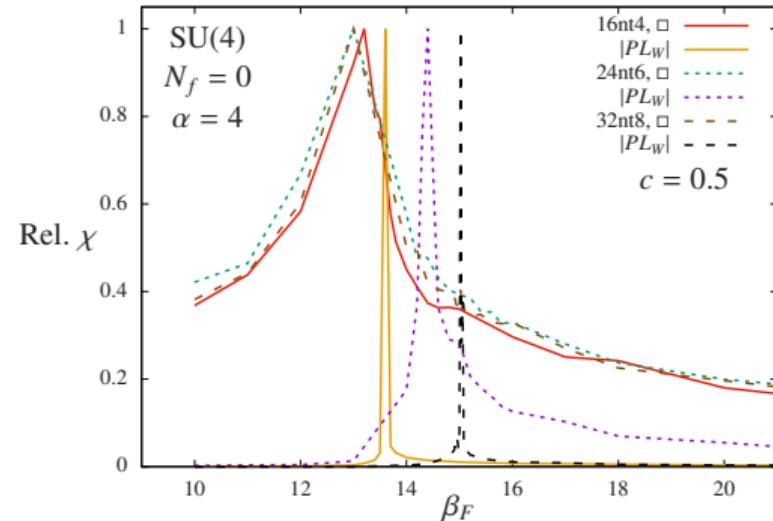
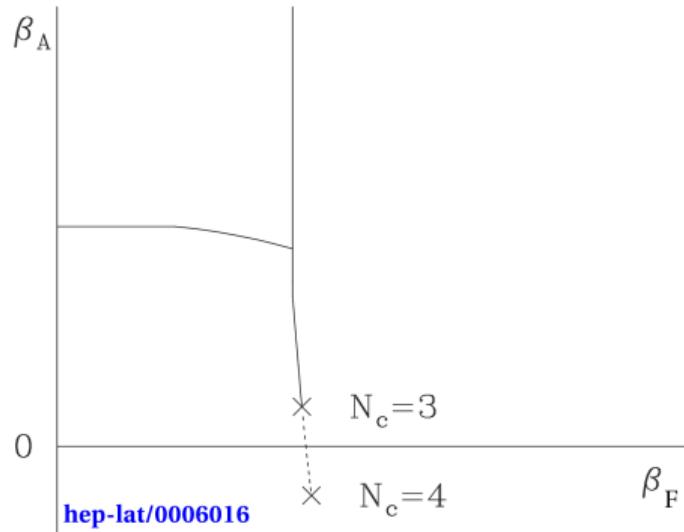
Rotor spectrum for spin- J baryons: $M(N, J) = NM_0 + C + B \frac{J(J+1)}{N} + \mathcal{O}\left(\frac{1}{N^2}\right)$

Fit M_0 , C and B with nucleon, Δ and spin-0 baryon masses

→ predictions for $S = 1, 2$ baryons (diamonds)



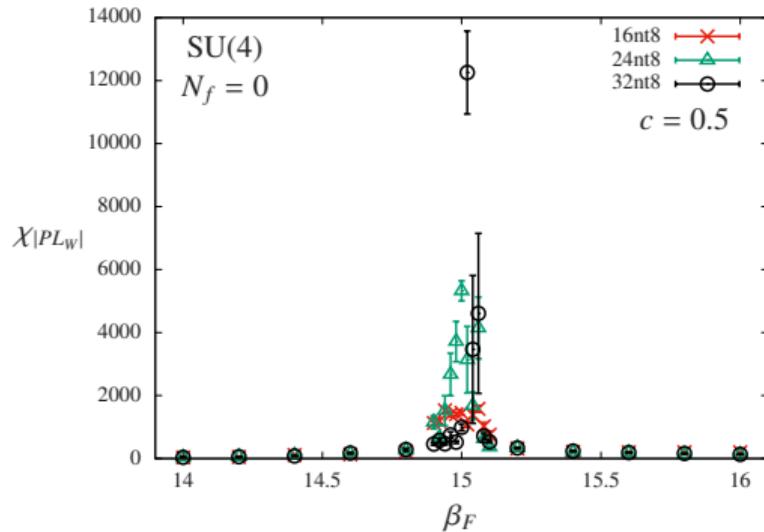
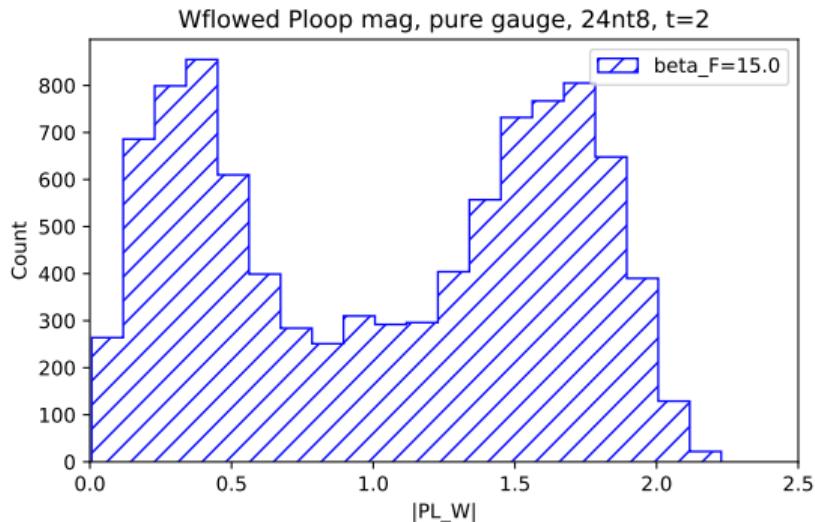
Backup: Thermal transition vs. bulk transition



Try to avoid bulk transition for small $L^3 \times N_T$ volumes \rightarrow use $\beta_A = -\beta_F/4$

Still need $N_T > 4$ for clear separation between bulk & thermal transitions

Backup: Compare with known first-order pure-gauge transition



Signals are stronger but qualitatively same as for $M_P/M_V \approx 0.96$

No clear hysteresis even in pure-gauge case