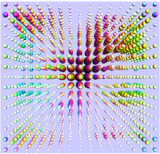
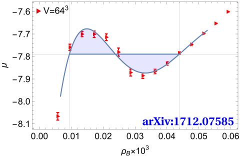
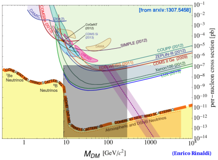
Physics Out Of The Box

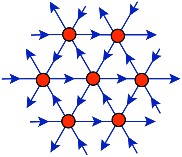
— Frontiers of Lattice Field Theory —



(Image credit: Claudio Rebbi)







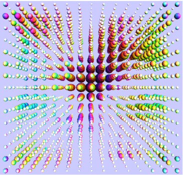
David Schaich (University of Bern)

Florida International University, 9 March 2018

schaich@itp.unibe.ch www.davidschaich.net

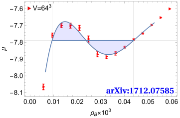
Overview

Lattice field theory is a broadly applicable tool
to study strongly coupled systems



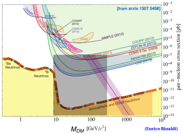
(Image credit: Claudio Rebbi)

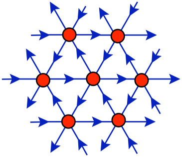
A high-level summary of lattice field theory



Applications — recent results & future plans

- Dense nuclear matter
- Composite dark matter
- Composite Higgs bosons
- Supersymmetry and holographic duality





Outlook

Lattice field theory in a nutshell: QFT

Lattice field theory is a broadly applicable tool

to study strongly coupled systems

Lattice field theory in a nutshell: QFT

Lattice field theory is a broadly applicable tool
to study strongly coupled quantum field theories (QFTs)

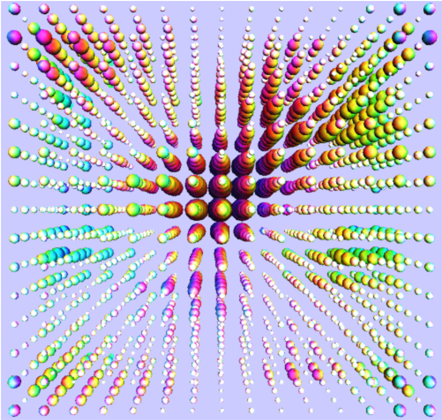
Lattice field theory in a nutshell: QFT

Lattice field theory is a broadly applicable tool
to study strongly coupled quantum field theories (QFTs)

QFT = quantum mechanics
+ special relativity

Picture relativistic quantum fields filling four-dimensional space-time

(Space and time on equal footing)



(Image credit: Claudio Rebbi)

The QFT / StatMech Correspondence

Generating functional (Feynman path integral)	Partition function
$\mathcal{Z} = \int \mathcal{D}\Phi \;\; e^{-S[\Phi]\;/\;\hbar}$	$\int \mathcal{D}q\, \mathcal{D}p \;\; e^{-H(q,p)\;/\;k_B T}$
Action $S[\Phi] = \int d^4x\, \mathcal{L}[\Phi(x)]$	Hamiltonian H
$\hbar \longleftrightarrow$ quantum fluctuations (natural units: $\hbar = 1$)	$k_B T \longleftrightarrow$ thermal fluctuations
$e^{-S[\Phi]}$	Boltzmann factor

Lattice field theory in a nutshell: Discretization

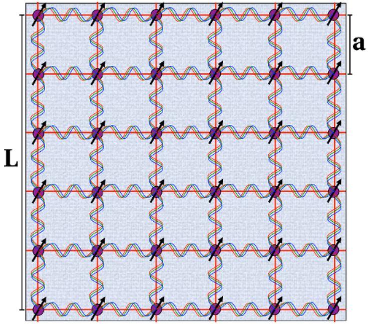
Formally $\quad \langle\mathcal{O}\rangle = \frac{1}{\mathcal{Z}}\int\mathcal{D}\Phi\;\; \mathcal{O}(\Phi)\;\; e^{-S[\Phi]} = \frac{\int\mathcal{D}\Phi\;\; \mathcal{O}(\Phi)\;\; e^{-S[\Phi]}}{\int\mathcal{D}\Phi\;\; e^{-S[\Phi]}}$

Lattice field theory in a nutshell: Discretization

Formally $\quad \langle \mathcal{O} \rangle = \frac{1}{\mathcal{Z}} \int \mathcal{D}\Phi \ \ \mathcal{O}(\Phi) \ \ e^{-S[\Phi]} = \frac{\int \mathcal{D}\Phi \ \ \mathcal{O}(\Phi) \ \ e^{-S[\Phi]}}{\int \mathcal{D}\Phi \ \ e^{-S[\Phi]}}$

Infinite-dimensional integrals in general intractable

Formulate theory in finite, discrete space-time $\longrightarrow$ **the lattice**



P. Vranas LLNL

Spacing between lattice sites ("a")
$\longrightarrow$ UV cutoff scale $1/a$

Removing cutoff: $a \to 0$ (with $L/a \to \infty$)

Hypercubic $\longrightarrow$ automatic symmetries

Numerical lattice field theory calculations

High-performance computing
$\longrightarrow$ evaluate up to
$\sim$billion-dimensional integrals

Importance sampling Monte Carlo

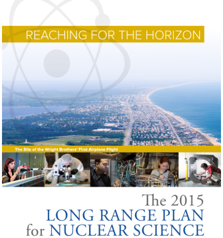
Algorithms sample field configurations with probability $\dfrac{1}{\mathcal{Z}} e^{-S[\Phi]}$

$$\langle \mathcal{O} \rangle = \frac{1}{\mathcal{Z}} \int \mathcal{D}\Phi \;\; \mathcal{O}(\Phi) \;\; e^{-S[\Phi]}$$

$$\longrightarrow \quad \frac{1}{N} \sum_{i=1}^{N} \mathcal{O}(\Phi_i) \;\; \text{with statistical uncertainty} \;\; \propto \sqrt{\frac{1}{N}}$$

Frontiers of lattice field theory

Lattice field theory is a broadly applicable framework
important in nuclear, particle and condensed matter physics

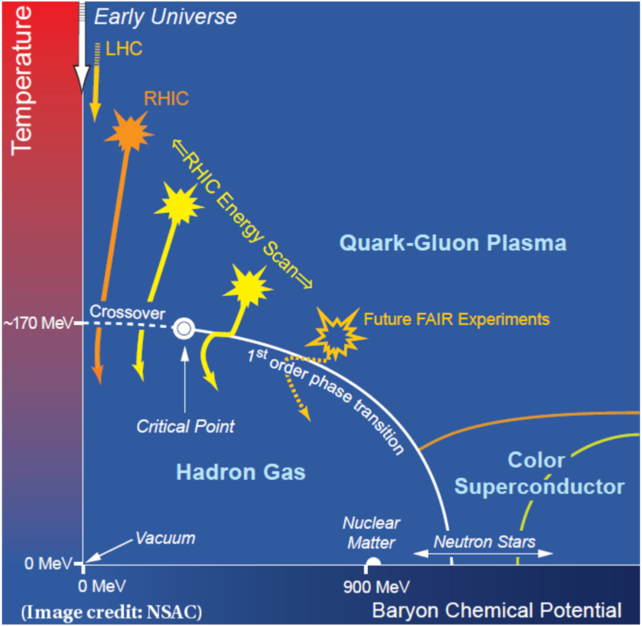


Executive Summary

- Use the Higgs boson as a new tool for discovery
- Pursue the physics associated with neutrino mass
- Identify the new physics of dark matter
- Understand cosmic acceleration: dark energy and inflation
- Explore the unknown: new particles, interactions, and physical principles.

Frontiers $\longrightarrow$ conceptual and practical challenges

Application: Dense nuclear matter

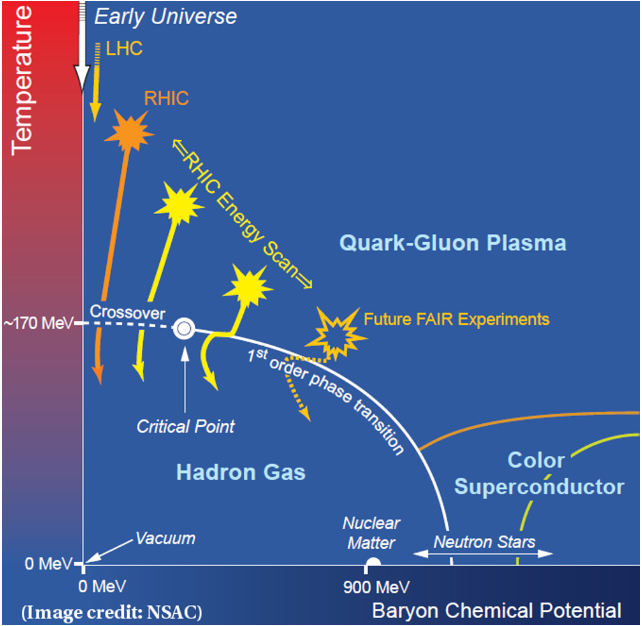


Strong nuclear force

$\updownarrow$

Dynamics of quarks and gluons (QCD)

Application: Dense nuclear matter



Chemical potential
$\longrightarrow$ complex $S[\Phi]$

"Sign problem"

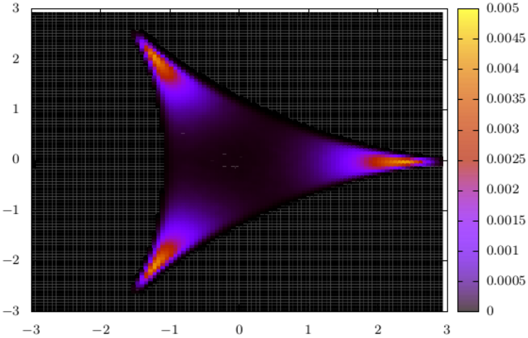
Complex $\frac{1}{\mathcal{Z}} e^{-S[\Phi_i]}$

can't be probability!

Can be solved
in some regimes

A canonical solution of the sign problem arXiv:1712.07585

Consider system with fixed number of (3-quark) baryons



Step 1: Simplify system

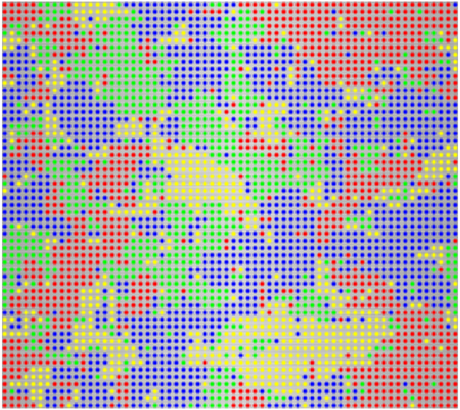
Quarks too heavy to move

$\longrightarrow$ Effective d.o.f.

$$\phi(x) \in \left\{1, e^{\pm 2\pi/3}\right\}$$

A canonical solution of the sign problem arXiv:1712.07585

Consider system with fixed number of (3-quark) baryons



(Image credit: SonEnvir)

Step 2: Divide space-time into clusters with constant ϕ

Complex contribution only if
1 or 2 extra quarks in cluster

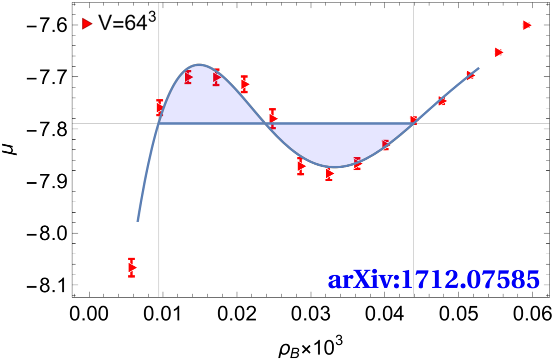
Sum to zero in path integral
$\longrightarrow$ sign problem solved

Benefit from physical intuition:
no free quarks in nature

Recent result: Critical chemical potential arXiv:1712.07585

Cost of adding baryon to system

$\longrightarrow$ chemical potential as function of baryon density



Maxwell construction

Deconfinement transition

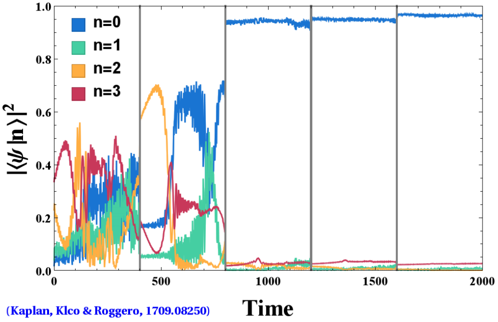
at critical $\mu_c \approx -7.8$

Next step: Apply canonical cluster intuition to less-simplified systems

Future plan: Quantum computing

Nature has no problem ‘computing’ how dense nuclear matter behaves

Can we employ the same (quantum) approach?



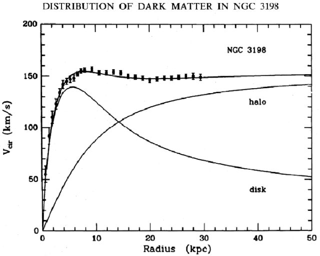
(Kaplan, Klco & Roggero, 1709.08250)

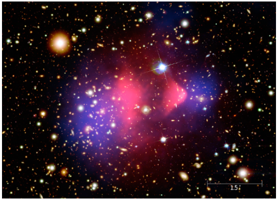
Quantum ‘spectral combing’ finds ground state of simple spin system

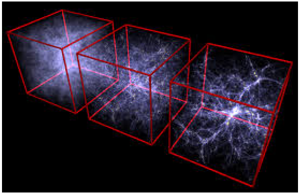
Algorithms and devices are being designed and tested

You will hear more about this in the future

Application: Dark matter

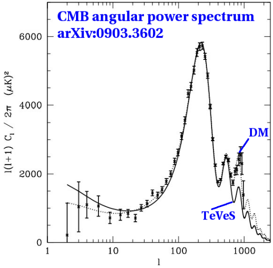






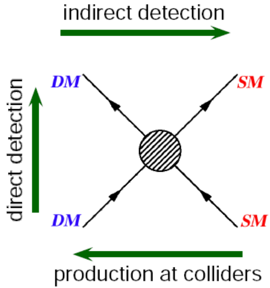
Consistent gravitational evidence spanning many scales

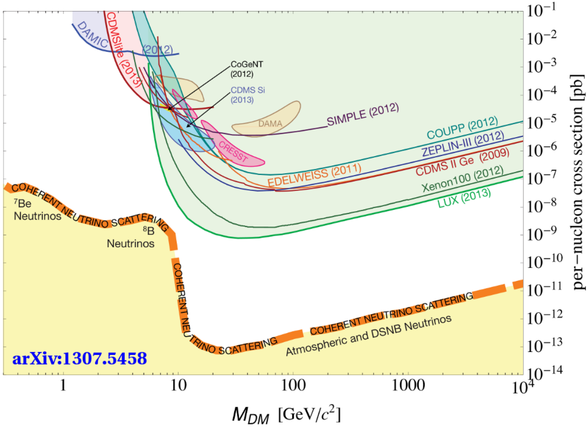
- Rotation curves of galaxies & clusters
- Gravitational lensing
- Large-scale structure formation
- Cosmic microwave background



Non-gravitational searches for dark matter

Colliders; cosmic rays; large underground detectors
$\longrightarrow$ no clear signals so far — fundamental nature unknown



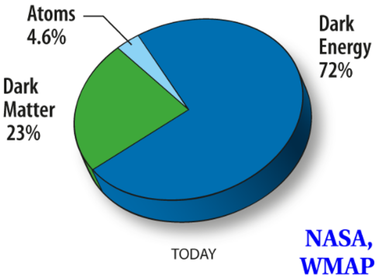


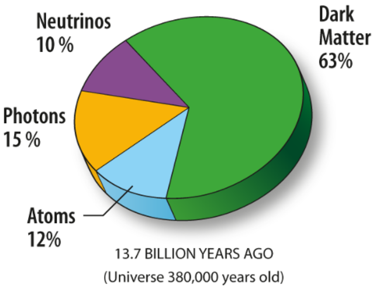
Motivation for non-gravitational interactions

Since the early universe,

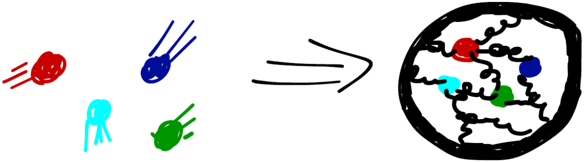
$$\frac{\Omega_{\text{dark}}}{\Omega_{\text{ordinary}}} \approx 5 \quad \ldots \text{not } 10^5 \text{ or } 10^{-5}$$

$\longrightarrow$ Non-gravitational interactions
between dark matter
and known particles





Composite dark matter



Deconfined charged fermions

$\longrightarrow$ non-gravitational interactions in early universe

Confined neutral 'dark baryon'

$\longrightarrow$ non-observation in current experiments

Direct detection signals from **form factors** of composite DM

(magnetic moment, charge radius, polarizability)

$\Longrightarrow$ Non-perturbative lattice calculations needed

Recent result: Lower bound for composite dark matter

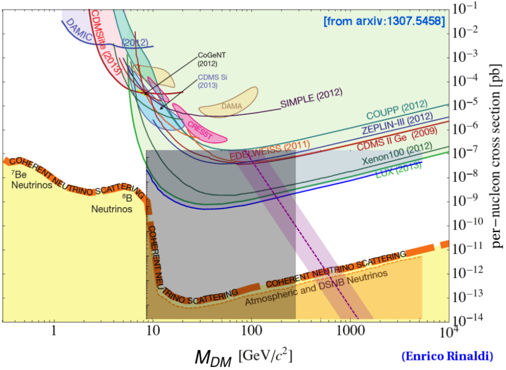
Investigate most general constraint

Lattice computation of electric polarizability

$\longrightarrow$ lower bound on the direct detection rate

Results specific to Xenon

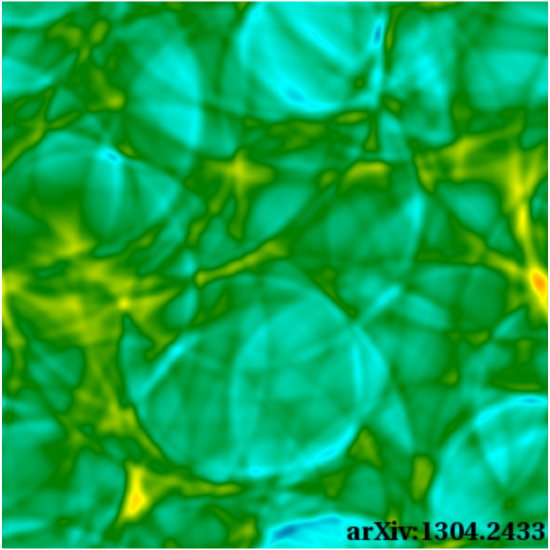
Uncertainties dominated
by Xenon matrix element



Shaded region is complementary constraint from particle colliders

Future plan: Gravitational wave signals

Gravitational wave observatories opening new window on cosmology



First-order DM transition
$\longrightarrow$ colliding bubbles
$\longrightarrow$ gravitational waves

Lattice calculations predict
properties of transition
& resulting signals

Application: Composite Higgs bosons

Large Hadron Collider priority

Determine fundamental nature of the Higgs boson

Composite Higgs boson — analogous to pion
could arise from **new strong dynamics**

Would protect Higgs from extreme sensitivity to quantum effects,
solving 'hierarchy' problem

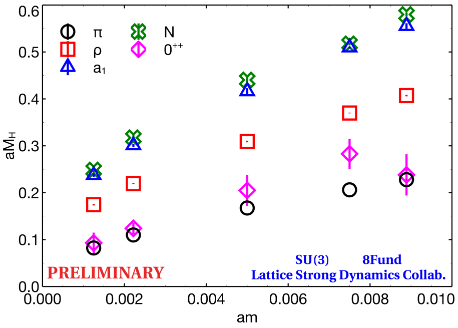
Lattice field theory is exploring this possibility

Recent result: Light composite Higgs

arXiv:180?.?????

Recent lattice studies observe interesting behavior
when more light fermions are added to QCD

Resulting composite Higgs is much lighter than in QCD,
closer to experiment



Large separation between
Higgs and resonances
as demanded by LHC

Need to extrapolate
fermion mass $m \to 0$
via effective field theory

Future plan: Interactions of light Higgs

There are many candidate effective field theories (EFTs)

that include both pions and a light Higgs

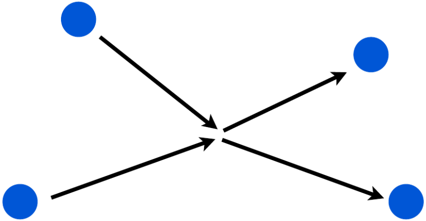
(linear σ model; Goldberger–Gristein–Skiba; Soto–Talavera–Tarrus; Matsuzaki–Yamawaki;

Golterman–Shamir; Hansen–Langaeble–Sannino; Appelquist–Ingoldby–Piai)

Need lattice computations of more observables to test EFTs

Studying interactions of light Higgs and pions

starting with $2 \to 2$ elastic scattering



Application: Lattice supersymmetry

Non-perturbative supersymmetry extremely interesting

Widely studied potential roles in new physics at the LHC

More generally, symmetries simplify systems
$\longrightarrow$ insight into strongly coupled dynamics

Many different methods have been brought to bear:

- Perturbation theory at weak coupling $\lambda \ll 1$,
  'dual' to strong coupling in some systems
- Holographic dualities with gravitational systems ("AdS / CFT")
- Conformal field theory techniques ("the bootstrap")

Only lattice field theory
provides non-perturbative predictions from first principles

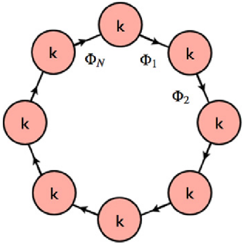
A brief history of lattice supersymmetry

Supersymmetries are "square roots" of infinitesimal translations which do not exist in discrete space-time

Problem can be solved for some systems

Use clever tricks to preserve **subset** of supersymmetries

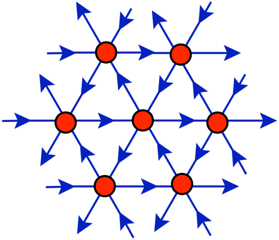
$\Longrightarrow$ recover others in continuum limit



Review:

Catterall, Kaplan & Ünsal

arXiv:0903.4881



Recent result: Testing holographic duality arXiv:1709.07025

2d supersymmetric Yang–Mills (SYM) theory with gauge group SU(N)

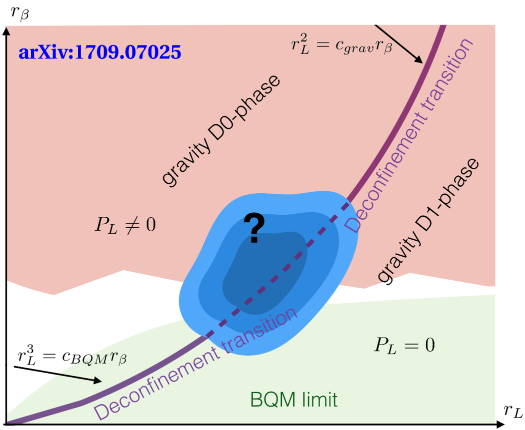
Low temperatures $t = 1/r_\beta \longleftrightarrow$ black holes in dual supergravity

For decreasing r_L at large N

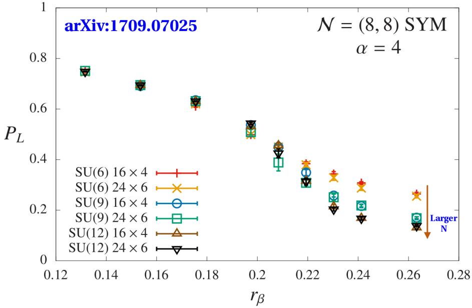
homogeneous black string (D1)
$\longrightarrow$ localized black hole (D0)

$\updownarrow$

“spatial deconfinement”
signalled by Wilson line P_L



Two-dimensional SYM lattice phase diagram



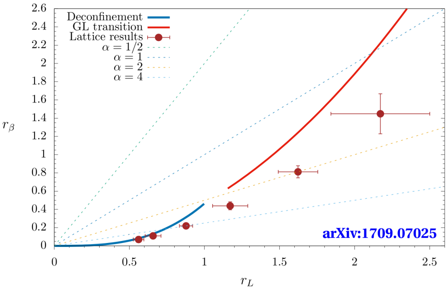
Fix aspect ratio $\alpha = r_L / r_\beta$,

scan in $r_\beta = r_L / \alpha = \beta\sqrt{\lambda}$

Clear transition in Wilson line and its susceptibility

Lower-temperature transitions at smaller $\alpha \longrightarrow$ larger errors

Results consistent with holography and high-temp. bosonic QM

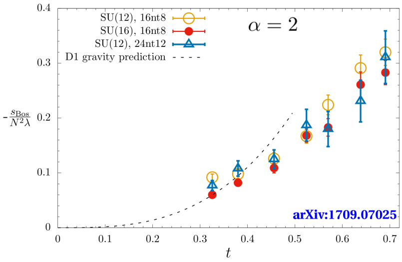


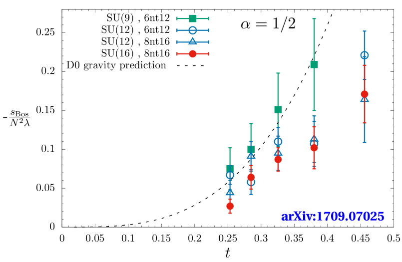
Dual black hole thermodynamics

Holography relates black holes' energy to action of SYM field theory

$\propto t^3$ for large-r_L D1 phase $\qquad\qquad \propto t^{3.2}$ for small-r_L D0 phase

Lattice results consistent with holography for sufficiently low $t \lesssim 0.4$





Need larger $N > 16$ to avoid instabilities at lower temperatures

Future plan: Supersymmetric QCD

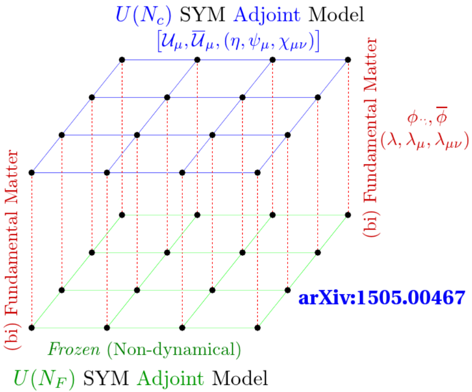
Supersymmetric Yang–Mills involves only analog of the gluon

Analogs of quarks involved in other dualities,
spontaneous supersymmetry breaking and more

Quarks arise from links between two SYM lattices

Produces supersymmetric QCD in $d-1$ dimensions

Numerical investigations only just beginning



arXiv:1505.00467

Outlook: An exciting time for lattice field theory

Lattice field theory is a broadly applicable tool
to study strongly coupled systems

- Solving the sign problem of simplified dense nuclear matter
- Predicting experimental signals of composite dark matter
- Exploring features of composite Higgs bosons
- Testing holographic dualities of supersymmetric gauge theories

Thank you!

schaich@itp.unibe.ch
www.davidschaich.net

Outlook: An exciting time for lattice field theory

Lattice field theory is a broadly applicable tool
to study strongly coupled systems

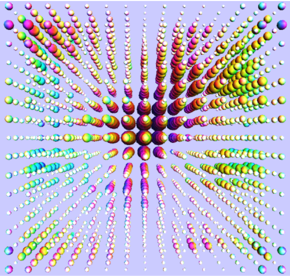
- Solving the sign problem of simplified dense nuclear matter
- Predicting experimental signals of composite dark matter
- Exploring features of composite Higgs bosons
- Testing holographic dualities of supersymmetric gauge theories

Thank you!

schaich@itp.unibe.ch
www.davidschaich.net

Backup: Hybrid Monte Carlo (HMC) algorithm

Goal: Sample field configurations Φ with probability $\frac{1}{\mathcal{Z}} e^{-S[\Phi]}$



(Image credit: Claudio Rebbi)

HMC is Markov process based on Metropolis–Rosenbluth–Teller

Fermions $\longrightarrow$ extensive action computation

$\Longrightarrow$ Global updates using fictitious molecular dynamics

1. Introduce fictitious random momenta and “MD time” τ
2. Inexact MD evolution along trajectory in τ $\longrightarrow$ new four-dimensional field configuration
3. Accept/reject test on MD discretization error

Lattice field theory helps to advance high-performance computing

IBM Blue Gene/Q @Livermore

USQCD cluster @Fermilab

Results shown above are from
state-of-the-art lattice calculations

$\mathcal{O}$(500M core-hours) invested overall

Many thanks to DOE, NSF
and computing centers!

Cray *Blue Waters* @NCSA

Backup:
Lattice Strong Dynamics Collaboration

Argonne	Xiao-Yong Jin, James Osborn
Bern	DS
Boston	Rich Brower, Claudio Rebbi, Evan Weinberg
Colorado	Anna Hasenfratz, Ethan Neil, Oliver Witzel
UC Davis	Joseph Kiskis
Livermore	Pavlos Vranas
Oregon	Graham Kribs
RBRC	Enrico Rinaldi
Yale	Thomas Appelquist, George Fleming, Andrew Gasbarro

Exploring the range of possible phenomena
in strongly coupled field theories

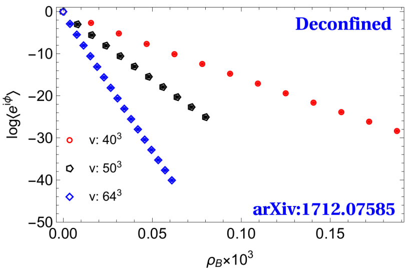
Backup: Phase reweighting

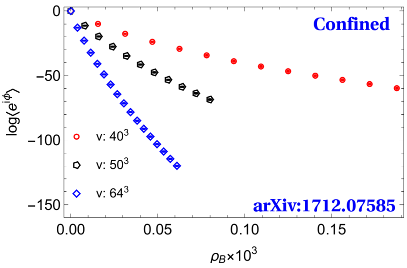
Importance sampling can still work with complex $e^{-S} = |e^{-S}| e^{i\alpha}$

$$\langle \mathcal{O} \rangle = \frac{\int \mathcal{D}\Phi \;\; \mathcal{O}(\Phi) \;\; e^{-S[\Phi]}}{\int \mathcal{D}\Phi \;\; e^{-S[\Phi]}} = \frac{\int \mathcal{D}\Phi \;\; \mathcal{O} e^{i\alpha} \;\; |e^{-S}|}{\int \mathcal{D}\Phi \;\; e^{i\alpha} \;\; |e^{-S}|} = \frac{\left\langle \mathcal{O} e^{i\alpha} \right\rangle_{||}}{\left\langle e^{i\alpha} \right\rangle_{||}}$$

However $\left\langle e^{i\alpha} \right\rangle_{||} = \mathcal{Z} / \mathcal{Z}_{||} = \exp\left[-V(f - f_{||})/T\right]$ and $f > f_{||}$

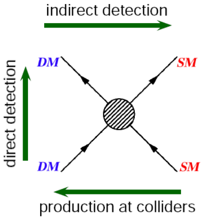
Sign problem $\longleftrightarrow$ exponential signal-to-noise problem





Backup: Thermal freeze-out for relic density

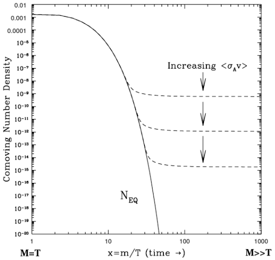
Requires coupling between ordinary matter and dark matter



$T \gtrsim M_{DM}$: DM $\longleftrightarrow$ SM
Thermal equilibrium

$T \lesssim M_{DM}$: DM $\longrightarrow$ SM
Rapid depletion of Ω_{DM}

Hubble expansion
$\longrightarrow$ dilution $\longrightarrow$ freeze-out



$2 \to 2$ scattering relates coupling and mass as $200\alpha \sim \dfrac{M_{DM}}{100\text{ GeV}}$

Strong $\alpha \sim 16$ $\longrightarrow$ 'natural' mass scale $M_{DM} \sim 300$ TeV

Smaller $M_{DM} \gtrsim 1$ TeV possible from $2 \to n$ scattering or asymmetry

Backup: Two roads to natural asymmetric dark matter

Idea: Dark matter relic density related to baryon asymmetry

$$\Omega_D \approx 5\Omega_B$$
$$\Longrightarrow M_D n_D \approx 5 M_B n_B$$

$n_D \sim n_B \quad \Longrightarrow \quad M_D \sim 5M_B \approx 5$ GeV

High-dim. interactions relate baryon# and DM# violation

$M_D \gg M_B \quad \Longrightarrow \quad n_B \gg n_D \sim \exp\left[-M_D/T_s\right] \qquad T_s \sim 200$ GeV

EW sphaleron processes above T_s distribute asymmetries

Both require coupling between ordinary matter and dark matter

Backup: Composite dark matter interactions

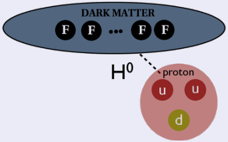
Photon exchange via electromagnetic form factors

Interactions suppressed by powers of confinement scale $\Lambda \sim M_{DM}$

- **Dimension 5:** Magnetic moment $\longrightarrow$ $(\overline{\psi}\sigma^{\mu\nu}\psi)\, F_{\mu\nu}/\Lambda$
- **Dimension 6:** Charge radius $\longrightarrow$ $(\overline{\psi}\gamma^{\nu}\psi)\, \partial^{\mu} F_{\mu\nu}/\Lambda^2$
- **Dimension 7:** Polarizability $\longrightarrow$ $(\overline{\psi}\psi)\, F^{\mu\nu} F_{\mu\nu}/\Lambda^3$

Higgs exchange via scalar form factors

Higgs couples through $\langle B | m_{\psi} \overline{\psi}\psi | B \rangle$ (σ terms)



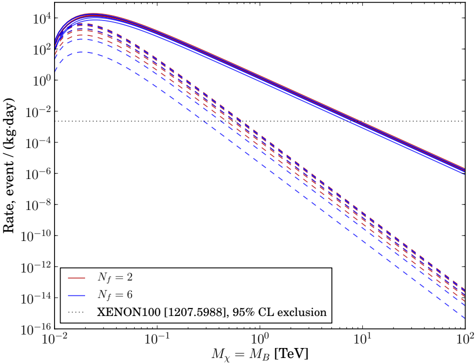
Needed for Big Bang nucleosynthesis
($\longrightarrow$ rapid charged 'meson' decay)

Non-perturbative form factors $\Longrightarrow$ lattice calculations

Backup: SU(3) direct detection constraints

Solid: Predicted event rate for SU(3) model vs. DM mass M_B

Dashed: Sub-leading charge radius contribution
suppressed $\sim 1/M_B^2$ compared to magnetic moment

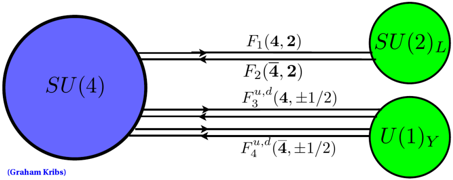


XENON100 (arXiv:1207.5988)
requires $M_B \gtrsim 10$ TeV

More recent LUX, PandaX
$\longrightarrow M_B \gtrsim 25$ TeV

SU(N) with even $N \geq 4$
forbids mag. moment...

Backup: Stealth dark matter model details



Field	$SU(N_D)$	$(SU(2)_L,\ Y)$	Q
$F_1 = \begin{pmatrix} F_1^u \\ F_1^d \end{pmatrix}$	$\mathbf{N}$	$(\mathbf{2},\ 0)$	$\begin{pmatrix} +1/2 \\ -1/2 \end{pmatrix}$
$F_2 = \begin{pmatrix} F_2^u \\ F_2^d \end{pmatrix}$	$\bar{\mathbf{N}}$	$(\mathbf{2},\ 0)$	$\begin{pmatrix} +1/2 \\ -1/2 \end{pmatrix}$
F_3^u	$\mathbf{N}$	$(\mathbf{1}, +1/2)$	$+1/2$
F_3^d	$\mathbf{N}$	$(\mathbf{1}, -1/2)$	$-1/2$
F_4^u	$\bar{\mathbf{N}}$	$(\mathbf{1}, +1/2)$	$+1/2$
F_4^d	$\bar{\mathbf{N}}$	$(\mathbf{1}, -1/2)$	$-1/2$

Mass terms $\quad m_V\left(F_1 F_2 + F_3 F_4\right) + y\left(F_1 \cdot H F_4 + F_2 \cdot H^\dagger F_3\right) + \text{h.c.}$

Vector-like masses evade Higgs-exchange direct detection bounds

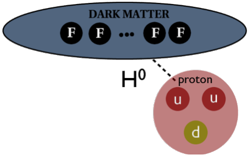
Higgs couplings $\rightarrow$ charged meson decay before Big Bang nucleosyn.

Both required

Backup: Effective Higgs interaction

$M_H = 125$ GeV $\longrightarrow$ Higgs exchange can dominate direct detection

$$\sigma_H \propto \left| \frac{\mu_{DM,N}}{M_H^2} \; y_\psi \langle DM|\overline{\psi}\psi|DM\rangle \; y_q \langle N|\overline{q}q|N\rangle \right|^2$$



Quark $\quad y_q = \dfrac{m_q}{v}$

Dark $\quad y_\psi = \alpha \dfrac{m_\psi}{v}$ suppressed by $\alpha \equiv \dfrac{v}{m_\psi} \dfrac{\partial m_\psi(h)}{\partial h}\Big|_{h=v} = \dfrac{yv}{yv + m_V}$

Can determine scalar form factors using Feynman–Hellmann theorem

$$\langle DM|\overline{\psi}\psi|DM\rangle = \frac{\partial M_{DM}}{\partial m_\psi}$$

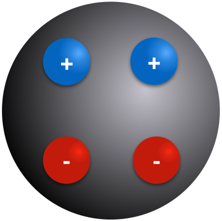
Backup: Stealth dark matter EM form factors

Lightest SU(4) dark baryon

Scalar $\longrightarrow$ no magnetic moment

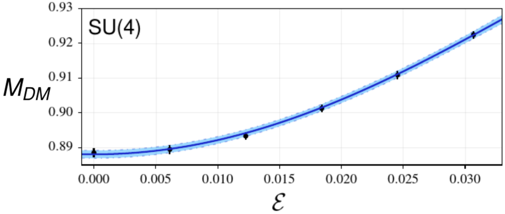
+/- charge symmetry $\longrightarrow$ no charge radius

Small α $\longrightarrow$ Higgs exchange suppressed

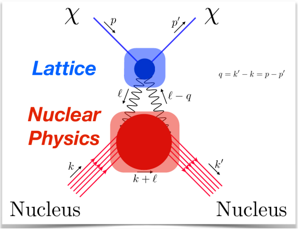


Polarizability $\longrightarrow$ lower bound on direct-detection cross section

Compute on lattice as dependence of M_{DM} on external field $\mathcal{E}$



$\rightsquigarrow$

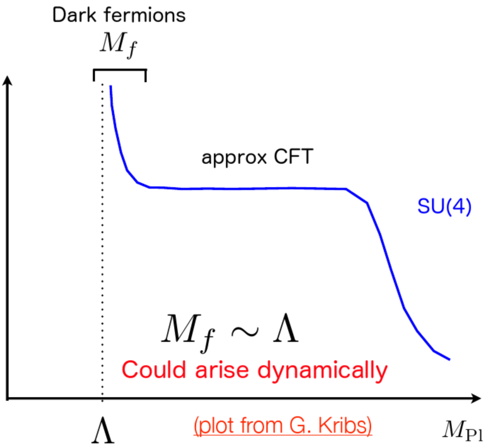


Backup: Stealth dark matter mass scales

Lattice studies focus on $m_\psi \simeq \Lambda_{DM}$ (effective theories least reliable)

$m_\psi \simeq \Lambda_{DM}$ could arise dynamically

Smaller $m_\psi \longrightarrow$ stronger collider constraints



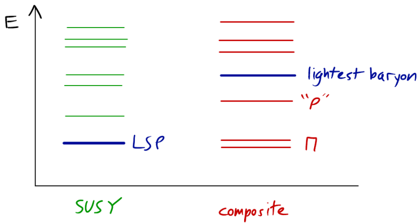
(plot from G. Kribs)

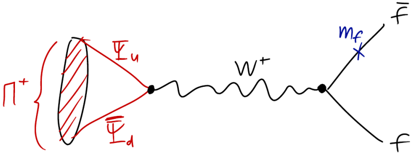
Backup: Stealth dark matter at colliders

Spectrum significantly different
from typical susy

$\longrightarrow$ Very little missing E_T

Main constraints from
much lighter **charged** "Π" states





Rapid Π decays, $\Gamma \propto m_f^2$

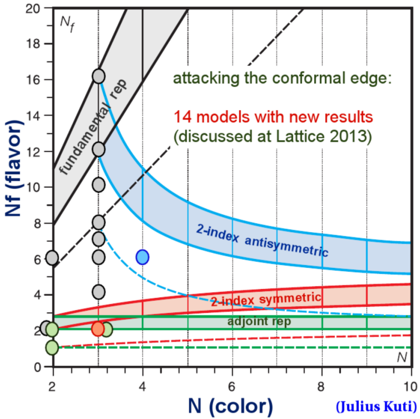
Best current constraints
recast LEP stau searches

LHC can search for $t\overline{b} + \overline{t}b$
from $\Pi^+\Pi^-$ Drell–Yan

Backup: Strategy for composite Higgs studies

Systematically depart from familiar ground of lattice QCD
($N = 3$ with $N_F = 2$ light flavors in fundamental rep)

Explore the range of possible phenomena in strongly coupled theories



Add more light flavors
$\longrightarrow N_F = 8$ fundamental

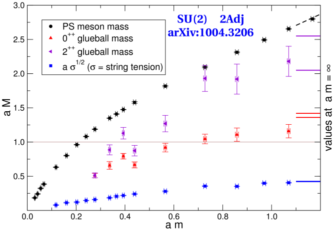
Enlarge fermion rep
$\longrightarrow N_F = 2$ two-index symmetric

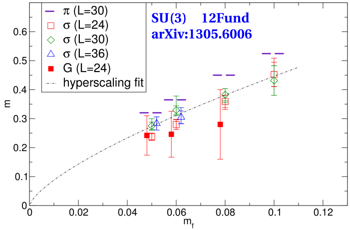
Explore $N = 2$ and 4
$\longrightarrow$ (pseudo)real reps for cosets
SU(n)/Sp(n) and SU(n)/SO(n)

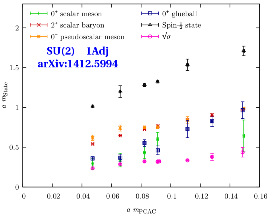
Backup: Light scalars beyond QCD

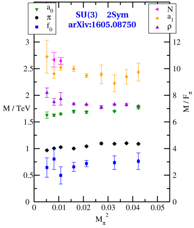
More fermionic d.o.f. $\longrightarrow$ near-conformal dynamics

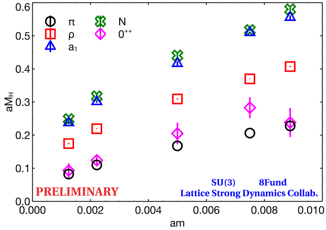
All lattice studies so far $\longrightarrow$ Higgs much lighter than in QCD









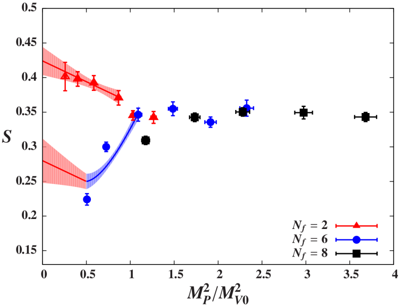


Backup: S parameter on the lattice

$$\mathcal{L}_\chi \supset \frac{\alpha_1}{2} g_1 g_2 B_{\mu\nu} \mathrm{Tr}\left[U\tau_3 U^\dagger W^{\mu\nu}\right] \longrightarrow$$

Lattice vacuum polarization calculation provides $S = -16\pi^2\alpha_1$

Non-zero masses and chiral extrapolation needed to avoid sensitivity to finite lattice volume



$S = 0.42(2)$ for $N_F = 2$ matches scaled-up QCD

Larger $N_F \longrightarrow$ significant reduction

Extrapolation to correct zero-mass limit becomes more challenging

Backup: Vacuum polarization from current correlator

$$S = 4\pi N_D \lim_{Q^2 \to 0} \frac{d}{dQ^2} \Pi_{V-A}(Q^2) - \Delta S_{SM}(M_H)$$

$$\Pi^{\mu\nu}_{V-A}(Q) = Z \sum_x e^{iQ\cdot(x+\widehat{\mu}/2)} \mathrm{Tr}\left[\left\langle \mathcal{V}^{\mu a}(x) V^{\nu b}(0) \right\rangle - \left\langle \mathcal{A}^{\mu a}(x) A^{\nu b}(0) \right\rangle\right]$$

$$\Pi^{\mu\nu}(Q) = \left(\delta^{\mu\nu} - \frac{\widehat{Q}^{\mu}\widehat{Q}^{\nu}}{\widehat{Q}^2}\right) \Pi(Q^2) - \frac{\widehat{Q}^{\mu}\widehat{Q}^{\nu}}{\widehat{Q}^2} \Pi^L(Q^2) \qquad \widehat{Q} = 2\sin\left(Q/2\right)$$

- Renormalization constant Z evaluated non-perturbatively
  Chiral symmetry of domain wall fermions $\Longrightarrow Z = Z_A = Z_V$
  $Z = 0.85$ [2f]; 0.73 [6f]; 0.70 [8f]

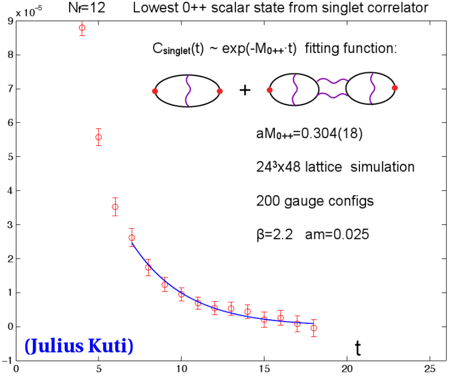
- Conserved currents $\mathcal{V}$ and $\mathcal{A}$ ensure that lattice artifacts cancel

Backup: Technical challenge for scalar on lattice

Only new strong sector included in lattice calculations

$\Longrightarrow$ flavor-singlet scalar mixes with vacuum

Leads to noisy data and relatively large uncertainties

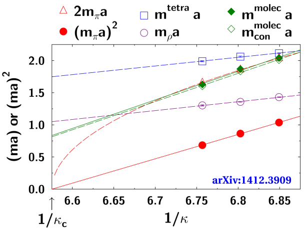


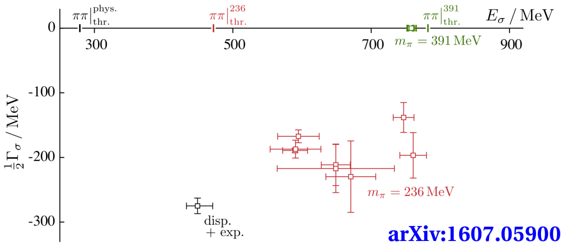
Fermion propagator computation relatively expensive

"Disconnected diagrams" formally need propagators at all L^4 sites

In practice estimate stochastically to control computational costs

Backup: Composite Higgs in QCD spectrum





QCD-like composite Higgs much heavier than pion

Generally $M_S \gtrsim 2M_P \quad \longrightarrow \quad M_S > M_V$ for heavy quarks

For a large range of quark masses m

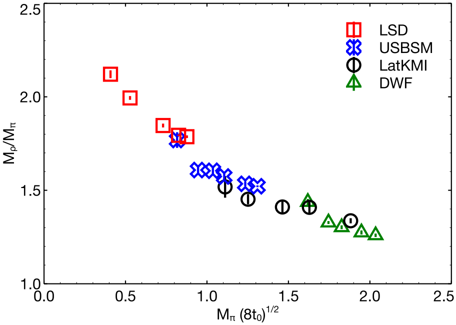
it mixes significantly with two-pion scattering states

Backup: More on composite Higgs spectrum

Higgs much lighter than in QCD-like systems

Degenerate with pions at accessible fermion masses $m > 0$

$\longrightarrow$ Next resonance roughly twice as heavy at these m,
and ratio is growing rapidly towards physical $m \to 0$ limit



Recent work using lattice volumes up to $64^3 \times 128$

Large separation between Higgs and resonances

Scale setting suggests resonance masses $\sim$2–3 TeV

Backup: 2 $\rightarrow$ 2 elastic scattering on the lattice

No asymptotic 'in' or 'out' states in lattice calculations

Instead extract scattering information from multi-particle energies

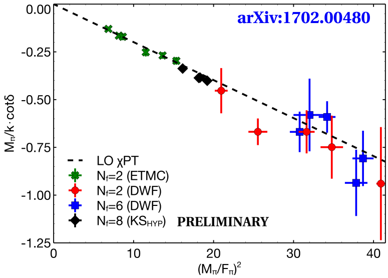
Measure both E_{PP} and M_P $\longrightarrow$ $k = \sqrt{(E_{PP}/2)^2 - M_P^2}$

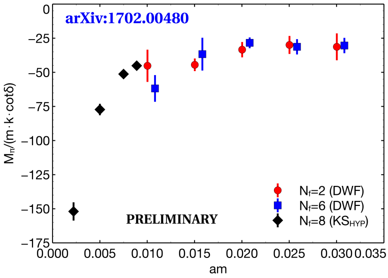
s-wave scattering phase shift: $\cot\delta_0(k) = \frac{1}{\pi k L} S\left(\frac{k^2 L^2}{4\pi}\right)$

involving regularized ζ function $S(\eta) = \sum_{j \neq 0}^{\Lambda} \frac{1}{j^2 - \eta} - 4\pi\Lambda$

Eff. range expansion: $k \cot\delta_0(k) = \frac{1}{a_{PP}} + \frac{1}{2} M_P^2 r_{PP} \left(\frac{k^2}{M_P^2}\right) + \mathcal{O}\left(\frac{k^4}{M_P^4}\right)$

Backup: Initial 2 $\to$ 2 elastic scattering results





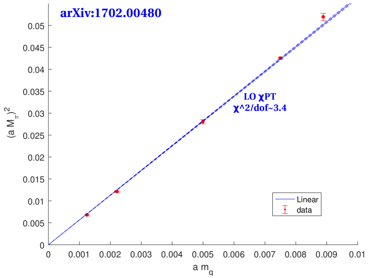
Simplest case: Analog of QCD $I = 2$ $\pi\pi$ scattering
(no fermion-line-disconnected diagrams)

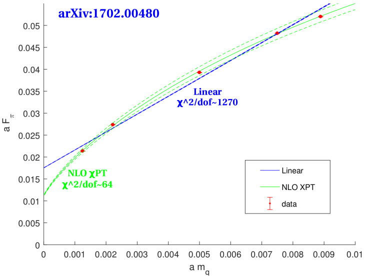
Simplest observable: Scattering length $a_{PP} \approx 1/(k \cot\delta)$

Left: $M_P a_{PP}$ vs. M_P^2/F_P^2 curiously close to leading-order χPT

Right: Divide by fermion mass m $\longrightarrow$ tension with χPT as expected
(predicts constant at LO; involves 8 LECs at NLO)

Backup: 8f chiral perturbation theory (χPT) fits





χPT omits the light scalar **and** suffers from large expansion parameter

$$5.8 \leq \frac{2N_F Bm}{16\pi^2 F^2} \leq 41.3 \qquad \text{for} \qquad 0.00125 \leq m \leq 0.00889$$

$\sim 50\sigma$ shift in F between linear extrapolation vs. NLO χPT

Poor fit quality, especially for NLO joint fit $\left(\chi^2/\text{d.o.f.} > 10^4\right)$

Backup: Discrete space-time breaks Leibnitz rule

$\left\{ Q_\alpha, \overline{Q}_{\dot{\alpha}} \right\} = 2\sigma^\mu_{\alpha\dot{\alpha}} P_\mu = 2i\sigma^\mu_{\alpha\dot{\alpha}} \partial_\mu$ is problematic

$\longrightarrow$ try $\left\{ Q_\alpha, \overline{Q}_{\dot{\alpha}} \right\} = 2i\sigma^\mu_{\alpha\dot{\alpha}} \nabla_\mu$ for a discrete translation

$$\nabla_\mu \phi(x) = \frac{1}{a}\left[\phi(x + a\widehat{\mu}) - \phi(x)\right] = \partial_\mu \phi(x) + \frac{a}{2}\partial_\mu^2 \phi(x) + \mathcal{O}(a^2)$$

Essential difference between ∂_μ and lattice ∇_μ with $a > 0$

$$\begin{aligned} \nabla_\mu \left[\phi(x)\eta(x)\right] &= a^{-1}\left[\phi(x + a\widehat{\mu})\eta(x + a\widehat{\mu}) - \phi(x)\eta(x)\right] \\ &= \left[\nabla_\mu \phi(x)\right]\eta(x) + \phi(x)\nabla_\mu \eta(x) + a\left[\nabla_\mu \phi(x)\right]\nabla_\mu \eta(x) \end{aligned}$$

Only recover Leibnitz rule $\partial_\mu(fg) = (\partial_\mu f)g + f\partial_\mu g$ when $a \to 0$

$\Longrightarrow$ "Discrete supersymmetry" breaks down on the lattice

(Dondi & Nicolai, "Lattice Supersymmetry", 1977)

Backup: $\mathcal{N} = 4$ SYM — the fruit fly of QFT

Widely used to develop continuum QFT tools & techniques,
from scattering amplitudes to holography

Arguably simplest non-trivial 4d field theory

SU(N) gauge theory with four fermions Ψ^{I} and six scalars Φ^{IJ},
all massless and in adjoint rep.

Action consists of kinetic, Yukawa and four-scalar terms
with coefficients related by symmetries $\longrightarrow$ single coupling $\lambda = g^2 N$

Maximal 16 supersymmetries Q_α^{I} and $\overline{Q}_{\dot\alpha}^{\mathrm{I}}$ $(\mathrm{I} = 1, \cdots, 4)$
transforming under global SU(4) $\sim$ SO(6) R symmetry

Conformal: β function is zero for any λ

Backup: Dimensional reduction to $\mathcal{N} = (8, 8)$ SYM

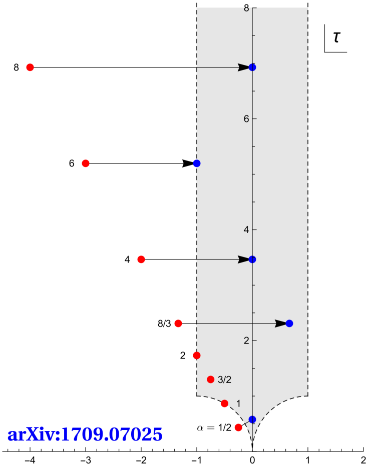
Naive for now: 4d $\mathcal{N} = 4$ SYM code with $N_y = N_z = 1$

A_4^* lattice $\longrightarrow$ A_2^* (triangular) lattice

Torus **skewed** depending on $\alpha = N_x / N_t$

Modular trans. into fundamental domain
can make skewed torus rectangular

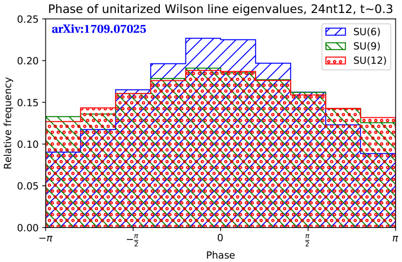
Also need to stabilize compactified links
to ensure broken center symmetries

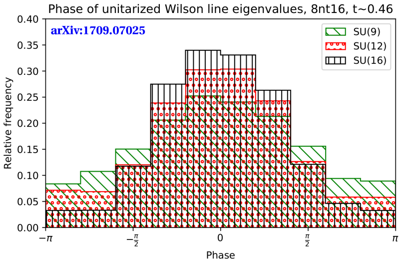


arXiv:1709.07025

Backup: $\mathcal{N} = (8, 8)$ SYM Wilson line eigenvalues

Check 'spatial deconfinement' through histograms of Wilson line eigenvalue phases





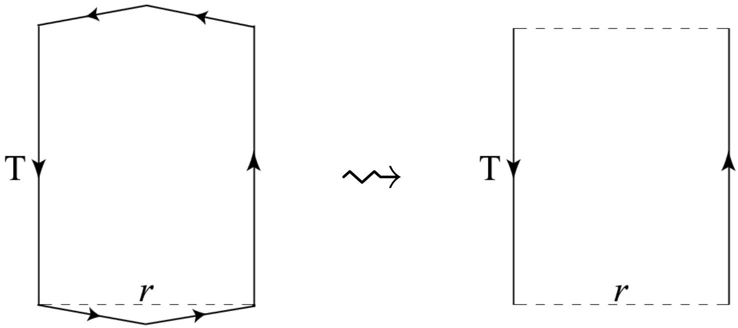
Left: $\alpha = 2$ distributions more extended as N increases
$\longrightarrow$ dual gravity describes homogeneous black string (D1 phase)

Right: $\alpha = 1/2$ distributions more compact as N increases
$\longrightarrow$ dual gravity describes localized black hole (D0 phase)

Backup: Static potential $V(r)$

Static probes $\longrightarrow$ $r \times T$ Wilson loops $W(r, T) \propto e^{-V(r)\,T}$

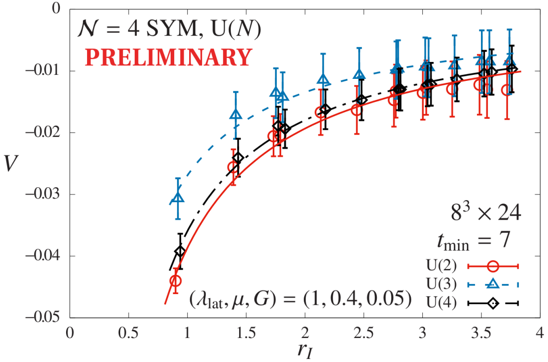
Coulomb gauge trick reduces A_4^* lattice complications



Backup: Static potential is Coulombic at all λ

Fits to confining $V(r) = A - C/r + \sigma r \longrightarrow$ vanishing string tension σ

$\Longrightarrow$ Fit to just $V(r) = A - C/r$ to extract Coulomb coefficient $C(\lambda)$

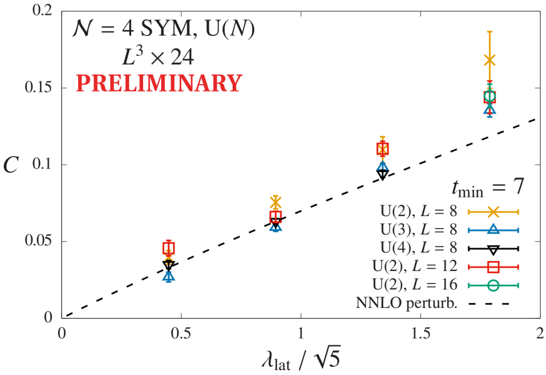


Recent progress: Incorporating tree-level improvement into analysis

Backup: Coupling dependence of Coulomb coefficient

Continuum perturbation theory predicts $C(\lambda) = \lambda/(4\pi) + \mathcal{O}(\lambda^2)$

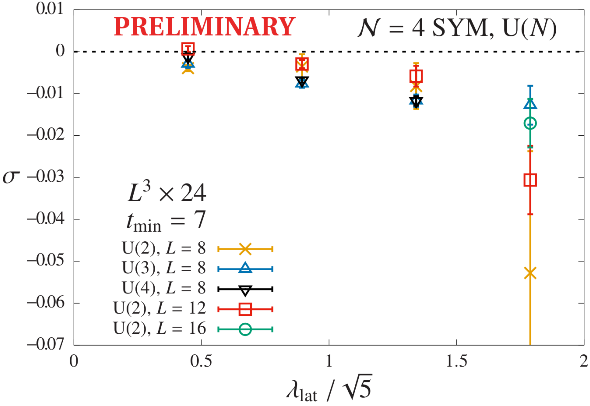
Holography predicts $C(\lambda) \propto \sqrt{\lambda}$ for $N \to \infty$ and $\lambda \to \infty$ with $\lambda \ll N$



Surprisingly good agreement with perturbation theory for $\lambda_{\text{lat}} \leq 4$

Backup: Static potential is Coulombic at all λ

String tension σ from fits to confining form $V(r) = A - C/r + \sigma r$



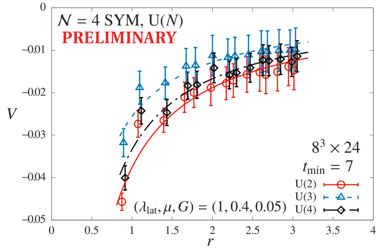
Slightly negative values flatten $V(r_I)$ for $r_I \lesssim L/2$

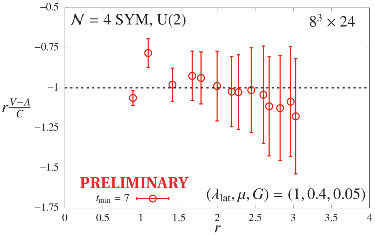
$\Longrightarrow$ $\sigma \to 0$ as accessible range of r_I increases on larger volumes

Backup: Discretization artifacts in static potential

Discretization artifacts visible at short distances
where Coulomb term in $V(r) = A - C/r$ is most significant

Right: Highlight artifacts by extracting fluctuations around Coulomb fit





Danger of potential contamination in results for Coulomb coefficient C

Backup: Tree-level improvement

Classic trick to reduce discretization artifacts in static potential

(Lang & Rebbi '82; Sommer '93; Necco '03)

Associate $V(r)$ data with r from Fourier transform of gluon propagator

Recall $\displaystyle \frac{1}{4\pi^2 r^2} = \int_{-\pi}^{\pi} \frac{d^4k}{(2\pi)^4} \frac{e^{ir\cdot k}}{k^2}$ where $\displaystyle \frac{1}{k^2} = G(k)$ in continuum

On A_4^* lattice $\longrightarrow$ $$\frac{1}{r_I^2} \equiv 4\pi^2 \int_{-\pi}^{\pi} \frac{d^4\widehat{k}}{(2\pi)^4} \frac{\cos\left(ir_I \cdot \widehat{k}\right)}{4\sum_{\mu=1}^{4} \sin^2\left(\widehat{k} \cdot \widehat{e}_\mu \;/\; 2\right)}$$

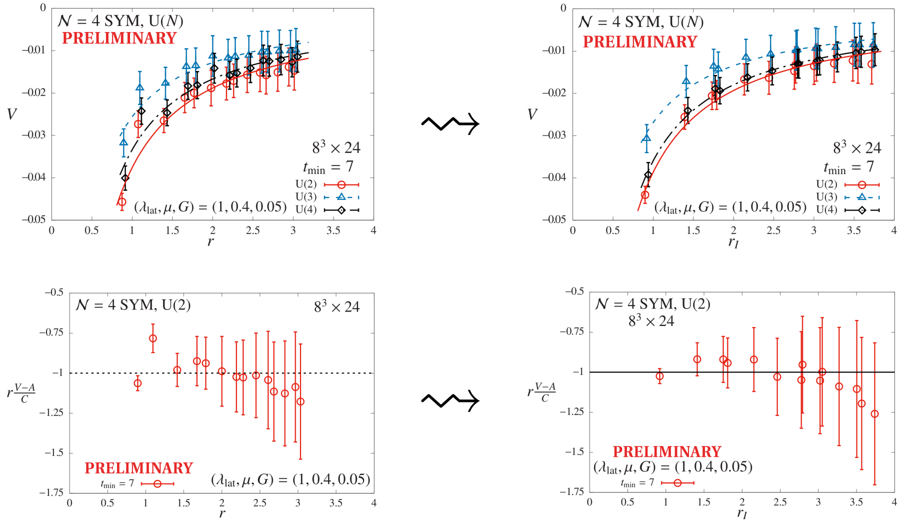
Tree-level perturbative lattice propagator from arXiv:1102.1725

$\widehat{e}_\mu$ are A_4^* lattice basis vectors

while momenta $\widehat{k} = \frac{2\pi}{L} \sum_{\mu=1}^{4} n_\mu \widehat{g}_\mu$ depend on dual basis vectors

Backup: Tree-level-improved static potential

Tree-level improvement significantly reduces discretization artifacts



Backup: Konishi operator scaling dimension

Conformality $\longrightarrow$ spectrum of scaling dimensions $\Delta(\lambda)$
govern power-law decays of correlation functions

Konishi is simplest conformal primary operator

$$\mathcal{O}_K(x) = \sum_{\mathrm{I}} \mathrm{Tr}\left[\Phi^{\mathrm{I}}(x)\Phi^{\mathrm{I}}(x)\right] \qquad C_K(r) \equiv \mathcal{O}_K(x+r)\mathcal{O}_K(x) \propto r^{-2\Delta_K}$$

Predictions for Konishi scaling dimension $\Delta_K(\lambda) = 2 + \gamma_K(\lambda)$

- $\gamma_K(\lambda) = \frac{3\lambda}{4\pi^2} + \mathcal{O}(\lambda^2)$ from weak-coupling perturbation theory,
  related to strong coupling by $\frac{4\pi N}{\lambda} \longleftrightarrow \frac{\lambda}{4\pi N}$ S duality
- $\Delta_K(\lambda) = 2\lambda^{1/4} + \mathcal{O}(\lambda^{-1/4})$ from holography for $N \to \infty$
- Upper bounds from conformal bootstrap

Lattice gauge theory can access nonperturbative λ at moderate N

Backup: Konishi operator on the lattice

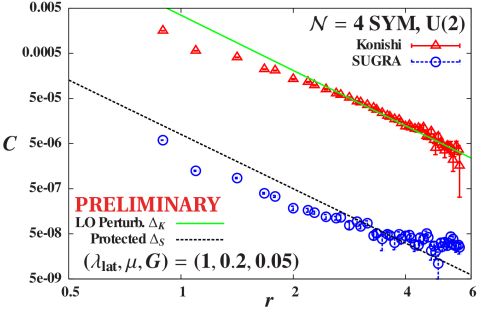
Lattice scalars $\varphi(n)$ from polar decomposition of complexified links

$$\mathcal{U}_a(n) \longrightarrow e^{\varphi_a(n)} U_a(n) \qquad \mathcal{O}_K^{\text{lat}}(n) = \sum_a \text{Tr}\left[\varphi_a(n)\varphi_a(n)\right] - \text{vev}$$

$$C_K(r) \equiv \mathcal{O}_K(x+r)\mathcal{O}_K(x) \propto r^{-2\Delta_K}$$

'SUGRA' is 20′ $\mathcal{O}_S \sim \varphi_{\{a}\varphi_{b\}}$

with protected $\Delta_S = 2$



To handle systemics, comparing

direct power-law decays vs. finite-size scaling vs. Monte Carlo RG

Backup: MCRG stability matrix

System $\longleftrightarrow$ (infinite) sum of operators $H = \sum_i c_i\, \mathcal{O}_i$

Couplings c_i flow under **symmetry-preserving** RG blocking R_b

n-times-blocked system $H^{(n)} = R_b H^{(n-1)} = \sum_i c_i^{(n)} \mathcal{O}_i^{(n)}$

Fixed point $H^\star = R_b H^\star$ with couplings $c_i^\star$

Linear expansion around fixed point $\longrightarrow$ stability matrix $T_{ij}^\star$

$$c_i^{(n)} - c_i^\star = \sum_k \left.\frac{\partial c_i^{(n)}}{\partial c_k^{(n-1)}}\right|_{H^\star} \left(c_k^{(n-1)} - c_k^\star\right) \equiv \sum_j T_{ik}^\star \left(c_k^{(n-1)} - c_k^\star\right)$$

Correlators of $\mathcal{O}_i$, $\mathcal{O}_k$ $\longrightarrow$ elements of stability matrix [Swendsen, 1979]

Eigenvalues of $T_{ik}^\star$ $\longrightarrow$ scaling dimensions of corresponding operators

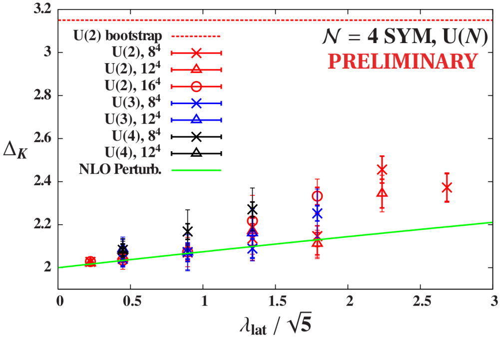
Backup: Preliminary Δ_K results from Monte Carlo RG

MCRG stability matrix

includes both $\mathcal{O}_K^{\text{lat}}$ and $\mathcal{O}_S^{\text{lat}}$

Impose protected $\Delta_S = 2$

Systematic uncertainties from different amounts of smearing



Complication: Twisted $SO(4)_{tw}$ involves only $SO(4)_R \subset SO(6)_R$

$\Longrightarrow$ Lattice Konishi operator mixes with $SO(4)_R$-singlet part of the $SO(6)_R$-nonsinglet SUGRA operator

Current work: Variational analyses to disentangle operators

Backup: Real-space RG for lattice $\mathcal{N} = 4$ SYM

Must preserve $\mathcal{Q}$ and S_5 symmetries $\longleftrightarrow$ geometric structure

Simple transformation constructed in arXiv:1408.7067

$$\mathcal{U}'_a(n') = \xi\,\mathcal{U}_a(n)\mathcal{U}_a(n+\widehat{\mu}_a) \qquad\qquad \eta'(n') = \eta(n)$$
$$\psi'_a(n') = \xi\left[\psi_a(n)\mathcal{U}_a(n+\widehat{\mu}_a) + \mathcal{U}_a(n)\psi_a(n+\widehat{\mu}_a)\right] \qquad\qquad \text{etc.}$$

Doubles lattice spacing $a \longrightarrow a' = 2a$, with tunable rescaling factor ξ

Scalar fields from polar decomposition $\mathcal{U}(n) = e^{\varphi(n)}U(n)$
are shifted, $\varphi \longrightarrow \varphi + \log\xi$, since blocked U must remain unitary

$\mathcal{Q}$-preserving RG blocking needed
to show only one log. tuning to recover continuum $\mathcal{Q}_a$ and $\mathcal{Q}_{ab}$

Backup: Smearing for Konishi analyses

As for glueballs, smear to enlarge operator basis

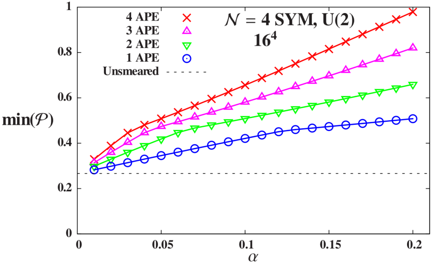
APE-like smearing: $\;—\; \longrightarrow \; (1-\alpha)— \; + \; \frac{\alpha}{8}\sum \sqcap$

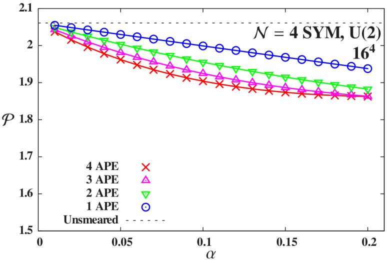
Staples built from unitary parts of links but no final unitarization

(unitarized smearing — e.g. stout — doesn't affect Konishi)

Average plaquette stable upon smearing (**right**)

while minimum plaquette steadily increases (**left**)





Backup: Potential sign problem of $\mathcal{N} = 4$ SYM

Integrating over a single Kähler–Dirac fermion Ψ in adjoint rep.,

$$\langle \mathcal{O} \rangle = \frac{1}{\mathcal{Z}} \int \mathcal{D}\mathcal{U}\ \mathcal{D}\overline{\mathcal{U}}\ \ \mathcal{O}(\mathcal{U}, \overline{\mathcal{U}})\ \ e^{-S_B[\mathcal{U}, \overline{\mathcal{U}}]}\ \ \text{pf}\, \mathcal{D}[\mathcal{U}, \overline{\mathcal{U}}]$$

Pfaffian can be complex for lattice $\mathcal{N} = 4$ SYM, $\text{pf}\, \mathcal{D} = |\text{pf}\, \mathcal{D}| e^{i\alpha}$

Complicates interpretation of $\left\{ e^{-S_B}\, \text{pf}\, \mathcal{D} \right\}$ as Boltzmann factor

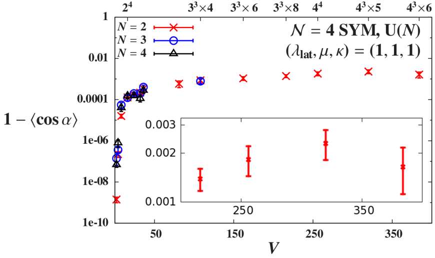
As for finite-density QCD, rewrite $\text{pf}\, \mathcal{D} = |\text{pf}\, \mathcal{D}| e^{i\alpha}$

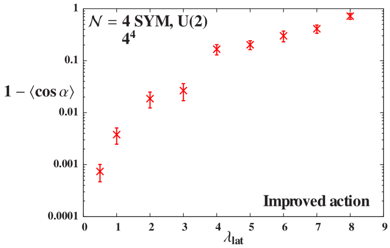
and monitor $\left\langle e^{i\alpha} \right\rangle_{||}$ as function of volume, coupling, N

Backup: Pfaffian dependence on volume and coupling

Left: $1 - \langle\cos(\alpha)\rangle_{pq} \ll 1$ independent of volume and N at $\lambda_{\mathrm{lat}} = 1$

Right: New 4^4 results at $4 \leq \lambda_{\mathrm{lat}} \leq 8$ show much larger fluctuations





Next step: Analyze more volumes, N, λ_{lat}

Extremely expensive computation despite new parallel algorithm:

$\mathcal{O}(n^3)$ scaling $\longrightarrow$ ~50 hours for single U(2) 4^4 measurement

Backup: $\mathcal{N} = 4$ SYM sign problem puzzles

Periodic temporal boundary conditions for the fermions

$\longrightarrow$ obvious sign problem, $\left\langle e^{i\alpha} \right\rangle_{pq} \approx 0$

Anti-periodic BCs $\longrightarrow$ $e^{i\alpha} \approx 1$, phase reweighting negligible

Why such sensitivity to the BCs?

Other $\langle \mathcal{O} \rangle_{pq}$ are nearly identical for these two ensembles

Why doesn't sign problem affect other observables?