Lattice field theory for composite dark matter

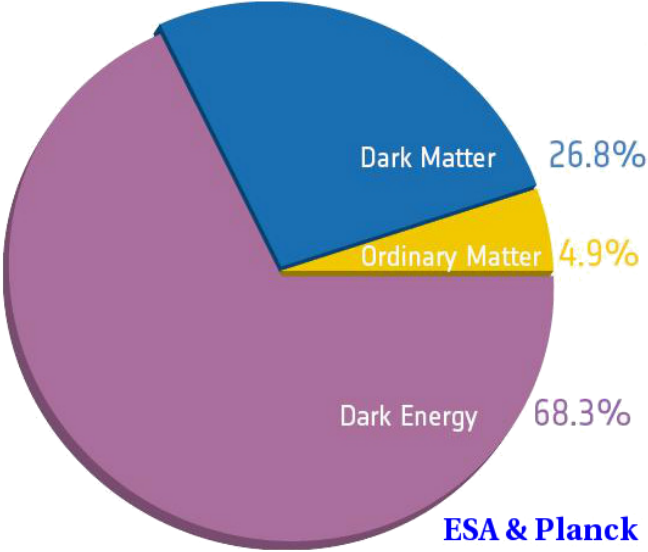
David Schaich (Liverpool)

Southampton High Energy Theory Seminar, 29 November 2019

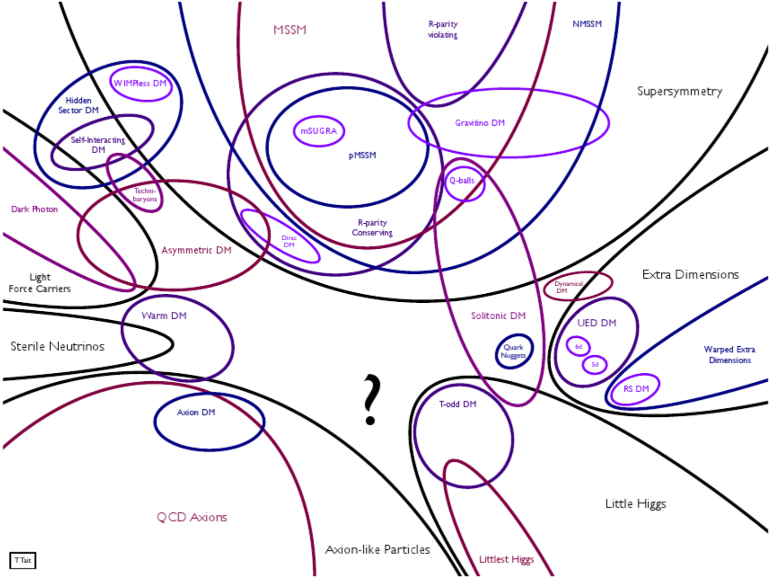
PRD **89**, 094508 PRL **115**, 171803 PRD **92**, 075030

and more to come with the Lattice Strong Dynamics Collaboration

Dark matter — we observe it...



...we don't yet know what it is



Overview

Composite dark matter is an attractive possibility

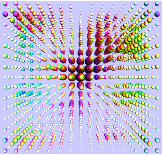
Lattice field theory is needed
to constrain models from experimental results

Dark matter & compositeness

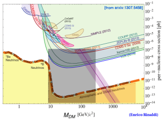
Lattice field theory

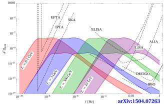
Experiments

- Large underground detectors
- High-energy particle colliders
- Gravitational-wave observatories



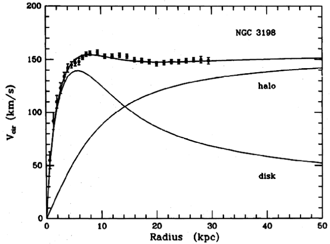
(Image credit: Claudio Rebbi)



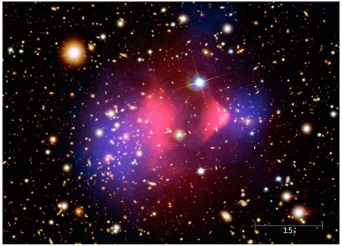


Gravitational evidence for dark matter

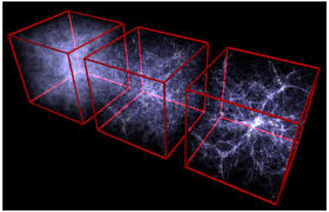
Rotation $\sim 10^3$–10^6 light-years



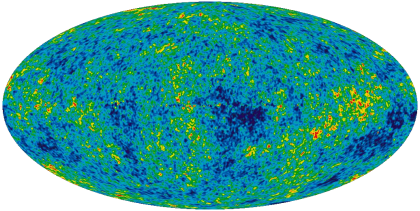
Lensing $\sim 10^6$ light-years



Structure $\sim 10^9$ light-years



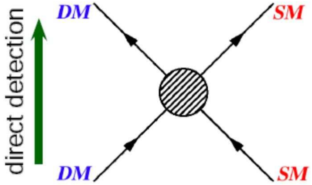
Cosmic background $\sim 10^{10}$ ly

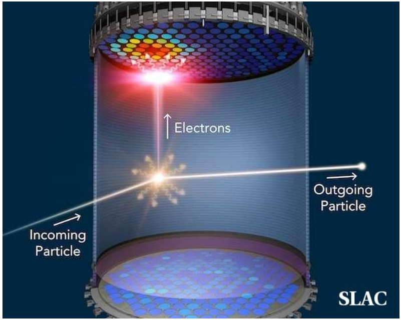


Non-gravitational dark matter interactions

Three search strategies

Direct scattering in underground detectors



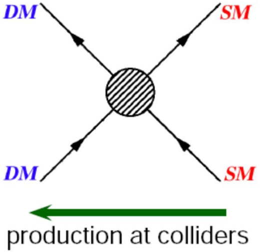


Non-gravitational dark matter interactions

Three search strategies

Direct scattering in underground detectors

Collider production at high energies



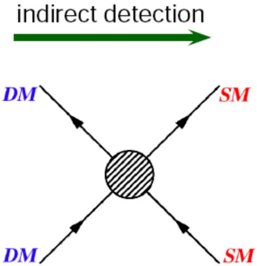
Non-gravitational dark matter interactions

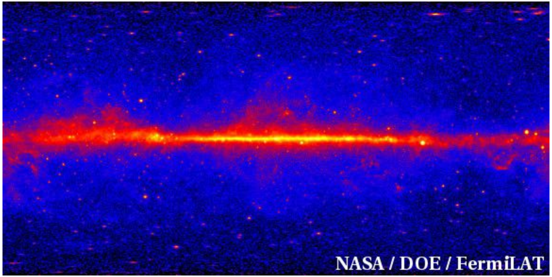
Three search strategies

Direct scattering in underground detectors

Collider production at high energies

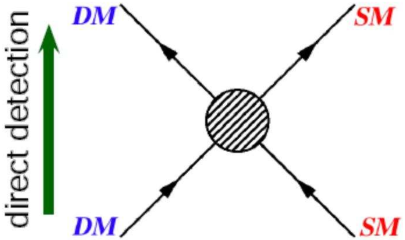
Indirect annihilation into cosmic rays

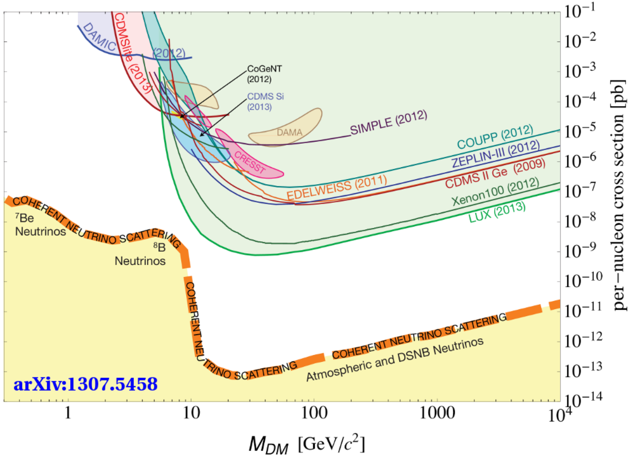




Non-gravitational dark matter interactions

No clear signals so far

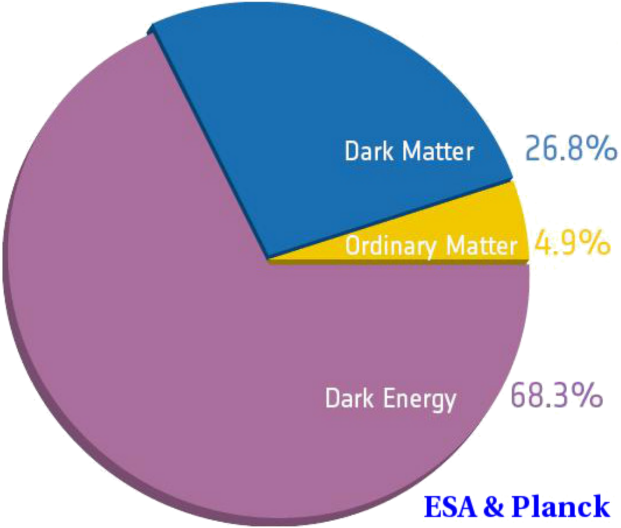




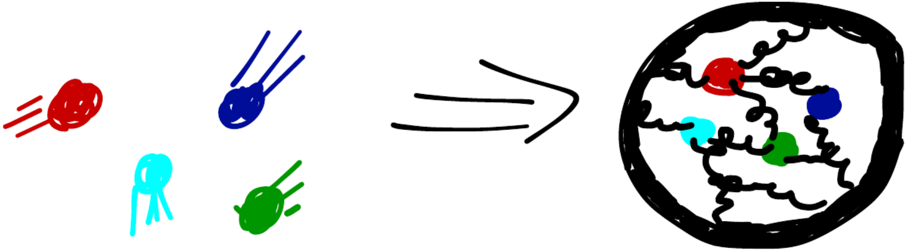
Why we expect non-gravitational interactions

$$\frac{\Omega_{\text{dark}}}{\Omega_{\text{ordinary}}} \approx 5 \quad \ldots \text{not } 10^5 \text{ or } 10^{-5}$$

Explained by non-gravitational interactions with known particles



Composite dark matter



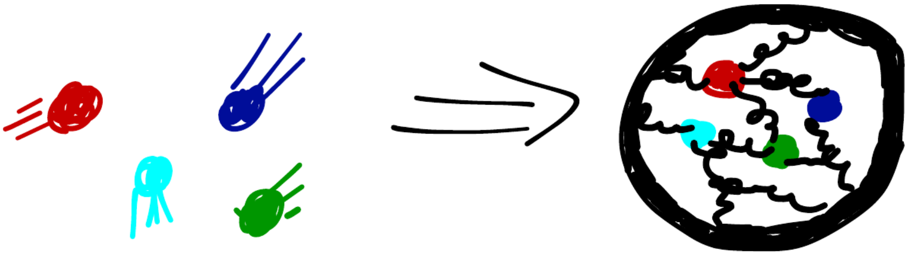
Early universe

Deconfined charged fermions $\longrightarrow$ non-gravitational interactions

Present day

Confined neutral 'dark baryons' $\longrightarrow$ no experimental detections

Composite dark matter



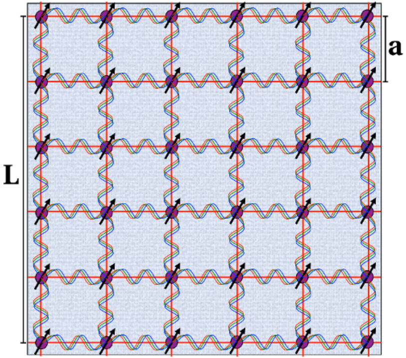
Even neutral composites interact, via charged constituents

$\longrightarrow$ need **lattice calculations** for quantitative predictions

Lattice field theory in a nutshell

Formally $\quad \langle \mathcal{O} \rangle = \frac{1}{\mathcal{Z}} \int \mathcal{D}\Phi \;\; \mathcal{O}(\Phi) \;\; e^{-S[\Phi]}$

Regularize by formulating theory in finite, discrete space-time $\longrightarrow$ **the lattice**



P. Vranas LLNL

Spacing between lattice sites (“a”)

$\longrightarrow$ UV cutoff scale $1/a$

Remove cutoff: $\quad a \to 0 \quad (L/a \to \infty)$

Hypercubic $\longrightarrow$ automatic symmetries

Numerical lattice field theory calculations

High-performance computing
$\longrightarrow$ evaluate up to
$\sim$billion-dimensional integrals

Importance sampling Monte Carlo

Algorithms sample field configurations with probability $\frac{1}{\mathcal{Z}}e^{-S[\Phi]}$

$$\langle\mathcal{O}\rangle = \frac{1}{\mathcal{Z}}\int\mathcal{D}\Phi\ \ \mathcal{O}(\Phi)\ \ e^{-S[\Phi]} \longrightarrow \frac{1}{N}\sum_{i=1}^{N}\mathcal{O}(\Phi_i)\ \text{with stat. uncertainty}\ \propto\frac{1}{\sqrt{N}}$$

Lattice Strong Dynamics Collaboration

Argonne	Xiao-Yong Jin, James Osborn
Bern	Andrew Gasbarro
Boston	Rich Brower, Dean Howarth, Claudio Rebbi
Colorado	**Ethan Neil**, Oliver Witzel
UC Davis	Joseph Kiskis
Livermore	Pavlos Vranas
Liverpool	**DS**
Nvidia	Evan Weinberg
Oregon	**Graham Kribs**
RIKEN	**Enrico Rinaldi**
Yale	Thomas Appelquist, Kimmy Cushman, George Fleming

Exploring the range of possible phenomena in strongly coupled field theories

Direct detection of composite dark matter

Charged constituents $\longrightarrow$ **form factors** $\longrightarrow$ experimental signals

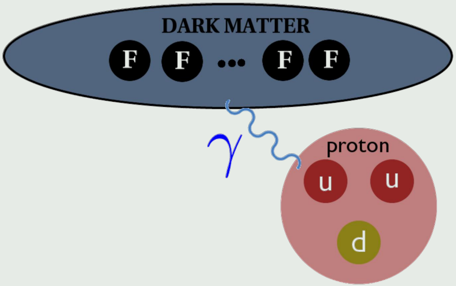
Photon exchange from electromagnetic form factors

Effective interactions suppressed by powers of dark matter mass

$$\text{Magnetic moment} \sim \frac{1}{M_{DM}}$$

$$\text{Charge radius} \sim \frac{1}{M_{DM}^2}$$

$$\text{Polarizability} \sim \frac{1}{M_{DM}^3}$$

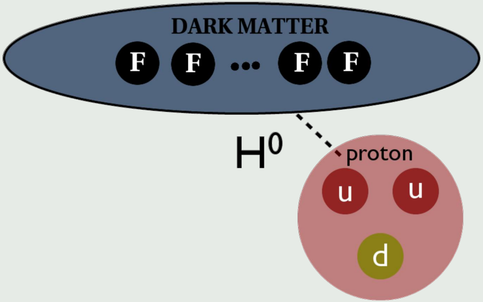


Direct detection of composite dark matter

Charged constituents $\longrightarrow$ **form factors** $\longrightarrow$ experimental signals

Higgs exchange from scalar form factor

Can dominate cross section. . . **if** F mass comes from Higgs

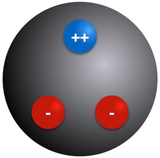


Direct detection of composite dark matter

Charged constituents $\longrightarrow$ **form factors** $\longrightarrow$ experimental signals

Simple first case: Dark matter like a "more-neutral neutron"

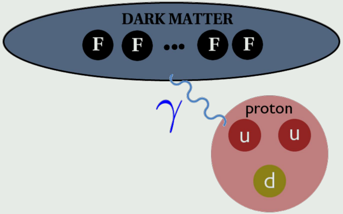
SU(3) with weak singlets $\longrightarrow$ no Higgs-exchange interaction



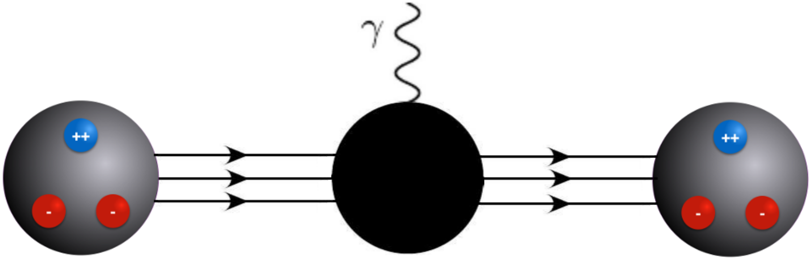
Investigate leading photon-exchange contributions

$$\text{Magnetic moment} \sim \frac{1}{M_{DM}}$$

$$\text{Charge radius} \sim \frac{1}{M_{DM}^2}$$



Magnetic moment and charge radius



$$\left\langle DM(p') \left| \Gamma_\mu(q^2) \right| DM(p) \right\rangle \sim F_1(q^2)\,\gamma_\mu + F_2(q^2)\,\frac{i\sigma_{\mu\nu}q^\nu}{2M_{DM}}, \qquad q = p' - p$$

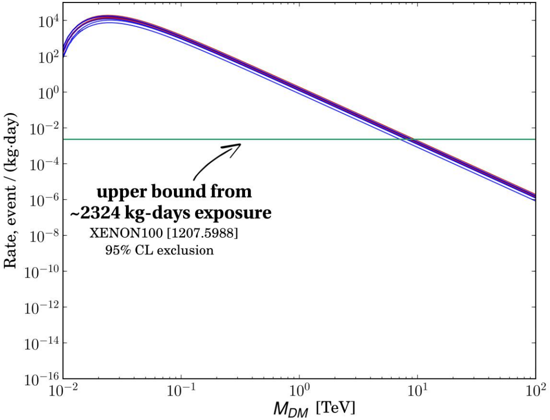
Electric charge: $F_1(0) = 0$

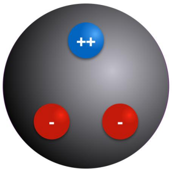
Magnetic moment: $F_2(0)$

Charge radius: $-6\left.\frac{dF_1(q^2)}{dq^2}\right|_{q^2=0} + \frac{3F_2(0)}{2M_{DM}^2}$

Resulting direct detection constraints

Lattice calculations of magnetic moment and charge radius
$\longrightarrow$ event rate vs. dark matter mass





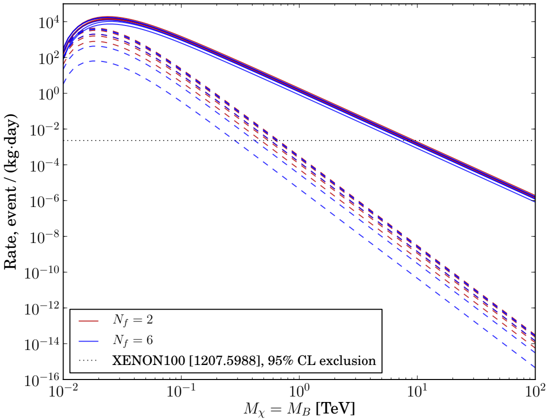
XENON100 $\longrightarrow$ $M_B \gtrsim 10$ TeV

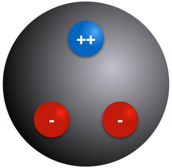
XENON1T $\longrightarrow$ $M_B \gtrsim 30$ TeV [1805.12562]

Little effect from varying model params

Magnetic moment dominates event rate

Charge radius contributions (dashed) are suppressed $\sim 1/M_{DM}^2$





Symmetries can forbid both magnetic moment and charge radius

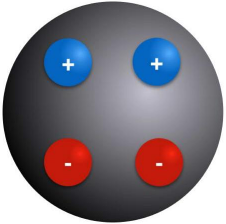
$\longrightarrow$ More freedom for resulting model

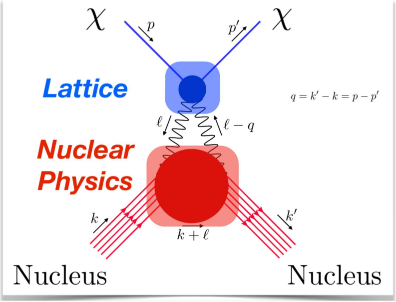
Smarter second case: Stealth Dark Matter

SU(4) composite dark matter with four F

Scalar particle $\longrightarrow$ no magnetic moment ✓

+/- charge symmetry $\longrightarrow$ no charge radius ✓





(Tiny) Coupling to Higgs needed for nucleosynthesis

Polarizability $\sim 1/M_{DM}^3$ dominates direct detection

$\longrightarrow$ Unavoidable lower bound
on broad class of composite dark matter models

‘Stealth’ composites constructed from conspicuous constituents

Direct detection cross section (pb)

Neutrino

$\sigma \sim 10^{-2}$

Radar cross section (m^2)

747

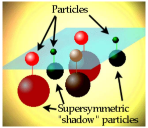
$\sigma \sim 10^{2}$

‘Stealth’ composites constructed from conspicuous constituents

Direct detection cross section (pb)

Neutrino

$\sigma \sim 10^{-2}$



SUSY neutralino

$10^{-6} \lesssim \sigma \lesssim 10^{-5}$

Radar cross section (m^2)

747

$\sigma \sim 10^2$

Falcon

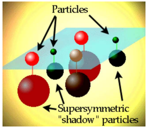
$\sigma \sim 10^{-2}$

'Stealth' composites constructed from conspicuous constituents

Direct detection cross section (pb)

Neutrino

$\sigma \sim 10^{-2}$



SUSY neutralino

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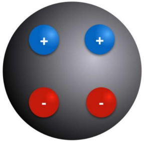


Stealth Dark Matter

$$\sigma \sim \left(\frac{200\text{ GeV}}{M_{DM}}\right)^6 \times 10^{-9}$$

Radar cross section (m^2)

747

$\sigma \sim 10^2$

Falcon

$\sigma \sim 10^{-2}$

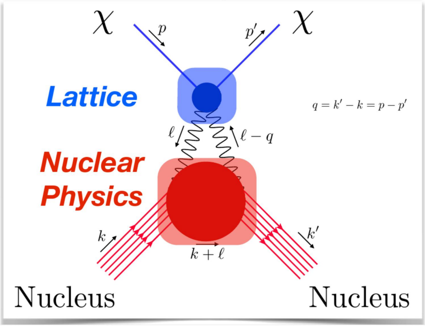
Stealth F-22

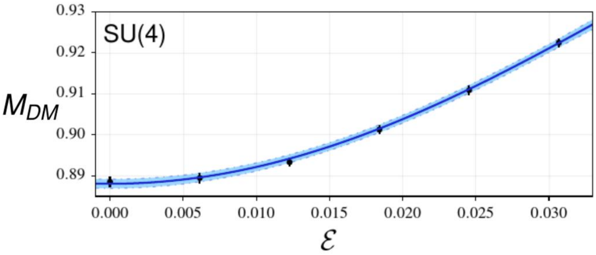
$\sigma < 10^{-3}$

Polarizability of Stealth Dark Matter

Unavoidable lower bound
on broad class of composite dark matter models

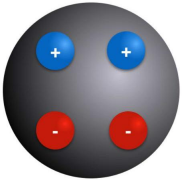
Nuclear physics very complicated
with large uncertanties





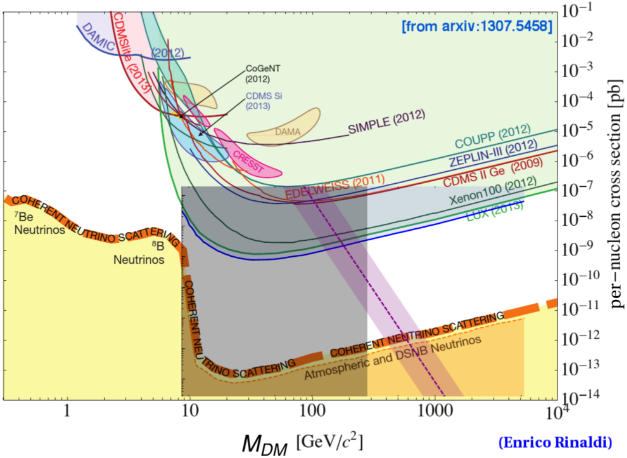
Polarizability is dependence
of lattice M_{DM} on external field $\mathcal{E}$

Lower bound on direct detection



Results specific
to Xenon detectors

Uncertainty dominated
by Xenon nuclear physics

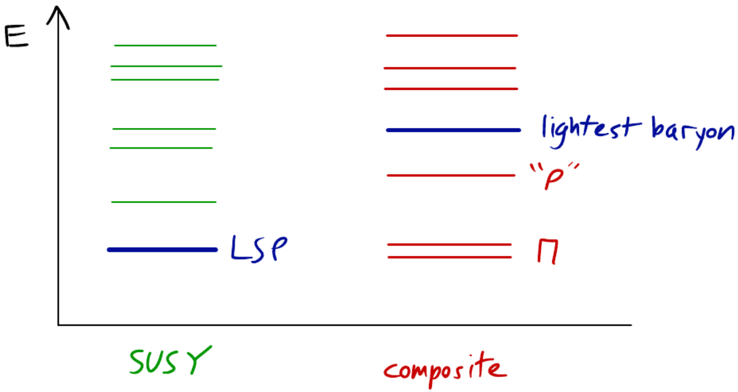


Shaded region is complementary constraint from particle colliders

Stealth Dark Matter at colliders

arXiv:1809.10184

The dark matter is the only stable composite particle, **not** the lightest



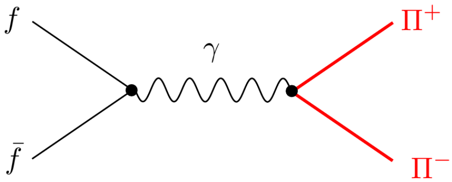
Main constraints from much lighter **charged** "Π"

$\longrightarrow$ standard 'missing energy' searches not efficient

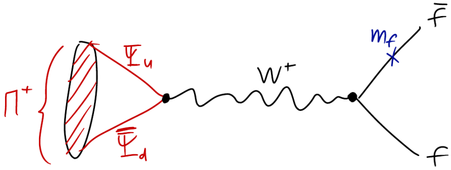
Stealth Dark Matter collider detection

arXiv:1809.10184

Production



Decay



"Particularly tricky" at the LHC: Current bounds only $M_\Pi \gtrsim 130$ GeV

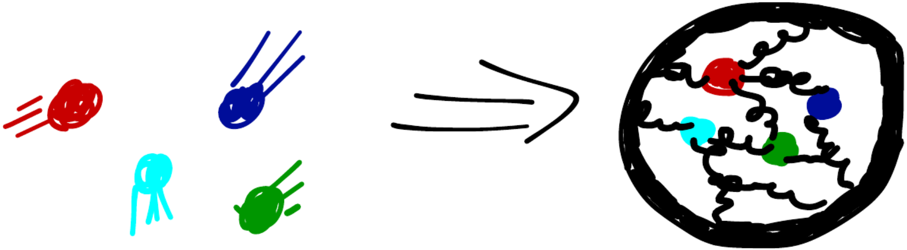
similar to $M_\Pi \gtrsim 100$ GeV from LEP searches for SUSY tau-partner

Lattice calculation of $M_{DM}/M_\Pi \longrightarrow M_{DM} \gtrsim 300$ GeV

More form factors to compute: $F_1(4M_\Pi^2)$ for Π and decay constant F_Π

Gravitational waves

Gravitational-wave observatories opening new window on cosmology



First-order confinement transition $\longrightarrow$ stochastic background of grav. waves

$\Longrightarrow$ Lattice studies of stealth dark matter phase transition

Phase diagram expectations

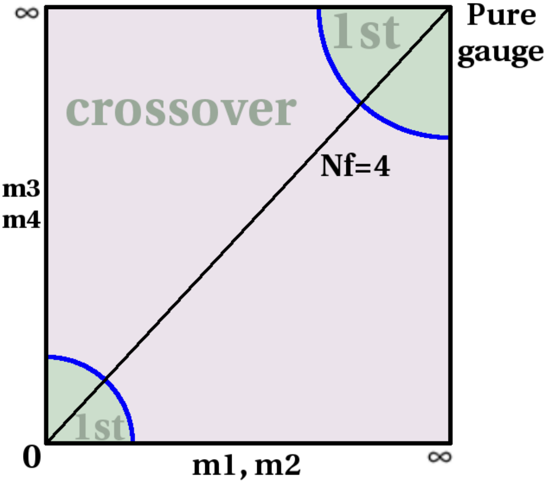
Pure-gauge transition is first order

Becomes stronger as N increases

First-order transition persists for sufficiently heavy fermions

Preliminary: Seem to need $M_P/M_V \gtrsim 0.9$

Form factor calculations considered $0.55 \leq M_P/M_V \leq 0.77$



From first-order transition to gravitational wave signal

First-order transition $\longrightarrow$ gravitational wave background will be produced

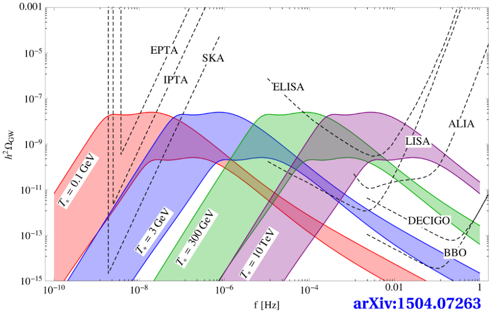
Four key parameters

Transition temperature $T_* \lesssim T_c$

Vacuum energy fraction from **latent heat**

Bubble nucleation rate (transition duration)

Bubble wall speed



arXiv:1504.07263

BSM transitions $\longrightarrow$ low frequencies requiring space-based observatories

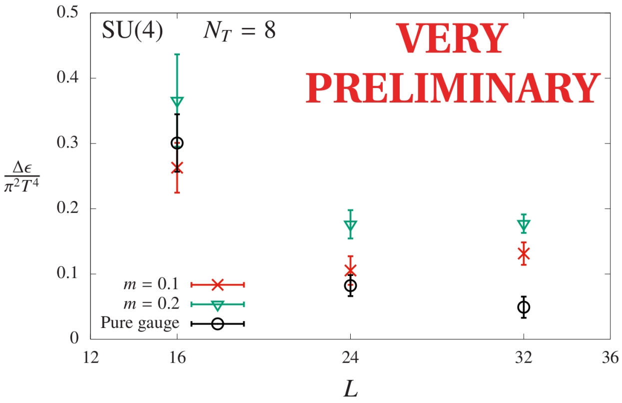
Next step: Latent heat $\Delta\epsilon$

First-order transition $\longrightarrow$ gravitational wave background will be produced

Vacuum energy fraction

$$\alpha \approx \frac{30}{4N(N^2-1)} \frac{\Delta\epsilon}{\pi^2 T_*^4}$$

Latent heat $\Delta\epsilon$
is change in energy density
at transition



Recapitulation and outlook

Composite dark matter is an attractive possibility

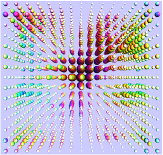
Lattice field theory is needed
to constrain models from experimental results

Minimize EM form factors for direct detection
$\longrightarrow$ Stealth Dark Matter

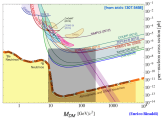
Collider constraints on dark sector

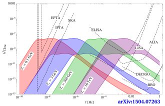
Future searches for gravitational waves

And more: relic abundance; indirect detection; ...



(Image credit: Claudio Rebbi)





Thank you!

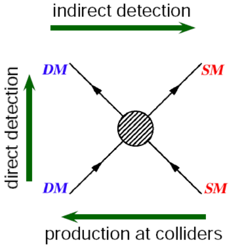
Lattice Strong Dynamics Collaboration

Especially Graham Kribs, Ethan Neil, Enrico Rinaldi

Funding and computing resources

Backup: Thermal freeze-out for relic density

Requires non-gravitational interactions with known particles



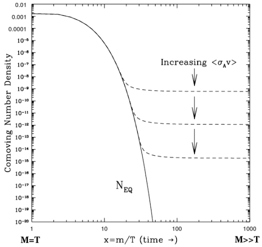
DM $\longleftrightarrow$ SM for $T \gtrsim M_{DM}$

DM $\longrightarrow$ SM for $T \lesssim M_{DM}$

$\Longrightarrow$ rapid depletion of Ω_{DM}

Hubble expansion

$\Longrightarrow$ dilution $\longrightarrow$ freeze-out



$2 \to 2$ scattering relates coupling and mass, $200\alpha \sim \dfrac{M_{DM}}{100\text{ GeV}}$

Strong $\alpha \sim 16 \longrightarrow$ 'natural' mass scale $M_{DM} \sim 300$ TeV

Smaller $M_{DM} \gtrsim 1$ TeV possible from $2 \to n$ scattering or asymmetry

Backup: Two roads to natural asymmetric dark matter

Idea: Dark matter relic density related to baryon asymmetry

$$\Omega_D \approx 5\Omega_B$$
$$\Longrightarrow M_D n_D \approx 5 M_B n_B$$

$n_D \sim n_B \quad \Longrightarrow \quad M_D \sim 5 M_B \approx 5$ GeV

High-dim. interactions relate baryon# and DM# violation

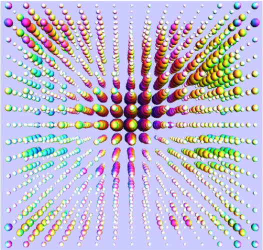
$M_D \gg M_B \quad \Longrightarrow \quad n_B \gg n_D \sim \exp\left[-M_D/T_s\right] \qquad T_s \sim 200$ GeV

EW sphaleron processes above T_s distribute asymmetries

Both require non-gravitational interactions with known particles

Backup: Hybrid Monte Carlo (HMC) algorithm

Goal: Sample field configurations Φ with probability $\frac{1}{\mathcal{Z}} e^{-S[\Phi]}$


(Image credit: Claudio Rebbi)

HMC is Markov process based on
Metropolis–Rosenbluth–Teller

Fermions $\longrightarrow$ extensive action computation

$\Longrightarrow$ Global updates via fictitious molecular dynamics

1. Introduce fictitious random momenta and "MD time" τ
2. Inexact MD evolution along trajectory in τ $\longrightarrow$ new configuration
3. Accept/reject test on MD discretization error

Backup: More details about form factors

Photon exchange via electromagnetic form factors

Interactions suppressed by powers of confinement scale $\Lambda \sim M_{DM}$

Dimension 5: Magnetic moment $\longrightarrow$ $\left(\overline{X}\sigma_{\mu\nu}X\right) F^{\mu\nu}/\Lambda$

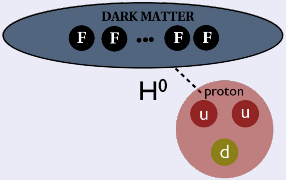
Dimension 6: Charge radius $\longrightarrow$ $\left(\overline{X}X\right) v_{\mu}\partial_{\nu}F^{\mu\nu}/\Lambda^2$

Dimension 7: Polarizability $\longrightarrow$ $\left(\overline{X}X\right) v_{\mu}v_{\nu}F^{\mu\alpha}F_{\alpha}^{\ \nu}/\Lambda^3$

Higgs exchange via scalar form factors

Higgs couples through σ terms $\left\langle B \middle| m_{\psi}\overline{\psi}\psi \middle| B\right\rangle$

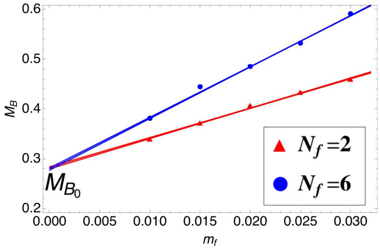
Produces rapid charged 'Π' decay
needed for Big Bang nucleosynthesis



Backup: More details about SU(3) composite dark matter model

Same SU(3) gauge group as QCD

Re-analyze existing data sets:
$32^3 \times 64$ lattices, domain wall fermions



Scan relatively heavy fermion masses $m_F \longrightarrow 0.55 \lesssim M_\Pi / M_V \lesssim 0.75$

Compare $N_F = 2$ or 6 degenerate flavors with same $M_{B_0} \equiv \lim_{m_F \to 0} M_B$

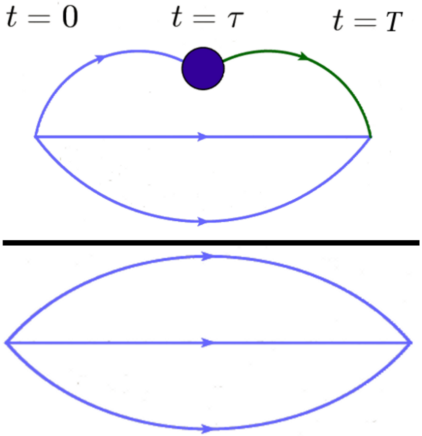
Unlike QCD, fermions are all $\mathrm{SU(2)}_L$ singlets $\longrightarrow Q = Y$

Half have $Q_\mathrm{P} = 2/3$, half $Q_\mathrm{M} = -1/3$

Dark matter candidate is singlet "dark baryon" $B = \mathrm{PMM}$

Backup: Form factor calculations on the lattice

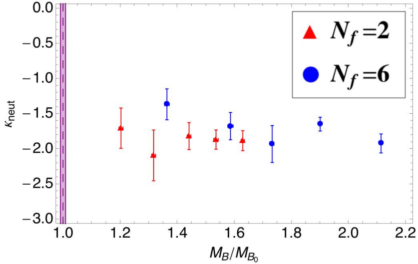
$R(\tau, T, p, p') \sim$



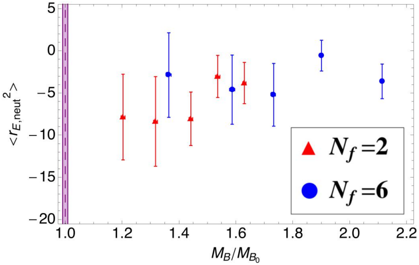
$$R_\Gamma(\tau, T, p, p') \longrightarrow \left\langle DM(p') \left| \Gamma_\mu(q^2) \right| DM(p) \right\rangle + \mathcal{O}\left(e^{-\Delta\tau},\ e^{-\Delta T},\ e^{-\Delta(T-\tau)}\right)$$

Backup: Electromagnetic form factor results

Magnetic moment κ



Charge radius $\langle r^2 \rangle$



Little dependence on N_F or on $m_F \sim M_B/M_{B_0}$

κ comparable to neutron's $\kappa_N = -1.91$

$\langle r^2 \rangle$ smaller than neutron's $\langle r^2 \rangle_N \approx -38$ (related to larger M_Π/M_V)

Insert into standard event rate formulas...

Backup: Event rate formulas and lattice input

$$\text{Rate} = \frac{M_{detector}}{M_T}\frac{\rho_{DM}}{M_{DM}}\int_{E_{min}}^{E_{max}} dE_R\ \mathcal{A}cc(E_R)\ \left\langle v_{DM}\frac{d\sigma}{dE_R}\right\rangle_f$$

$$\frac{d\sigma}{dE_R} = \frac{\overline{|\mathcal{M}_{SI}|^2} + \overline{|\mathcal{M}_{SD}|^2}}{16\pi\,(M_{DM}+M_T)^2\,E_R^{max}} \qquad E_R^{max} = \frac{2M_{DM}^2 M_T v_{col}^2}{(M_{DM}+M_T)^2}$$

From **magnetic moment** κ and **charge radius** $\langle r^2\rangle$

$$\frac{\overline{|\mathcal{M}_{SI}|^2}}{e^4\,[ZF_c(Q)]^2} = \left(\frac{M_T}{M_{DM}}\right)^2\left[\frac{4}{9}M_{DM}^4\left\langle r^2\right\rangle^2 + \frac{\kappa^2\,(M_T+M_{DM})^2\,(E_R^{max}-E_R)}{M_T^2 E_R}\right]$$

$$\overline{|\mathcal{M}_{SD}|^2} = e^4\frac{2}{3}\left(\frac{J+1}{J}\right)\left[\left(A\frac{\mu_T}{\mu_n}\right)F_s(Q)\right]^2\kappa^2$$

Backup: Event rate formulas and lattice input

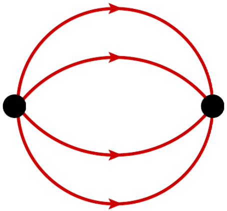
$$\text{Rate} = \frac{M_{detector}}{M_T}\frac{\rho_{DM}}{M_{DM}}\int_{E_{min}}^{E_{max}} dE_R\, \mathcal{A}cc(E_R)\, \left\langle v_{DM}\frac{d\sigma}{dE_R}\right\rangle_f$$

$$\frac{d\sigma}{dE_R} = \frac{\overline{|\mathcal{M}_{SI}|^2} + \overline{|\mathcal{M}_{SD}|^2}}{16\pi\left(M_{DM}+M_T\right)^2 E_R^{max}} \qquad E_R^{max} = \frac{2M_{DM}^2 M_T v_{col}^2}{\left(M_{DM}+M_T\right)^2}$$

From **polarizability** C_F

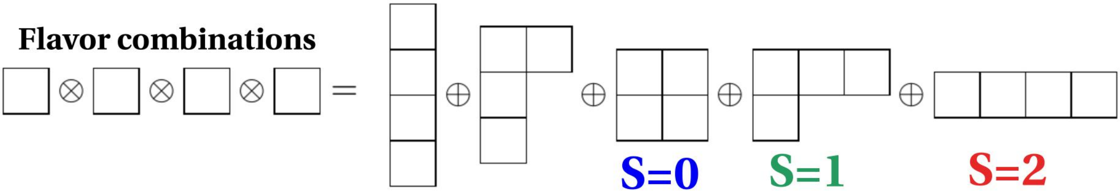
$$\sigma_{SI} = \frac{Z^4}{A^2}\frac{144\pi\alpha_{em}^4 \widetilde{M}_{n,DM}^2}{M_{DM}^6 R^2} C_F^2 \propto \frac{Z^4}{A^2} \qquad \text{per nucleon}$$

Backup: More details about SU(4) Stealth Dark Matter



Quenched SU(4) lattice ensembles

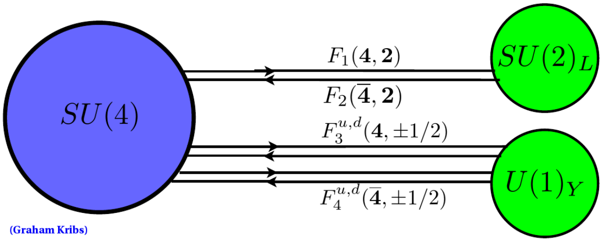
Lattice volumes up to $64^3 \times 128$,
several lattice spacings to check systematic effects



Dark matter candidate is spin-zero baryon $\longrightarrow$ no magnetic moment

Need at least two flavors to anti-symmetrize $\longrightarrow$ no charge radius

Backup: Even more details about SU(4) Stealth Dark Matter



Field	$SU(N_D)$	$(SU(2)_L, Y)$	Q
$F_1 = \begin{pmatrix} F_1^u \\ F_1^d \end{pmatrix}$	$\mathbf{N}$	$(\mathbf{2}, 0)$	$\begin{pmatrix} +1/2 \\ -1/2 \end{pmatrix}$
$F_2 = \begin{pmatrix} F_2^u \\ F_2^d \end{pmatrix}$	$\bar{\mathbf{N}}$	$(\mathbf{2}, 0)$	$\begin{pmatrix} +1/2 \\ -1/2 \end{pmatrix}$
F_3^u	$\mathbf{N}$	$(\mathbf{1}, +1/2)$	$+1/2$
F_3^d	$\mathbf{N}$	$(\mathbf{1}, -1/2)$	$-1/2$
F_4^u	$\bar{\mathbf{N}}$	$(\mathbf{1}, +1/2)$	$+1/2$
F_4^d	$\bar{\mathbf{N}}$	$(\mathbf{1}, -1/2)$	$-1/2$

Mass terms $\quad m_V\left(F_1 F_2 + F_3 F_4\right) + y\left(F_1 \cdot H F_4 + F_2 \cdot H^\dagger F_3\right) + \text{h.c.}$

Vector-like masses evade Higgs-exchange direct detection bounds

Higgs couplings $\longrightarrow$ charged meson decay before Big Bang nucleosynthesis

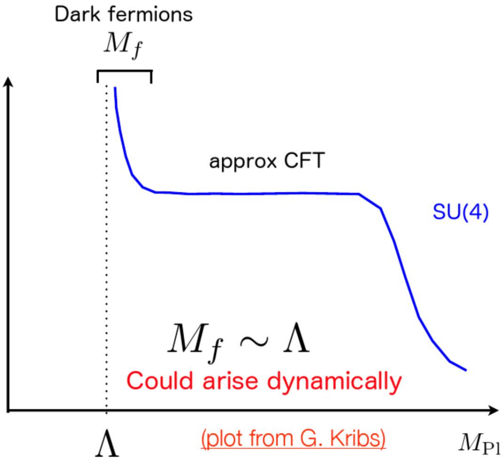
Both required $\longrightarrow$ four flavors

Backup: Stealth Dark Matter mass scales

Lattice studies focus on $m_\psi \simeq \Lambda_{DM}$ where effective theories least reliable

$m_\psi \simeq \Lambda_{DM}$ could arise dynamically

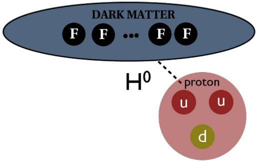
Collider constraints on M_{DM}
become stronger as m_ψ decreases



Backup: Effective Higgs interaction

$M_H = 125$ GeV $\longrightarrow$ Higgs exchange can dominate direct detection

$$\sigma_H^{(SI)} \propto \left| \frac{\widetilde{M}_{DM,N}}{M_H^2} \; y_\psi \left\langle DM \left| \overline{\psi}\psi \right| DM \right\rangle \; y_q \left\langle N \left| \overline{q}q \right| N \right\rangle \right|^2$$



Quark $\quad y_q = \dfrac{m_q}{v}$

Dark $\quad y_\psi = \alpha \dfrac{m_\psi}{v}$ suppressed by $\alpha \equiv \dfrac{v}{m_\psi} \dfrac{\partial m_\psi(h)}{\partial h}\Big|_{h=v} = \dfrac{yv}{yv + m_V}$

Determine using Feynman–Hellmann theorem $\quad \left\langle DM \left| \overline{\psi}\psi \right| DM \right\rangle = \dfrac{\partial M_{DM}}{\partial m_\psi}$

Backup: Feynman–Hellmann theorem

$m_\psi \overline{\psi}\psi$ is the only term in the hamiltonian that depends on m_ψ

$$\Longrightarrow \left\langle B \left| \frac{\partial \widehat{H}}{\partial m_\psi} \right| B \right\rangle = \left\langle B \left| \overline{\psi}\psi \right| B \right\rangle$$

Since $\widehat{H}\,|B\rangle = M_B\,|B\rangle$ and $\langle B|\,\widehat{H} = \langle B|\,M_B$ we have

$$\frac{\partial}{\partial m_\psi} M_B = \frac{\partial}{\partial m_\psi} \left\langle B \left| \widehat{H} \right| B \right\rangle = \left\langle \frac{\partial B}{\partial m_\psi} \left| \widehat{H} \right| B \right\rangle + \left\langle B \left| \widehat{H} \right| \frac{\partial B}{\partial m_\psi} \right\rangle + \left\langle B \left| \frac{\partial \widehat{H}}{\partial m_\psi} \right| B \right\rangle$$

$$= M_B \langle \frac{\partial B}{\partial m_\psi} | B \rangle + M_B \langle B | \frac{\partial B}{\partial m_\psi} \rangle + \left\langle B \left| \overline{\psi}\psi \right| B \right\rangle$$

$$= M_B \frac{\partial}{\partial m_\psi} \langle B | B \rangle + \left\langle B \left| \overline{\psi}\psi \right| B \right\rangle = \left\langle B \left| \overline{\psi}\psi \right| B \right\rangle \qquad \square$$

Backup: Lattice results for Higgs exchange constrain α

$$\sigma_H^{(SI)} \propto \left| y_\psi \left\langle DM \left| \overline{\psi}\psi \right| DM \right\rangle \right|^2$$

Matrix element $\propto \dfrac{\partial M_{DM}}{\partial m_\psi}$

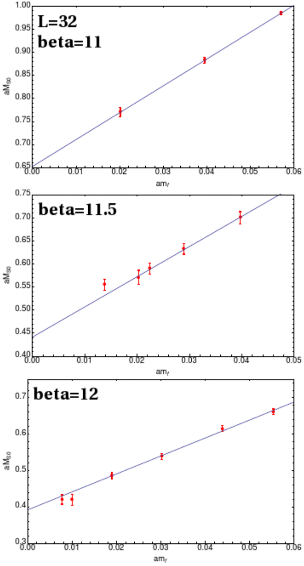
(Feynman–Hellmann)

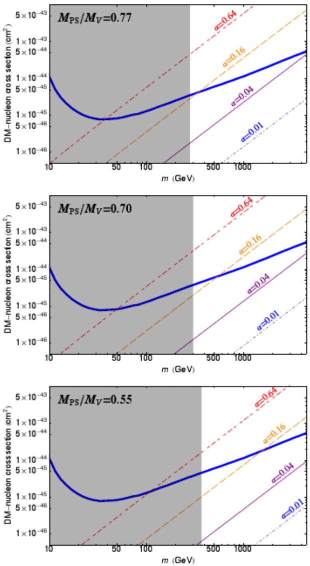
Stealth Dark Matter:

$$0.15 \lesssim \frac{m_\psi}{M_{DM}} \frac{\partial M_{DM}}{\partial m_\psi} \lesssim 0.34$$

Larger than QCD

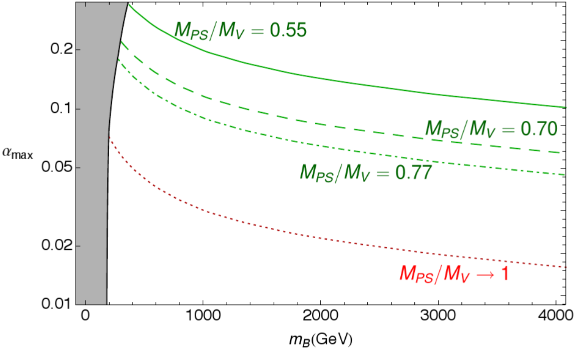
$$0.04 \lesssim \frac{m_q}{M_N} \frac{\partial M_N}{\partial m_q} \lesssim 0.08$$





Backup: Bounds on effective Higgs coupling

Higgs-exchange cross section $\longrightarrow$ maximum α allowed by LUX [1310.8214]



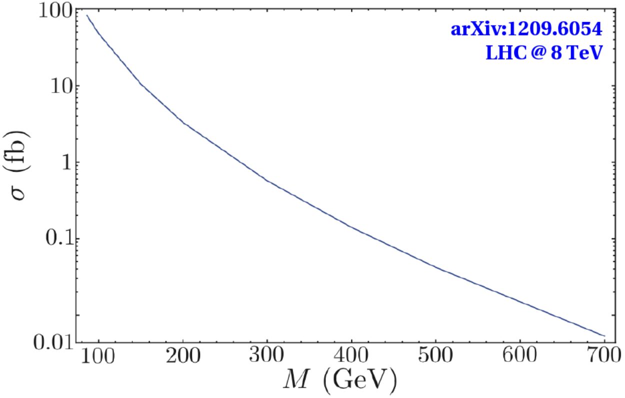
Maximum α depends on M_Π/M_V and dark matter mass

Smaller $M_\Pi/M_V \longleftrightarrow m_F$
$\longrightarrow$ stronger constraints from colliders

Effective Higgs interaction tightly constrained

$\alpha \lesssim 0.3$ for $M_\Pi/M_V \gtrsim 0.55$ $\longrightarrow$ fermion masses must be mainly vector-like

Backup: More about Stealth Dark Matter at the LHC



Π pair production cross section

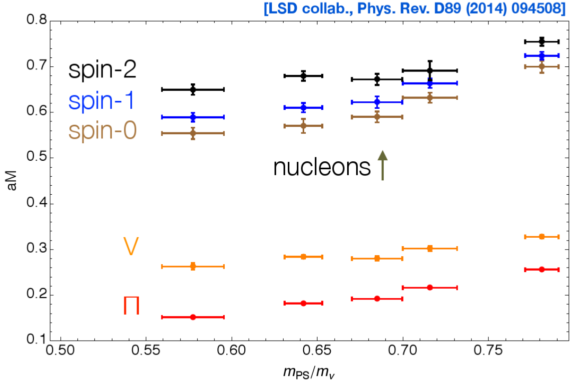
Integrate over proton parton dist.,

set $F_1(4M_\Pi^2) = 1$

LHC can search for $\Pi^+\Pi^- \longrightarrow t\bar{b} + \bar{t}b$ in addition to $\tau^+\tau^- + \not{E}_T$

Should eventually surpass $M_\Pi \gtrsim 100$ GeV from LEP

Backup: Indirect detection



Lattice results for composite spectrum

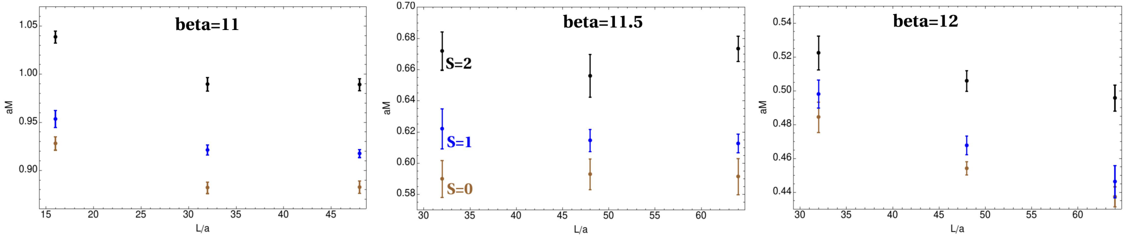
Predict γ-rays from splitting between baryons with spin $S = 0$, 1 and 2

Much more challenging future work

DM–$\overline{\text{DM}}$ annihilation into (many) lighter Π that then decay

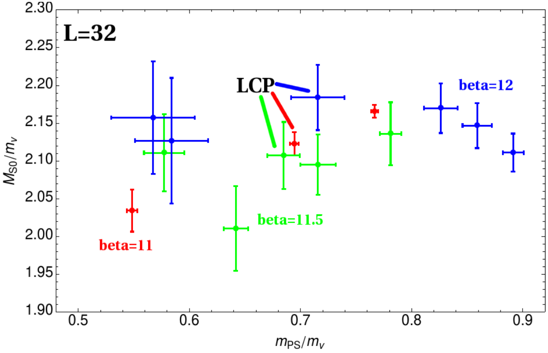
Backup: Volume and discretization effects

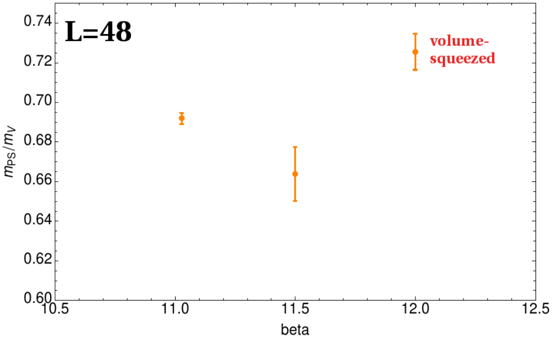
Baryon masses vs. L at fixed lattice spacing (set by $\beta \simeq 8/g_0^2$) and fermion mass



Backup: Volume and discretization effects

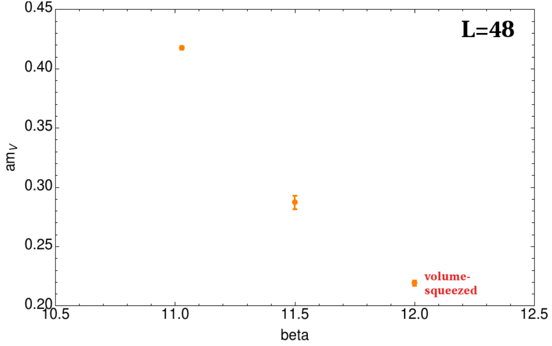
Edinburgh-style plot of $\frac{M_{S0}}{M_V}$ vs. $\frac{M_\Pi}{M_V}$ and line of constant physics (LCP)

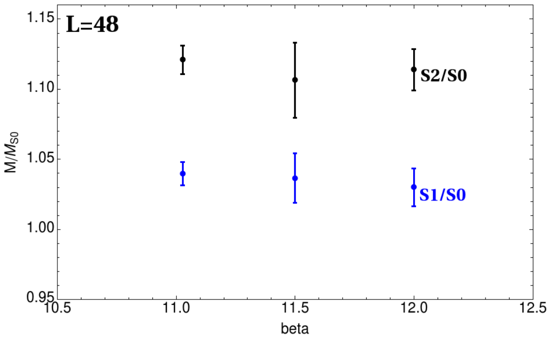




Backup: Volume and discretization effects

Lattice spacing and discretization effects for $\dfrac{M_{S2,S1}}{M_{S0}}$ on line of constant physics





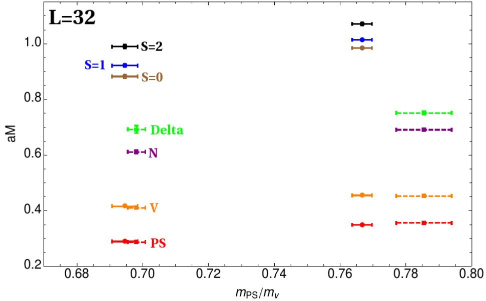
Backup: Large-N predictions for SU(4) baryons

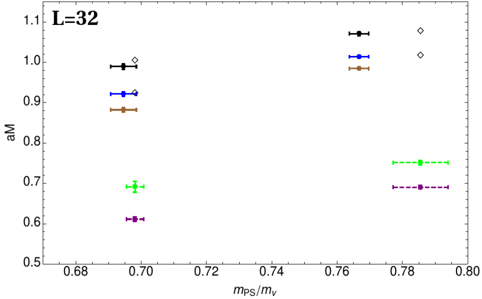
Tune (β, m_F) to match SU(3) M_Π and M_V (dashed)

Rotor spectrum for spin-J baryons: $M(N,J) = NM_0 + C + B\frac{J(J+1)}{N} + \mathcal{O}\left(\frac{1}{N^2}\right)$

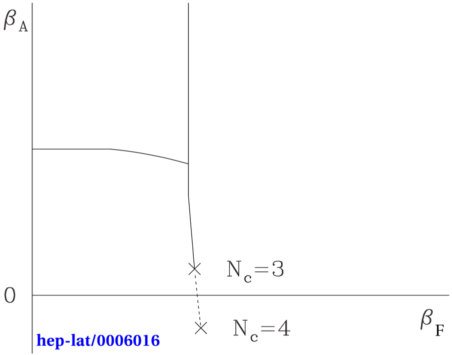
Fit M_0, C and B with nucleon, Δ and spin-0 baryon masses

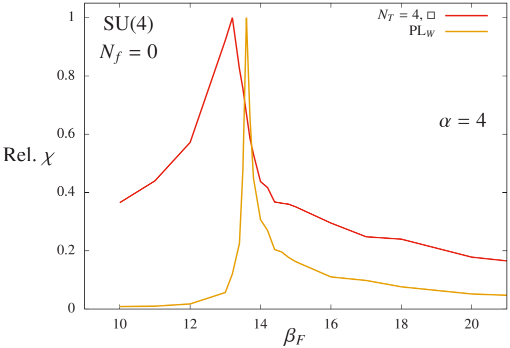
$\longrightarrow$ predictions for $S = 1, 2$ baryons (diamonds)





Backup: Pure gauge checks — Bulk and thermal transitions

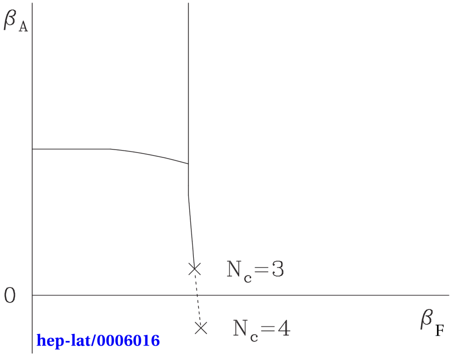


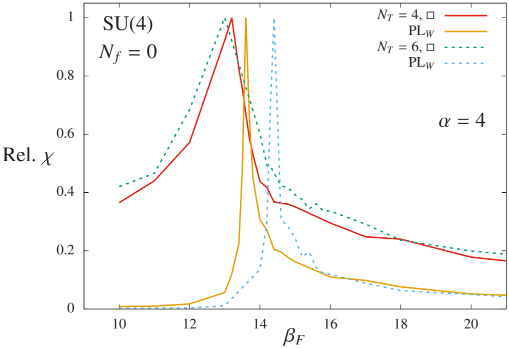


Try to avoid bulk transition for small N_T $\longrightarrow$ use $\beta_A = -\beta_F/4$

Still need $N_T > 4$ for clear separation between bulk & thermal transitions

Backup: Pure gauge checks — Bulk and thermal transitions





Try to avoid bulk transition for small $N_T \longrightarrow$ use $\beta_A = -\beta_F/4$

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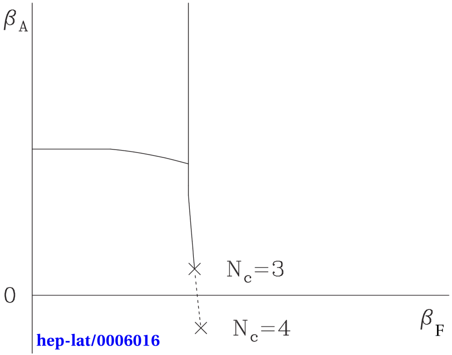
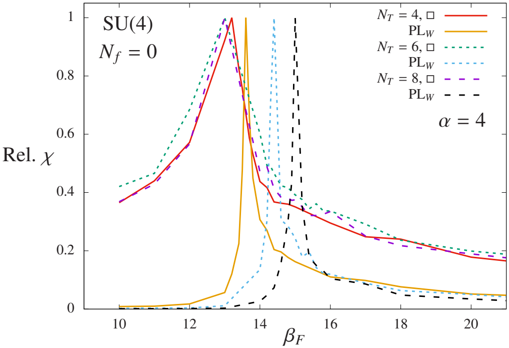
Backup: Pure gauge checks — Bulk and thermal transitions

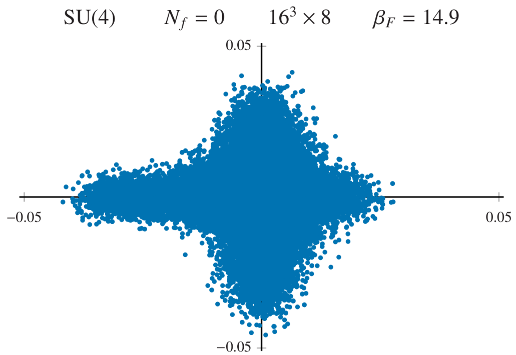


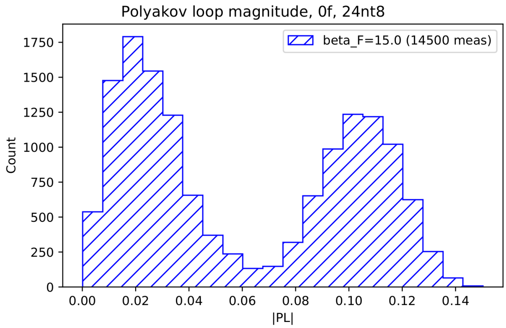


Try to avoid bulk transition for small N_T $\longrightarrow$ use $\beta_A = -\beta_F/4$

Still need $N_T > 4$ for clear separation between bulk & thermal transitions

Backup: Pure gauge checks — Order of thermal transition





Two peaks in Polyakov loop magnitude histogram $\longrightarrow$ first-order transition ✓

Hysteresis not clearly visible even in pure-gauge case