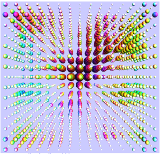
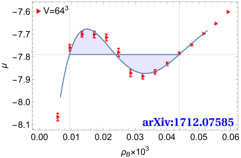
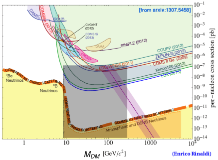
Physics Out Of The Box
— The Impact of Lattice Field Theory —

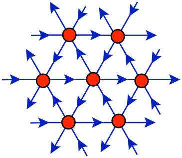


(Image credit: Claudio Rebbi)





(Enrico Rinaldi)



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University of Liverpool, 5 July 2018

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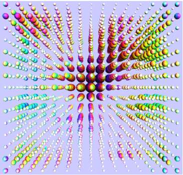
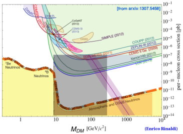
Overview

Lattice field theory is a broadly applicable tool
to study strongly coupled quantum field theories



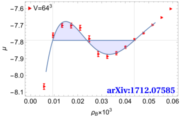
(Image credit: Claudio Rebbi)

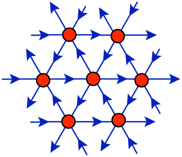
A high-level summary of lattice field theory



Applications — recent results & future plans

- Composite dark matter
- Dense nuclear matter
- Supersymmetry and holographic duality
- Composite Higgs boson



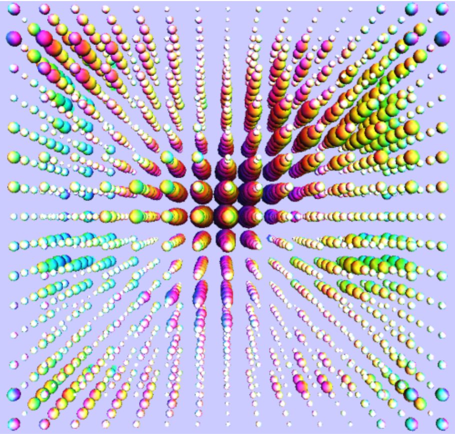


Outlook

Lattice field theory in a nutshell: QFT

Quantum **F**ield **T**heory = quantum mechanics + special relativity

Picture relativistic quantum fields filling four-dimensional space-time



(Image credit: Claudio Rebbi)

(Space and time on equal footing)

The QFT / StatMech Correspondence

Generating functional (Feynman path integral)	Partition function
$\mathcal{Z} = \int \mathcal{D}\Phi \;\; e^{-S[\Phi] \; / \; \hbar}$	$\int \mathcal{D}q \, \mathcal{D}p \;\; e^{-H(q,p) \; / \; k_B T}$
Action $S[\Phi] = \int d^4x \; \mathcal{L}[\Phi(x)]$	Hamiltonian H
$\hbar \;\longleftrightarrow\;$ quantum fluctuations (natural units: $\hbar = 1$)	$k_B T \;\longleftrightarrow\;$ thermal fluctuations

Lattice field theory in a nutshell: Discretization

Formally $\quad \langle\mathcal{O}\rangle = \dfrac{1}{\mathcal{Z}}\displaystyle\int \mathcal{D}\Phi\ \ \mathcal{O}(\Phi)\ \ e^{-S[\Phi]}$

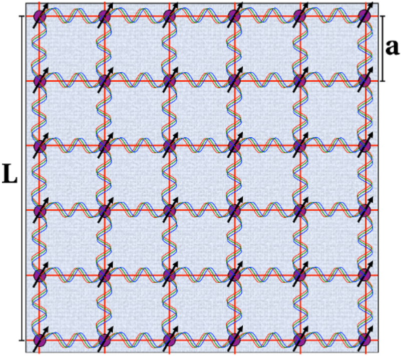
…but infinite-dimensional integrals in general intractable

Lattice field theory in a nutshell: Discretization

Formally $$\langle \mathcal{O} \rangle = \frac{1}{\mathcal{Z}} \int \mathcal{D}\Phi \;\; \mathcal{O}(\Phi) \;\; e^{-S[\Phi]}$$

...but infinite-dimensional integrals in general intractable

Formulate theory in finite, discrete space-time $\longrightarrow$ **the lattice**



P. Vranas LLNL

Spacing between lattice sites ("a")

$\longrightarrow$ UV cutoff scale $1/a$

Removing cutoff: $a \to 0 \quad (L/a \to \infty)$

Hypercubic $\longrightarrow$ automatic symmetries

Numerical lattice field theory calculations

High-performance computing
$\longrightarrow$ evaluate up to
$\sim$billion-dimensional integrals

Importance sampling Monte Carlo

Algorithms sample field configurations with probability $\frac{1}{\mathcal{Z}} e^{-S[\Phi]}$

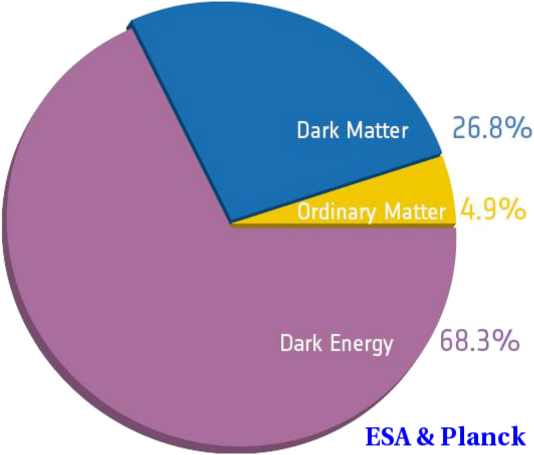
$$\langle \mathcal{O} \rangle = \frac{1}{\mathcal{Z}} \int \mathcal{D}\Phi \;\; \mathcal{O}(\Phi) \;\; e^{-S[\Phi]}$$

$$\longrightarrow \quad \frac{1}{N} \sum_{i=1}^{N} \mathcal{O}(\Phi_i) \;\; \text{with statistical uncertainty} \;\; \propto \frac{1}{\sqrt{N}}$$

Application: Dark matter

Abundant **gravitational** evidence spanning many scales

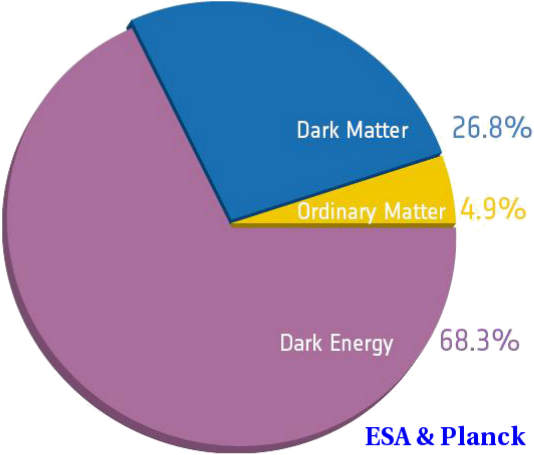
$\Longrightarrow$ most matter in the universe is **dark** — details **unknown**



Non-gravitational dark matter interactions

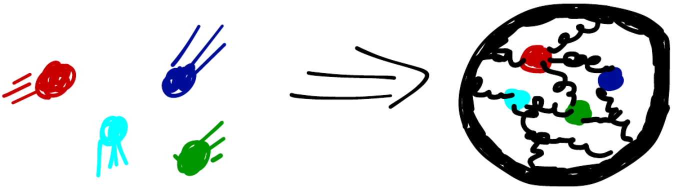
$$\frac{\Omega_{\text{dark}}}{\Omega_{\text{ordinary}}} \approx 5$$

$\ldots$not 10^5 or 10^{-5}



$\longrightarrow$ Non-gravitational interactions with known particles,
not yet detected by ongoing experiments

Composite dark matter



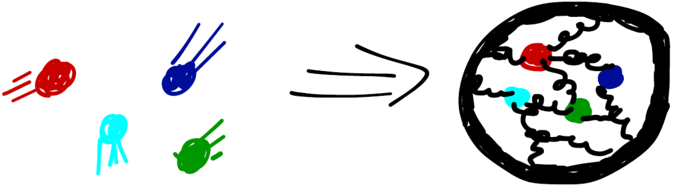
Early universe

Deconfined charged fermions $\longrightarrow$ non-gravitational interactions

Present day

Confined neutral 'dark baryons' $\longrightarrow$ no experimental detections

Composite dark matter



Direct detection signals

Depend on **form factors** of composite dark matter

(magnetic moment, charge radius, polarizability)

Need lattice calculations for quantitative predictions

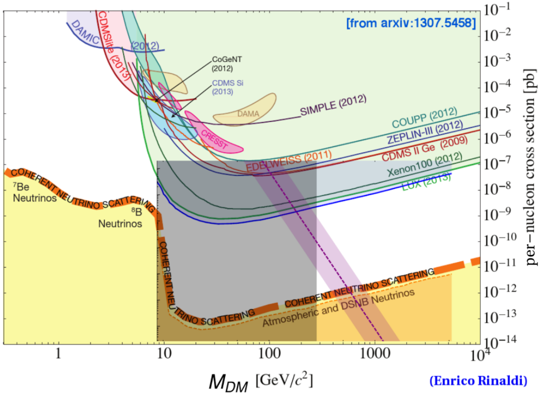
Recent result: Lower bound for composite dark matter

Stealth Dark Matter $\longrightarrow$ electric polarizability is leading interaction

Lattice calculation $\longrightarrow$ lower bound on the direct detection rate

Results specific
to Xenon detectors

Uncertainties dominated
by Xenon matrix element

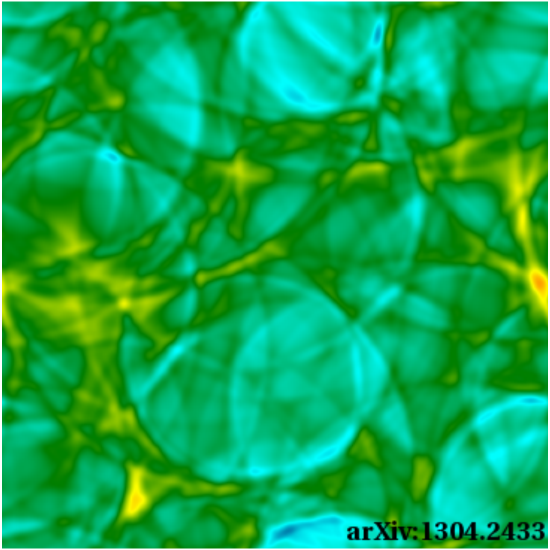


(Enrico Rinaldi)

Shaded region is complementary constraint from particle colliders

Future plan: Gravitational wave signals

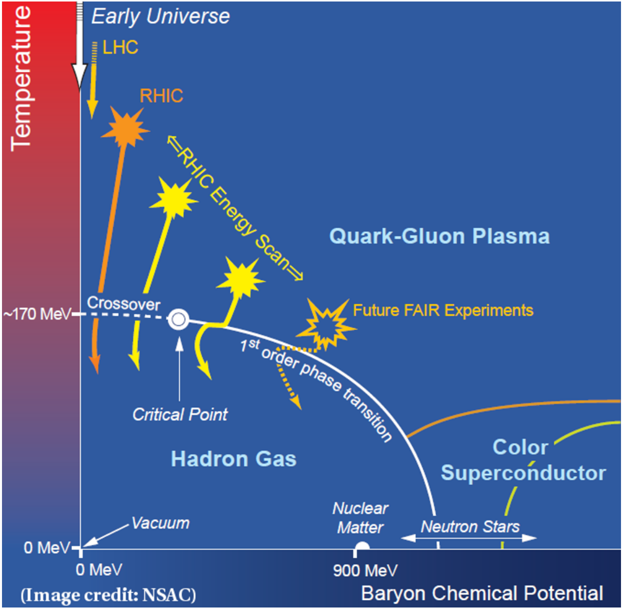
Gravitational wave observatories opening new window on cosmology



First-order dark transition
$\longrightarrow$ colliding bubbles
$\longrightarrow$ gravitational waves

Lattice calculations predict
properties of transition
& resulting signals

Application: Dense nuclear matter



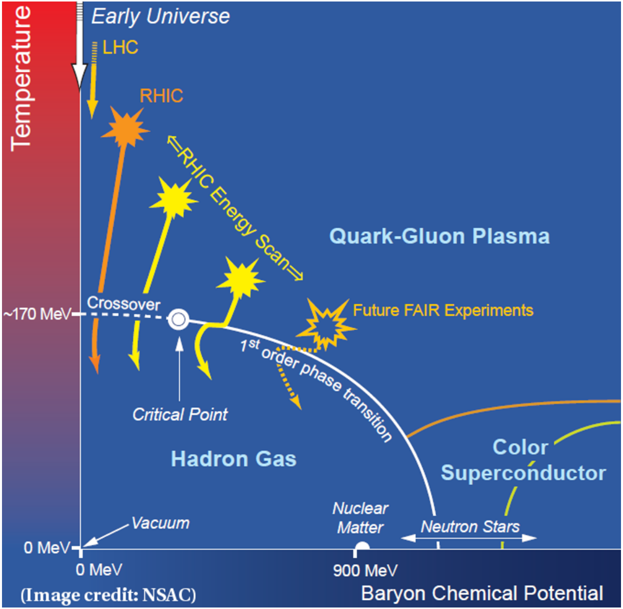
Strong nuclear force

$\updownarrow$

Dynamics of quarks and gluons (QCD)

Many features of phase diagram not well known

Application: Dense nuclear matter



Chemical potential
$\longrightarrow$ complex $S[\Phi]$

Sign problem

Complex $\frac{1}{\mathcal{Z}} e^{-S[\Phi_i]}$

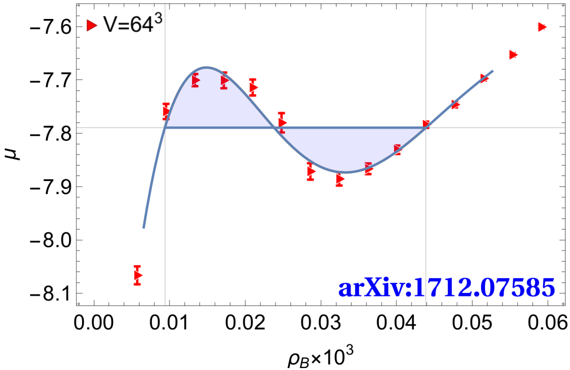
is not probability!

Brute-force approach
$\longrightarrow$ exponential costs

Recent result: Canonical cluster solution arXiv:1712.07585

Canonical formulation of simplified (Z_3 spin) model solves sign problem

Clusters correspond to 'baryons' $\longrightarrow$ exponential signal enhancement



Cost of adding baryon
$\longrightarrow$ chemical potential μ

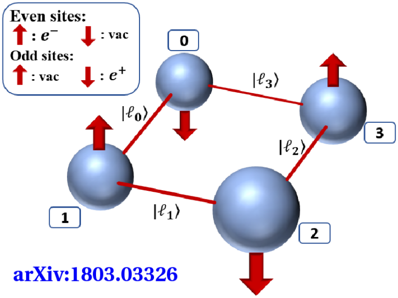
Phase transition
at critical $\mu_c \approx -7.8$

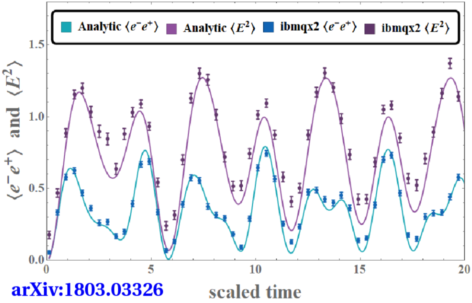
Work in progress to use for less-simplified systems

Future plan: Quantum computing

Nature can 'compute' how dense nuclear matter behaves

$\Longrightarrow$ we should use the same (quantum) methods





Algorithms and apparatus are being designed and tested

$\longrightarrow$ potential revolution in near future

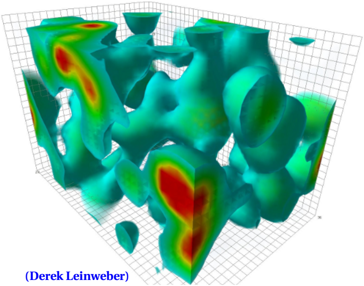
Application: Lattice supersymmetry

Lattice field theory promises first-principles predictions
for strongly coupled supersymmetric QFTs

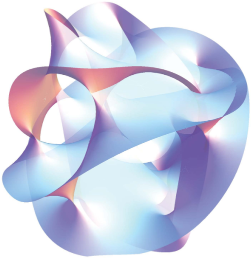
Many directions to be explored

BSM

QFT



Holography



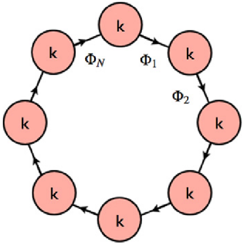
A brief history of lattice supersymmetry

Supersymmetries are "square roots" of infinitesimal translations

$\longrightarrow$ do not exist in discrete space-time

Solution: Reformulate theory to preserve subset of supersymmetries

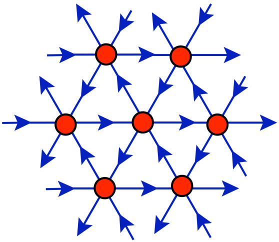
$\Longrightarrow$ recover others in continuum limit



Review:

Catterall, Kaplan & Ünsal

arXiv:0903.4881



Testing holographic duality

arXiv:1709.07025

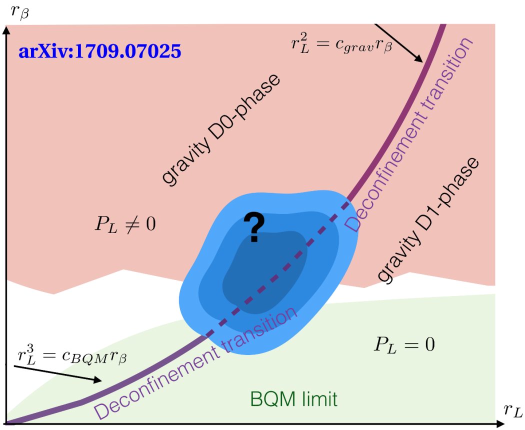
Holographic duality conjecture

Thermodynamics of supersymmetric QFT $\longleftrightarrow$ stringy black holes

Details: $\mathcal{N} = (8, 8)$ SYM
(Supersymmetric Yang–Mills)
involves gluon + superpartners

Gauge group SU(N),
$6 \leq N \leq 16$ 'colours'

Lives on $2d$ torus, size $r_\beta \times r_L$
$\longrightarrow$ temperature $t = 1/r_\beta$



Testing holographic duality

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Holographic duality conjecture

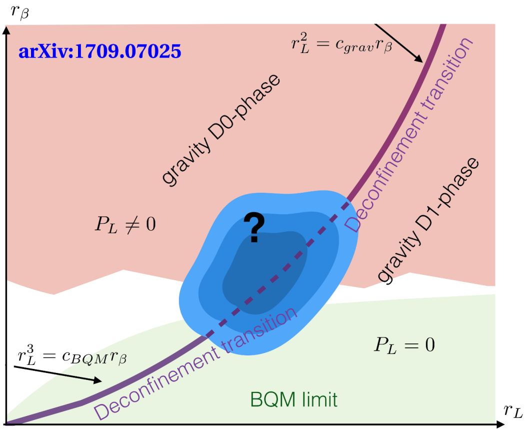
Thermodynamics of supersymmetric QFT $\longleftrightarrow$ stringy black holes

For decreasing r_L
at low $t = 1/r_\beta$ and large N

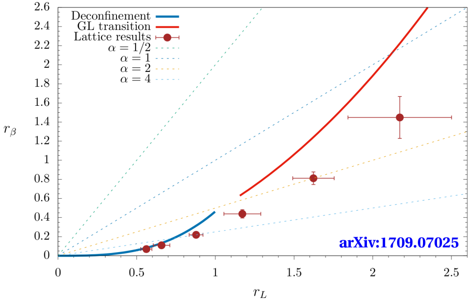
homogeneous black string (D1)
$\longrightarrow$ localized black hole (D0)

$\updownarrow$

"spatial deconfinement"
signalled by Wilson line

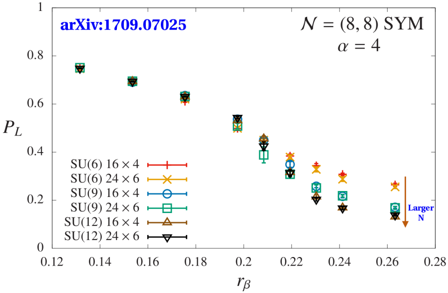


Recent result: 2*d* SYM lattice phase diagram



Consistent with holography
at low temperatures

Example spatial deconfinement
transition in Wilson line



Future plan: Supersymmetric QCD

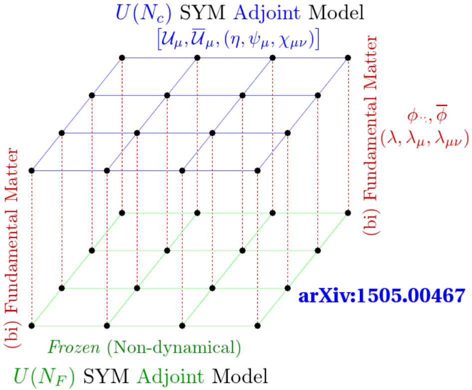
Supersymmetric Yang–Mills involves only analog of gluons

Adding analogs of quarks
$\longrightarrow$ other dualities, dynamical supersymmetry breaking and more

Possible in $d < 4$
through 'quiver construction'

Change SYM gauge group
between two lattice 'slices'
and decouple one

Numerical investigations
only just beginning



Application: Composite Higgs boson

Large Hadron Collider priority

Determine fundamental nature of the Higgs boson

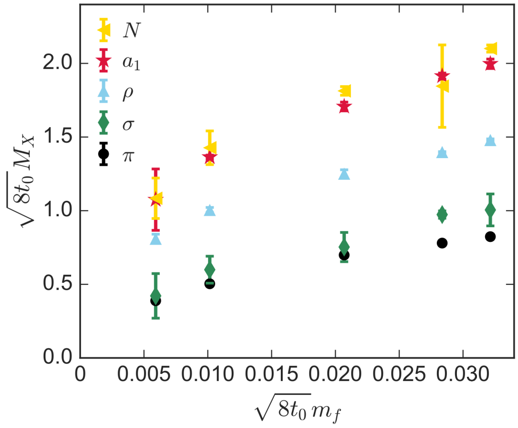
Composite Higgs boson could arise from **new strong dynamics**

Protects against extreme sensitivity to quantum effects, being investigated via lattice field theory

Recent result: Light composite Higgs

arXiv:180?.?????

Lattice studies of QFTs with many light 'quarks'
find composite Higgs boson much lighter than in QCD



Larger hierarchy between
Higgs and resonances
as demanded by LHC

Need to extrapolate
fermion mass $m_f \to 0$
via effective field theory

Future plan: Interactions of light Higgs

There are many candidate effective field theories

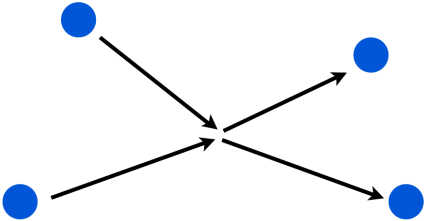
including pions and a light Higgs

Lattice computations of more observables will test

which are consistent with the non-perturbative dynamics

Now studying interactions of light Higgs and pions

starting with $2 \to 2$ elastic scattering



Outlook: An exciting time for lattice field theory

Lattice field theory is a broadly applicable tool
to study strongly coupled quantum field theories

- Predicting experimental signals of composite dark matter
- Solving the sign problem of simplified dense nuclear matter
- Testing holographic dualities of supersymmetric QFTs
- Exploring features of composite Higgs boson

Thank you!

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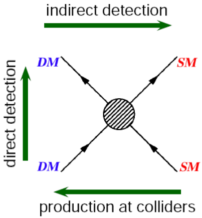
Backup: Thermal freeze-out for relic density

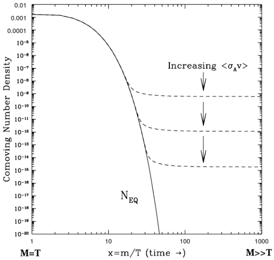
Requires coupling between ordinary matter and dark matter



$T \gtrsim M_{DM}$: DM $\longleftrightarrow$ SM
Thermal equilibrium

$T \lesssim M_{DM}$: DM $\longrightarrow$ SM
Rapid depletion of Ω_{DM}

Hubble expansion
$\longrightarrow$ dilution $\longrightarrow$ freeze-out



$2 \to 2$ scattering relates coupling and mass as $200\alpha \sim \dfrac{M_{DM}}{100\text{ GeV}}$

Strong $\alpha \sim 16$ $\longrightarrow$ 'natural' mass scale $M_{DM} \sim 300$ TeV

Smaller $M_{DM} \gtrsim 1$ TeV possible from $2 \to n$ scattering or asymmetry

Backup: Two roads to natural asymmetric dark matter

Idea: Dark matter relic density related to baryon asymmetry

$$\Omega_D \approx 5\Omega_B$$
$$\Longrightarrow M_D n_D \approx 5 M_B n_B$$

$n_D \sim n_B \quad \Longrightarrow \quad M_D \sim 5M_B \approx 5$ GeV

High-dim. interactions relate baryon# and DM# violation

$M_D \gg M_B \quad \Longrightarrow \quad n_B \gg n_D \sim \exp\left[-M_D/T_s\right] \qquad T_s \sim 200$ GeV

EW sphaleron processes above T_s distribute asymmetries

Both require coupling between ordinary matter and dark matter

Backup: Composite dark matter interactions

Photon exchange via electromagnetic form factors

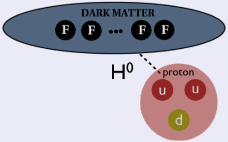
Interactions suppressed by powers of confinement scale $\Lambda \sim M_{DM}$

- **Dimension 5:** Magnetic moment $\longrightarrow$ $\left(\overline{\psi}\sigma^{\mu\nu}\psi\right) F_{\mu\nu}/\Lambda$
- **Dimension 6:** Charge radius $\longrightarrow$ $\left(\overline{\psi}\gamma^{\nu}\psi\right) \partial^{\mu} F_{\mu\nu}/\Lambda^2$
- **Dimension 7:** Polarizability $\longrightarrow$ $\left(\overline{\psi}\psi\right) F^{\mu\nu} F_{\mu\nu}/\Lambda^3$

Higgs exchange via scalar form factors

Higgs couples through $\langle B|m_{\psi}\overline{\psi}\psi|B\rangle$ (σ terms)

Needed for Big Bang nucleosynthesis
($\longrightarrow$ rapid charged 'meson' decay)



Non-perturbative form factors $\Longrightarrow$ lattice calculations

Backup:
Lattice Strong Dynamics Collaboration

Argonne	Xiao-Yong Jin, James Osborn
Bern	DS
Boston	Rich Brower, Claudio Rebbi, Evan Weinberg
Colorado	Anna Hasenfratz, Ethan Neil, Oliver Witzel
UC Davis	Joseph Kiskis
Livermore	Pavlos Vranas
Oregon	Graham Kribs
RBRC	Enrico Rinaldi
Yale	Thomas Appelquist, George Fleming, Andrew Gasbarro

Exploring the range of possible phenomena
in strongly coupled field theories

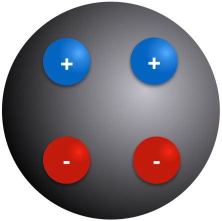
Backup: Stealth dark matter EM form factors

Lightest SU(4) dark baryon

Scalar $\longrightarrow$ no magnetic moment

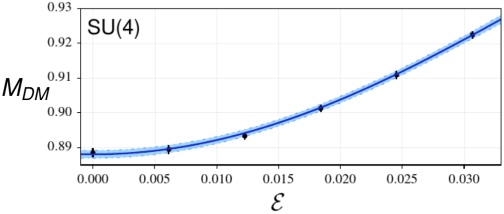
+/- charge symmetry $\longrightarrow$ no charge radius

Small α $\longrightarrow$ Higgs exchange suppressed



Polarizability $\longrightarrow$ lower bound on direct-detection cross section

Compute on lattice as dependence of M_{DM} on external field $\mathcal{E}$



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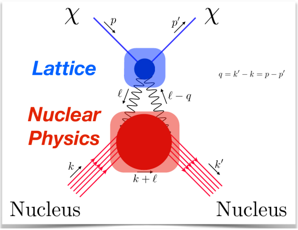
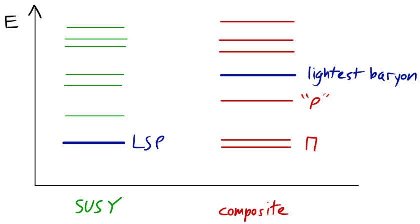


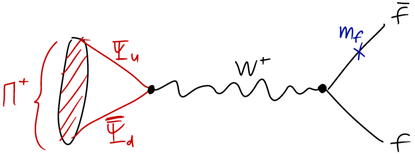
Backup: Stealth dark matter at colliders

Spectrum significantly different from typical susy

$\longrightarrow$ Very little missing E_T

Main constraints from much lighter **charged** "Π" states





Rapid Π decays, $\Gamma \propto m_f^2$

Best current constraints recast LEP stau searches

LHC can search for $t\overline{b} + \overline{t}b$ from $\Pi^+\Pi^-$ Drell–Yan

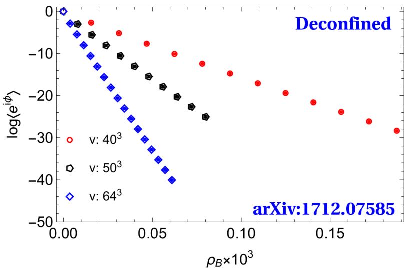
Backup: Phase reweighting

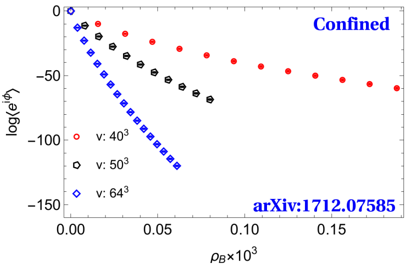
Idea: Move complex phase from $e^{-S} = |e^{-S}|e^{i\alpha}$ into observable

$$\langle\mathcal{O}\rangle = \frac{\int\mathcal{D}\Phi\ \ \mathcal{O}(\Phi)\ \ e^{-S[\Phi]}}{\int\mathcal{D}\Phi\ \ e^{-S[\Phi]}} = \frac{\int\mathcal{D}\Phi\ \ \mathcal{O}e^{i\alpha}\ \ |e^{-S}|}{\int\mathcal{D}\Phi\ \ e^{i\alpha}\ \ |e^{-S}|} = \frac{\left\langle\mathcal{O}e^{i\alpha}\right\rangle_{||}}{\left\langle e^{i\alpha}\right\rangle_{||}}$$

Issue: $\left\langle e^{i\alpha}\right\rangle_{||} = \mathcal{Z}/\mathcal{Z}_{||} = \exp\left[-V(f - f_{||})/T\right]$ and $f \geq f_{||}$

Sign problem $\longleftrightarrow$ exponential signal-to-noise problem

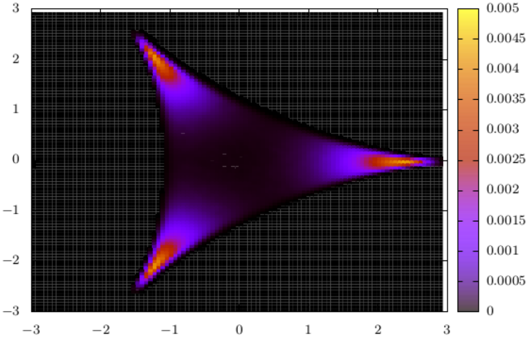




Backup: Canonical solution of the sign problem

Consider system with fixed number of (3-quark) baryons

[arXiv:1712.07585]



Step 1: Simplify system

Quarks too heavy to move

$\longrightarrow$ Effective d.o.f.

$$\phi(x) \in \left\{1, e^{\pm 2\pi/3}\right\}$$

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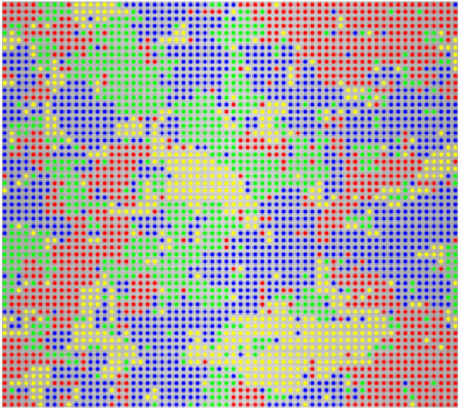
Consider system with fixed number of (3-quark) baryons

[arXiv:1712.07585]



(Image credit: SonEnvir)

Step 2: Divide space-time into clusters with constant ϕ

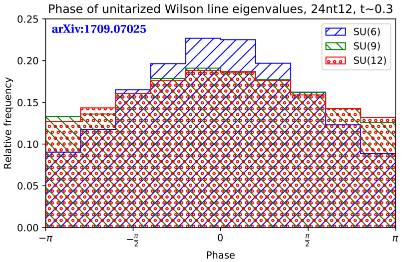
Complex contribution only if
1 or 2 extra quarks in cluster

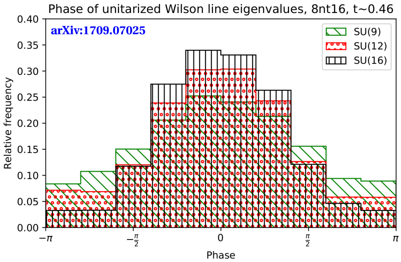
Sum to zero in path integral
$\longrightarrow$ sign problem solved

Benefit from physical intuition:
no free quarks in nature

Backup: $\mathcal{N} = (8,8)$ SYM Wilson line eigenvalues

Check 'spatial deconfinement' through histograms of Wilson line eigenvalue phases





Left: $\alpha = 2$ distributions more extended as N increases
$\longrightarrow$ dual gravity describes homogeneous black string (D1 phase)

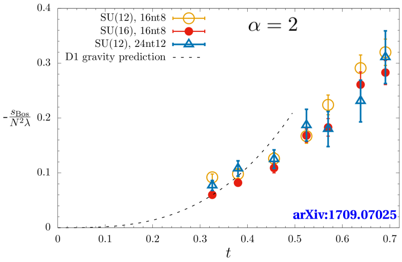
Right: $\alpha = 1/2$ distributions more compact as N increases
$\longrightarrow$ dual gravity describes localized black hole (D0 phase)

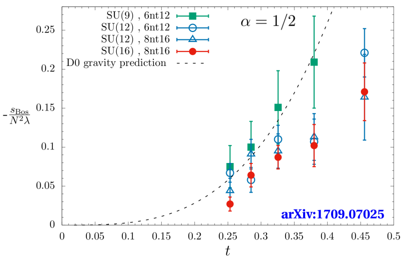
Backup: Dual black hole thermodynamics

Holography relates black holes' energy to action of SYM field theory

$\propto t^3$ for large-r_L D1 phase $\propto t^{3.2}$ for small-r_L D0 phase

Lattice results consistent with holography for sufficiently low $t \lesssim 0.4$



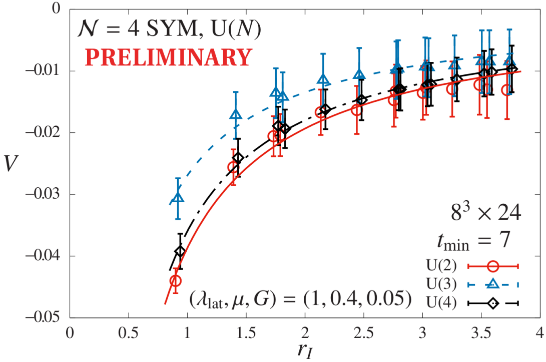


Need larger $N > 16$ to avoid instabilities at lower temperatures

Backup: Static potential is Coulombic at all λ

Fits to confining $V(r) = A - C/r + \sigma r \longrightarrow$ vanishing string tension σ

$\Longrightarrow$ Fit to just $V(r) = A - C/r$ to extract Coulomb coefficient $C(\lambda)$

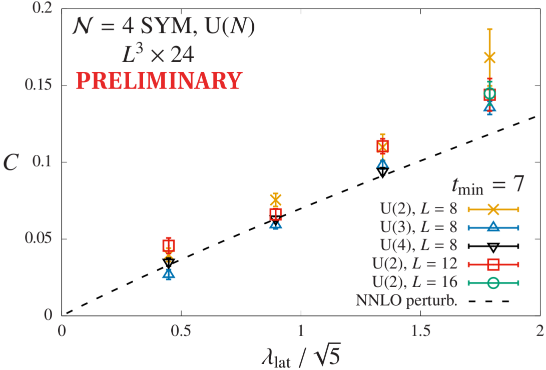


Recent progress: Incorporating tree-level improvement into analysis

Backup: Coupling dependence of Coulomb coefficient

Continuum perturbation theory predicts $C(\lambda) = \lambda/(4\pi) + \mathcal{O}(\lambda^2)$

Holography predicts $C(\lambda) \propto \sqrt{\lambda}$ for $N \to \infty$ and $\lambda \to \infty$ with $\lambda \ll N$



Surprisingly good agreement with perturbation theory for $\lambda_{\mathrm{lat}} \leq 4$

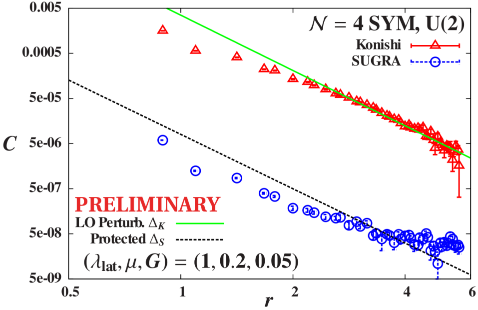
Backup: Konishi operator on the lattice

Lattice scalars $\varphi(n)$ from polar decomposition of complexified links

$$\mathcal{U}_a(n) \longrightarrow e^{\varphi_a(n)} U_a(n) \qquad\qquad \mathcal{O}_K^{\text{lat}}(n) = \sum_a \text{Tr}\left[\varphi_a(n)\varphi_a(n)\right] - \text{vev}$$

$$C_K(r) \equiv \mathcal{O}_K(x+r)\mathcal{O}_K(x) \propto r^{-2\Delta_K}$$

'SUGRA' is 20′ $\mathcal{O}_S \sim \varphi_{\{a}\varphi_{b\}}$
with protected $\Delta_S = 2$



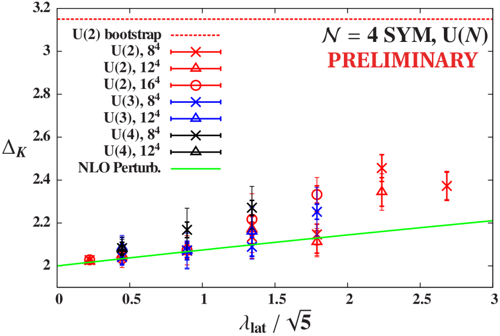
To handle systemics, comparing
direct power-law decays vs. finite-size scaling vs. Monte Carlo RG

Backup: Preliminary Δ_K results from Monte Carlo RG

MCRG stability matrix
includes both $\mathcal{O}_K^{\text{lat}}$ and $\mathcal{O}_S^{\text{lat}}$

Impose protected $\Delta_S = 2$

Systematic uncertainties from
different amounts of smearing



Complication: Twisted $SO(4)_{tw}$ involves only $SO(4)_R \subset SO(6)_R$

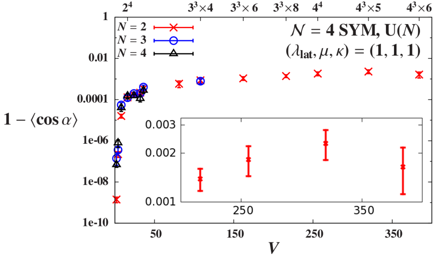
$\Longrightarrow$ Lattice Konishi operator mixes with $SO(4)_R$-singlet part
of the $SO(6)_R$-nonsinglet SUGRA operator

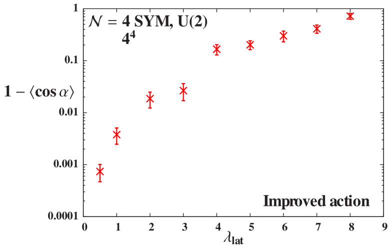
Current work: Variational analyses to disentangle operators

Backup: $\mathcal{N} = 4$ SYM phase vs. volume and coupling

Left: $1 - \langle\cos(\alpha)\rangle_{pq} \ll 1$ independent of volume and N at $\lambda_{\text{lat}} = 1$

Right: New 4^4 results at $4 \le \lambda_{\text{lat}} \le 8$ show much larger fluctuations





Next step: Analyze more volumes, N, λ_{lat}

Extremely expensive computation despite new parallel algorithm:

$\mathcal{O}(n^3)$ scaling $\longrightarrow$ ~50 hours for single U(2) 4^4 measurement

Backup: $\mathcal{N} = 4$ SYM sign problem puzzles

Periodic temporal boundary conditions for the fermions

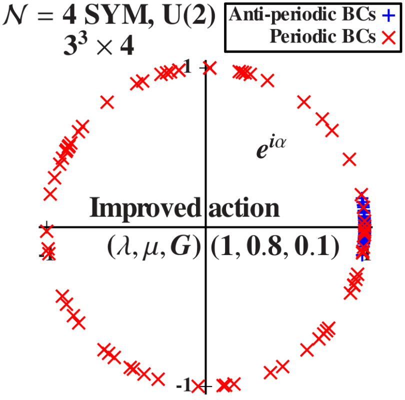
$\longrightarrow$ obvious sign problem, $\left\langle e^{i\alpha}\right\rangle_{pq} \approx 0$

Anti-periodic BCs $\longrightarrow$ $e^{i\alpha} \approx 1$, phase reweighting negligible

Why such sensitivity to the BCs?

Other $\langle\mathcal{O}\rangle_{pq}$ are nearly identical for these two ensembles

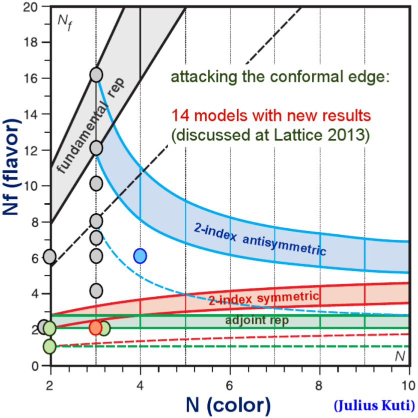
Why doesn't sign problem affect other observables?



Backup: Strategy for composite Higgs studies

Systematically depart from familiar ground of lattice QCD
($N = 3$ with $N_F = 2$ light flavors in fundamental rep)

Explore the range of possible phenomena in strongly coupled theories



Add more light flavors
$\longrightarrow N_F = 8$ fundamental

Enlarge fermion rep
$\longrightarrow N_F = 2$ two-index symmetric

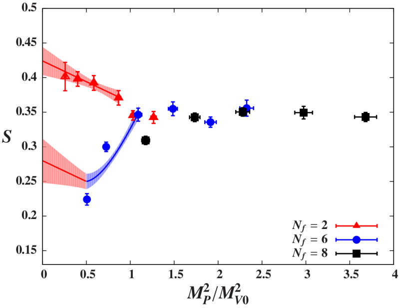
Explore $N = 2$ and 4
$\longrightarrow$ (pseudo)real reps for cosets
SU(n)/Sp(n) and SU(n)/SO(n)

Backup: S parameter on the lattice

$$\mathcal{L}_\chi \supset \frac{\alpha_1}{2} g_1 g_2 B_{\mu\nu} \mathrm{Tr}\left[U \tau_3 U^\dagger W^{\mu\nu}\right] \longrightarrow$$

Lattice vacuum polarization calculation provides $S = -16\pi^2\alpha_1$

Non-zero masses and chiral extrapolation needed
to avoid sensitivity to finite lattice volume



$S = 0.42(2)$ for $N_F = 2$
matches scaled-up QCD

Larger N_F $\longrightarrow$ significant reduction

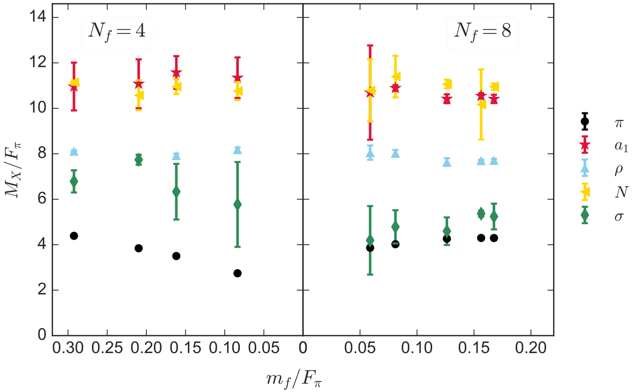
Extrapolation to correct zero-mass limit
becomes more challenging

Backup: More on composite Higgs spectrum

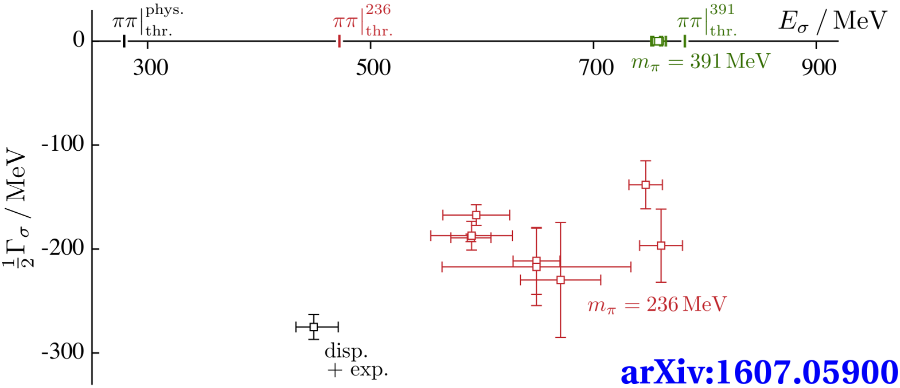
$N_F = 8$ Higgs much lighter than in QCD-like systems

Clear qualitative difference vs. QCD-like $N_F = 4$

Hierarchy between Higgs and resonances
increasing as fermion mass $m_F \to 0$



Backup: Composite Higgs in QCD spectrum



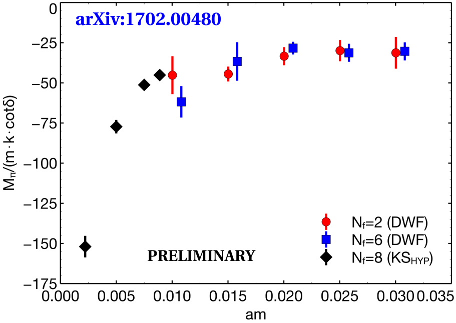
In lattice QCD, scalar mass $M_S \gtrsim 2M_P$

$\longrightarrow$ significant mixing with two-pion scattering states

Backup: Initial $2 \to 2$ elastic scattering results

Simplest case: Analog of QCD $I = 2$ $\pi\pi$ scattering
(no fermion-line-disconnected diagrams)

Simplest observable: Scattering length $a_{PP} \approx 1/(k \cot \delta)$



Chiral perturbation theory predicts $M_P a_{PP}/m \sim$ constant,
clearly not good description of $N_F = 8$ results