Supersymmetric lattice field theories — Classical simulations and quantum opportunities —



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Motivations

Pursuing first-principles predictions for strongly coupled supersymmetric QFTs



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Quantum opportunities for lattice susy

Overview

Significant progress currently being made in classical lattice studies of supersymmetric QFTs

Opportunities for near-future quantum simulations

✓ Motivations

Recent highlights of lower-dim'l classical simulations

- Super-Yang–Mills (SYM) quantum mechanics
- Plane-wave matrix model
- (1+1)-dimensional SYM

Quantum opportunities









Supersymmetry must be broken on the lattice

Supersymmetry is a space-time symmetry, adding spinor generators Q^{I}_{α} and $\overline{Q}^{I}_{\dot{\alpha}}$ to translations, rotations, boosts

 $\left\{ Q^{\mathrm{I}}_{\alpha}, \overline{Q}^{\mathrm{J}}_{\dot{\alpha}} \right\} = 2\delta^{\mathrm{IJ}} \sigma^{\mu}_{\alpha\dot{\alpha}} P_{\mu}$ broken in discrete space-time

 \longrightarrow relevant susy-violating operators



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Easier to handle in lower dimensions \longrightarrow active area for classical simulations

Super-Yang–Mills quantum mechanics

Holographic duality conjecture Maximal SYM at non-zero temperature $\leftrightarrow \rightarrow$ properties of stringy black holes

Ultimate simplification — compactify all spatial dimensions

 $(9+1)d \ U(N) \ SYM \longrightarrow$ quantum mechanics of interacting $N \times N$ matrices (16 'fermionic', 9 'scalar', 1 'gauge')

Large-N limit of SYM QM $\leftrightarrow \rightarrow$ strong-coupling limit of type IIA string theory [Banks–Fischler–Shenker–Susskind, hep-th/9610043]

Testing holography with lattice super-Yang-Mills QM

Dual black hole energy from maximal SYM QM

Monte Carlo String/M-Theory Collaboration [1606.04951]



Challenges for lattice super-Yang-Mills QM

Large-*N* continuum extrapolations under control Low-temperature instability due to emergence of flat directions



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Plane-wave matrix model

Deform SYM QM to lift flat directions while preserving all 16 supersymmetries [Berenstein–Maldacena–Nastase, hep-th/0202021]

Deformation parameter $\mu \longrightarrow$ dimensionless coupling $g \equiv \lambda/\mu^3 = g_{YM}^2 N/\mu^3$ dimensionless temperature T/μ

'Confinement' transition as T/μ decreases for fixed g



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Plane-wave matrix model transition signals

Peaks in Polyakov loop susceptibility match change in its magnitude |PL|, grow with size of SU(*N*) gauge group, comparing N = 8, 12, 16



Plane-wave matrix model phase diagram

Goal: Interpolate between $g \rightarrow 0$ perturbation theory

and large- $N \hspace{.1in} g
ightarrow \infty$ holographic prediction



Beyond quantum mechanics

Higher dimensions \longrightarrow more challenging fine-tuning

Address by preserving susy sub-algebra at non-zero lattice spacing

Equivalent constructions from topological twisting and deconstruction



Review: arXiv:0903.4881



Need 2^{d+1} supersymmetries in d+1 dimensions

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Topological twisting

Need 2^{d+1} supersymmetries in d+1 dimensions \longrightarrow global 'R' symmetry contains $SO(d+1)_R$ subgroup

Replace spinors by objects with integer spin under 'twisted rotation group' $SO(d+1)_{tw} \equiv diag \left[SO(d+1)_{euc} \otimes SO(d+1)_{R}\right]$

Discrete space-time $\{\mathcal{Q},\mathcal{Q}\}=2\mathcal{Q}^2=0$



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 \bigcirc

Discrete space-time

Can preserve closed sub-algebra

$$\{\mathcal{Q},\mathcal{Q}\}=2\mathcal{Q}^2=0$$

Expectations in (1+1) dimensions

Maximal SYM deconfined for any inverse temperature $r_{\beta} = 1/t$



For decreasing r_L at low t and large N

homogeneous black string (D1) \longrightarrow localized black hole (D0)

"spatial deconfinement" signalled by Wilson line

(1+1)-dimensional SYM transition signals

Peaks in Wilson line susceptibility match change in its magnitude |WL|, grow with size of SU(*N*) gauge group, comparing N = 6, 9, 12

Agreement for 16×4 vs. 24×6 lattices (aspect ratio $\alpha = r_L/r_\beta = 4$)



Phase diagram of (1+1)-dimensional SYM

Large $\alpha = r_L/r_\beta \gtrsim 4 \longrightarrow$ good agreement with high-temperature bosonic QM Small $\alpha \lesssim 2 \longrightarrow$ harder to control uncertainties with $6 \le N \le 16$



Dual black hole thermodynamics

Dual black hole energy from (1+1)-dimensional maximal SYM $\propto t^3$ for large- r_L D1 phase $\propto t^{3.2}$ for small- r_L D0 phase

Lattice results consistent with holography for sufficiently low $t \lesssim 0.4$



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Quantum opportunities for lattice susy

From SYM to superQCD

[Catterall–Veernala, arXiv:1505.00467]

Preserve twisted supersymmetry sub-algebra in 1+1 or 2+1 dimensions

2-slice lattice SYM with $U(N) \times U(F)$ gauge group Adj. fields on each slice Bi-fundamental in between Decouple U(F) slice

 \rightarrow U(*N*) superQCD with *F* fund. hypermultiplets



Dynamical susy breaking in (1+1)-dimensional lattice superQCD

U(N) superQCD with F fundamental hypermultiplets

Spontaneous susy breaking requires N > F



Dynamical susy breaking in (1+1)-dimensional lattice superQCD

U(N) superQCD with *F* fundamental hypermultiplets Spontaneous susy breaking requires N > F and zero Witten index



Quantum opportunities: Sign problems

All results above from phase-quenched rational hybrid Monte Carlo (RHMC)

$$\langle \mathcal{O} \rangle = \frac{\int [d\Phi] \ \mathcal{O} \ e^{-S_{B}} \ \mathsf{pf} \mathcal{D}}{\int [d\Phi] \ e^{-S_{B}} \ \mathsf{pf} \mathcal{D}} \longrightarrow \frac{\int [d\Phi] \ \mathcal{O} e^{i\alpha} \ e^{-S_{B}} \ |\mathsf{pf} \mathcal{D}|}{\int [d\Phi] \ e^{i\alpha} \ e^{-S_{B}} \ |\mathsf{pf} \mathcal{D}|} = \frac{\langle \mathcal{O} e^{i\alpha} \rangle_{pq}}{\langle e^{i\alpha} \rangle_{pq}}$$

"Witten index"
$$W = \text{Tr} \left[(-1)^F e^{-\beta H} \right] \propto \left\langle e^{i\alpha} \right\rangle_{pq}$$
 (at zero temperature)

 \implies W = 0 for dynamical supersymmetry breaking \longrightarrow severe sign problem

Quantum opportunities: Sign problems

Also sign problems without susy breaking

Phase fluctuations increase with coupling [figs for (3+1)d $\mathcal{N} = 4$ SYM]





Quantum opportunities: Real-time dynamics

Already several talks about quantum simulation for real-time dynamics

In near term, simpler supersymmetric systems more realistic targets [e.g., (1+1)-dimensional Wess–Zumino model]

 $\exp[-S] \longrightarrow \exp[iS]$

Recap: An exciting time for lattice supersymmetry

Significant progress currently being made in classical lattice studies of supersymmetric QFTs

Opportunities for near-future quantum simulations

Lower-dimensional studies test holography

- Super-Yang–Mills (SYM) quantum mechanics
- Plane-wave matrix model
- (1+1)-dimensional SYM

Quantum opportunities to address sign problems \longrightarrow supersymmetry breaking, real-time dynamics, ...







Thank you!

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Backup: Breakdown of Leibniz rule on the lattice

Crucial difference between ∂ and Δ

$$\Delta [\phi\eta] = a^{-1} [\phi(x+a)\eta(x+a) - \phi(x)\eta(x)]$$
$$= [\Delta\phi] \eta + \phi\Delta\eta + a[\Delta\phi] \Delta\eta$$

Full supersymmetry requires Leibniz rule $\partial [\phi \eta] = [\partial \phi] \eta + \phi \partial \eta$

only recoverd in $a \rightarrow 0$ continuum limit for any local finite difference

Backup: Another plane-wave matrix model confinement observable

Polyakov loop eigenvalue distribution evolves from localized to uniform



Backup: (1+1)-dimensional SYM Wilson line eigenvalues

Check 'spatial deconfinement' through Wilson line eigenvalue phases



Left: $\alpha = 2$ distributions more extended as *N* increases \longrightarrow D1 black string **Right:** $\alpha = 1/2$ distributions more compact as *N* increases \longrightarrow D0 black hole

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Backup: Dimensional reduction to (1+1)-dimensional SYM

Naive for now: 4d $\mathcal{N} = 4$ SYM code with $N_x = N_y = 1$

 $A_4^* \longrightarrow A_2^*$ (triangular) lattice

Torus **skewed** depending on $\alpha = N_t/L$

Modular trans. into fund. domain \longrightarrow some skewed tori actually rectangular

Also need to stabilize compactified links to ensure broken center symmetries



Backup: More on dynamical susy breaking

Spontaneous susy breaking means $\langle 0 | H | 0 \rangle > 0$ or equivalently $\langle \mathcal{QO} \rangle \neq 0$

Twisted superQCD auxiliary field e.o.m. \leftrightarrow Fayet–Iliopoulos D-term potential

$$\boldsymbol{d} = \overline{\mathcal{D}}_{\boldsymbol{a}} \mathcal{U}_{\boldsymbol{a}} + \sum_{i=1}^{F} \phi_{i} \overline{\phi}_{i} - \boldsymbol{r} \mathbb{I}_{\boldsymbol{N}} \qquad \longleftrightarrow \qquad \mathsf{Tr} \left[\left(\sum_{i} \phi_{i} \overline{\phi}_{i} - \boldsymbol{r} \mathbb{I}_{\boldsymbol{N}} \right)^{2} \right] \in \boldsymbol{H}$$

Have *F* scalar vevs to zero out *N* diagonal elements $\longrightarrow N > F$ suggests susy breaking, $\langle 0 | H | 0 \rangle > 0 \iff \langle Q \eta \rangle = \langle d \rangle \neq 0$

Backup: More on $\mathcal{N} = 4$ SYM sign problem in 3+1 dimensions





Fix 4⁴ volume Fluctuations increase with coupling Signal-to-noise becomes obstruction for $\lambda_{\text{lat}} \gtrsim 4$