Lower-dimensional lattice supersymmetry

David Schaich, 22 March 2018

Overview and plan

- **Presentation goal:** Survey of recent work
  on maximally supersymmetric Yang–Mills (SYM) theories in $d < 4$ dimensions
- **Research goal:** Reproduce known results in perturb., holographic, etc. regimes
  then use lattice to access new domains
- Review lattice supersymmetry and (4d) twisted lattice $\mathcal{N} = 4$ SYM
- 1d SYM bosonic action (others' work, 2007 through arXiv:1606.04951)
- 2d $\mathcal{N} = (8, 8)$ SYM phase diagram and bosonic action (arXiv:1709.07025)
- Work in progress: 1d supersymmetric mass deformation; max-SYM in 3d and 4d

Quick review of 4d lattice $\mathcal{N} = 4$ SYM

- 16 spinor generators (supercharges) Q^A_α and $\overline{Q}^A_{\dot\alpha}$ with $A = 1, \cdots, \mathcal{N}$
  $\left\{Q^A_\alpha, \overline{Q}^B_{\dot\alpha}\right\} = 2\delta^{AB}\sigma^\mu_{\alpha\dot\alpha}P_\mu \longrightarrow$ supersymmetry algebra broken on the lattice
- Two ways to avoid impractical fine-tuning (will use both):
  1) Work in lower dimensions where theories are super-renormalizable
  2) Preserve closed sub-algebra of supersymmetries
- Easiest to accomplish via topological twisting (introduced for curved manifolds)
  $Q^A_\alpha, \overline{Q}^B_{\dot\alpha} \longrightarrow \mathcal{Q}, \mathcal{Q}_\mu, \mathcal{Q}_{\mu\nu}, \overline{\mathcal{Q}}_\mu, \overline{\mathcal{Q}}$ in integer-spin reps of "twisted rotation group"

$$\text{SO}(4)_{tw} \equiv \text{diag}\left[\text{SO}(4)_{\text{euc}} \otimes \text{SO}(4)_R\right] \qquad \text{with} \qquad \text{SO}(4)_R \subset \text{SO}(6)_R$$

- More generally, for $2 \le d \le 5$ $\qquad \text{SO}(d)_{tw} \equiv \text{diag}\left[\text{SO}(d)_{\text{euc}} \otimes \text{SO}(d)_R\right]$
  $Q \ge 2^d$ supercharges $\longrightarrow \lfloor Q/2^d \rfloor \ge 1$ closed susy subalgebras $\mathcal{Q}^2 = 0$
- Reducing $\mathcal{N} = 1$ SYM in 10d $\longrightarrow$ SO$(10 - d)$ R symmetry
  16 fermions, d-component gauge field and $10 - d$ scalars
  all massless and in adjoint rep
- For $2 \le d \le 4$, discretize on $A_d^\star$ lattice with $d + 1$ basis vectors
  (familiar triangular lattice for $d = 2$)

“D0 brane quantum mechanics” (**Banks–Fischler–Shenker–Susskind, ’96**)

- Reduced to the point that twisting both impossible and unnecessary
  Only temporal component of gauge field remains, plus 9 scalars
  $$S_0 = \frac{N}{2\lambda}\int dt\ \mathrm{Tr}\left[\left(D_t X^A\right)^2 + \Psi^\alpha D_t \Psi^\alpha + \frac{1}{2}\left[X^A, X^B\right]^2 + i\Psi^\alpha \gamma^A_{\alpha\beta}\left[\Psi^\beta, X^A\right]\right]$$
  with $A, B = 1, \cdots, 9$ and $\alpha, \beta = 1, \cdots, 16$
- Finite-temperature system holographically dual to black holes
- Temperature and dimension-3 ’t Hooft coupling $\longrightarrow$ dim’less $T = T_{\mathrm{dim}}/\lambda^{1/3} \equiv 1/r_\beta$
- Low $T \ll 1$ and large number of colors N $\longrightarrow$ classical supergravity (SUGRA)
  Large N suppresses string quantum (g_s) corrections
  Low temperatures (large λ) suppress α' corrections (string size $\propto \sqrt{\alpha'}$)
- Numerical state of the art: gauge groups SU(16)–SU(32) with L up to 32
- Investigate dual black hole internal energy $\longleftrightarrow$ SYM bosonic action
  Fitting to powers predicted by SUGRA,
  reproduce leading SUGRA coefficient 7.4(5) vs. $E/N^2 = 7.41T^{2.8}$
  Also predict unknown coefficients of subleading correction terms
  $$E/N^2 = 7.41T^{2.8} + a_1 T^{4.6} + a_2 T^{5.8} \qquad \text{with} \quad a_1 = -10.0(4) \quad a_2 = 5.8(5)$$
- Correction pulls points down—if only one term, eventually dominates at larger t
- Aside: Gauge field may not matter (arXiv:1802.00428, arXiv:1802.02985)

Two-dimensional $\mathcal{N} = (8,8)$ SYM

- Naive dimensional reduction of twisted 4d $\mathcal{N} = 4$ SYM,
$$S_{tw} = \frac{N}{4\lambda}\mathcal{Q}\int d^4x \ \mathrm{Tr}\left[\chi_{ab}\mathcal{F}_{ab} + \eta[\overline{\mathcal{D}}_a, \mathcal{D}_a] - \frac{1}{2}\eta d + \epsilon_{abcde}\chi_{de}\overline{\mathcal{D}}_c\chi_{ab}\right],$$
$a, b = 1, \cdots, 5$ now include 'flavor' and three A_2 basis vectors

- Now have torus with $r_\beta = 1/t = \beta\sqrt{\lambda}$ and $r_L = L\sqrt{\lambda}$ — aspect ratio $\alpha = r_L/r_\beta$
$\longrightarrow$ more complicated phase diagram

- Solid expectations at both low and high temperatures
Same sort of transition in each limit, with different dependence on r_β vs. r_L
(all phases still thermally deconfined $\longleftrightarrow$ dual black holes)

- High-temperature bosonic quantum mechanics transition
with Wilson line order parameter $W_L = \frac{1}{N}\left\langle |\mathrm{Tr}\left[\mathcal{P}e^{i\oint_L A}\right]|\right\rangle$ around spatial circle
Large-r_L "spatial confinement" ($W_L \neq 0$) to "spatial deconf." ($W_L = 0$) at small r_L
Order of transition debated:
first-order vs. strong second-order plus nearby Gross–Witten–Wadia

- Low-temperature large-N classical SUGRA transition
from homogeneous D1 "black strings" with horizon $\mathbb{R} \times S^7$
to D0 black holes with horizon S^8 localized on spatial circle
(Radial direction U and time fill out 10 dimensions in total)

- T duality relates small $r_L \lesssim c_{GL} r_\beta^2$ Type IIB winding mode instability
to Type IIA classical Gregory–Laflamme transition

- Non-orthogonal basis vectors of triangular lattice
$\longrightarrow$ **skewed** tori, "generalized" thermal ensemble

- **Restricted** $\mathrm{SL}(2,\mathbb{Z})$ modular transformations describe same torus geometry
despite sometimes changing skewed $\longrightarrow$ rectangular
$$\begin{pmatrix} \vec{L}' \\ \vec{\beta}' \end{pmatrix} = M \cdot \begin{pmatrix} \vec{L} \\ \vec{\beta} \end{pmatrix} \qquad\qquad M = \begin{pmatrix} a & 2n \\ c & 2m-1 \end{pmatrix} \in \mathrm{SL}(2,\mathbb{Z})$$
with $n, m, c \in \mathbb{Z} \ \longrightarrow \ a \in 2\mathbb{Z} - 1$

- **Numerical results:** Horizon $\longleftrightarrow$ distribution of Wilson line eigenvalue phases
In D1 / spatially confined phase, distribution more uniform as N increases,
$E/(N^2\lambda) \propto t^3$ from leading-order SUGRA
In D0 / spatially deconfined phase, distribution more localized as N increases,
$E/(N^2\lambda) \propto t^{3.2}$ from leading-order SUGRA

- Obviously room for improvement compared to quantum mechanics case

Deformed quantum mechanics (Berenstein–Maldacena–Nastase, '02)

- Dimensional reduction of 10d plane-wave background
  that preserves all 16 supersymmetries

  $$S = S_0 - \delta S$$
  $$\delta S = \frac{N}{2\lambda}\int dt\ \mathrm{Tr}\left[\frac{\mu^2}{3}(X^i)^2 + \frac{\mu^2}{6}(X^a)^2 + \frac{\mu}{24}\Psi^\alpha \epsilon_{ijk}\left(\gamma^i\gamma^j\gamma^k\right)_{\alpha\beta}\Psi^\beta + i\frac{2\mu}{3}\epsilon_{ijk}X^iX^jX^k\right]$$

  with $i, j, k = 1, 2, 3$ and $a = 4, \cdots, 9$
  $\longrightarrow$ dim'ful $\mu \neq 0$ breaks SO(9) R symmetry to SO(3)×SO(6)

- Deformation lifts moduli space $\longrightarrow$ discrete set of vacua
  Can also regulate low-t instability, though may need small $\mu \sim 1/N$

- Now have non-trivial phase diagram in plane of T/μ vs. dim'less $g = \lambda/\mu^3$
  and can consider strong coupling at both large and small T/μ

- Phase diagram / transition "qualitatively similar" to 2d case (arXiv:1411.5541)
  although lose thermal deconfinement $\longrightarrow$ energy scales $\propto N^0$ rather than N^2
  no dual black holes(?)

- First-order Hagedorn transition at $g = 0$; Hawking–Page-like transition as $g \to \infty$

Higher dimensions

- Empirically, smaller N suffice to maintain stability for lower temperatures
  $\longrightarrow$ may amount to trading d.o.f. between space-time and
  Expect small $1/N^2$ corrections in 4d with adjoint fermions

- In uniform D2 phase, $E/(N^2\lambda^3) \propto t^{10/3}$ from leading-order SUGRA

- "3/4 problem" in four dimensions:
  Perturbative energy cN^2T^4 for small $\lambda \to 0$
  vs. holographic $\frac{3}{4} \times cN^2T^4$ for large $1 \ll \lambda \ll N$

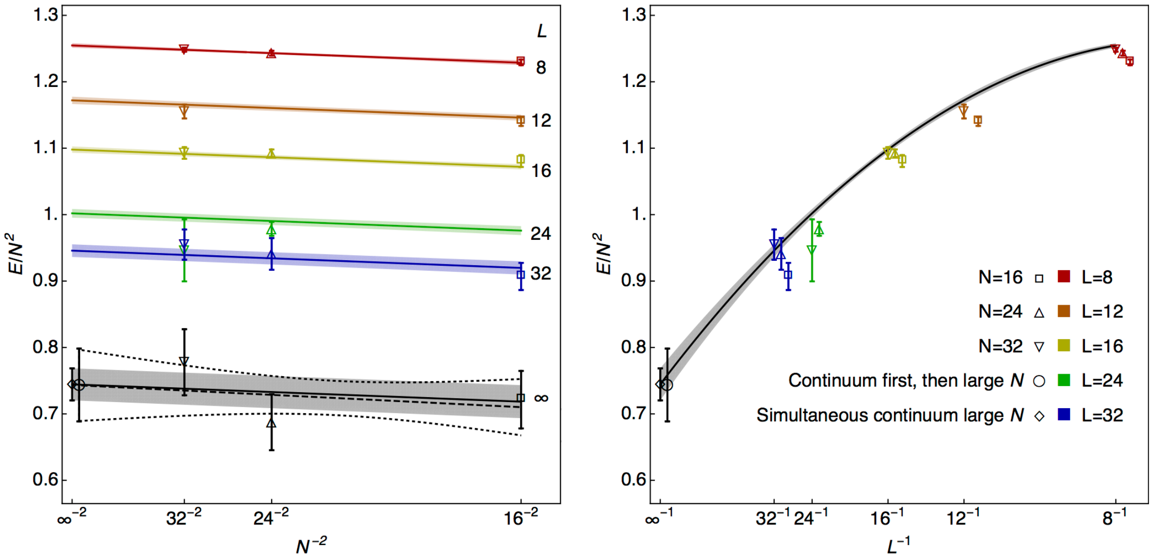

Figure 1: Lattice results for the dual black hole internal energy of D0 brane quantum mechanics, from arXiv:1606.04951. This work considers gauge groups SU(N) with $16 \leq N \leq 32$ on lattices with up to $L = 32$ sites. These two plots fix the dimensionless temperature $T = T_{\text{dim}}/\lambda^{1/3} = 0.5$ and show both individual and combined extrapolations to the limits $N^2 \to \infty$ and $L \to \infty$. The latter is the continuum limit in which the lattice UV cutoff is removed while the large-N limit suppresses string quantum (g_s) corrections in the dual gravitational calculation.

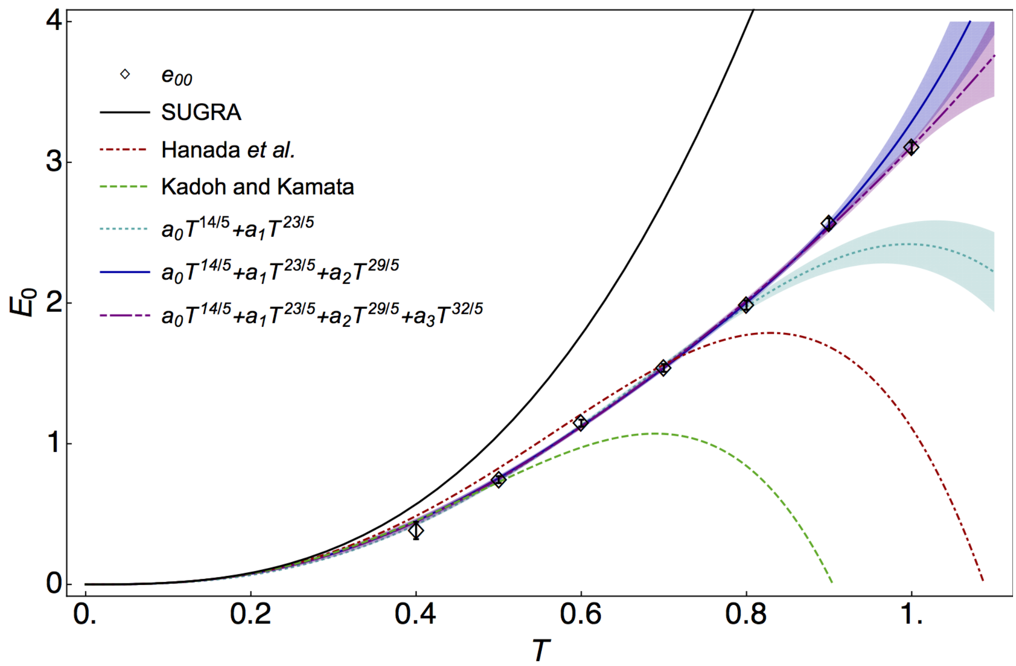

Figure 2: Extrapolated lattice results for the dual black hole internal energy of D0 brane quantum mechanics, from arXiv:1606.04951, using combined $N^2 \to \infty$ and $L \to \infty$ extapolations like the one shown in Fig. 1 for $T = 0.5$. The results are consistently below the leading-order SUGRA prediction (solid black line), but can be fit to expressions including subleading corrections (three colored curves with error bands). These fits correctly reproduce the leading $a_0 = 7.41$ predicted by SUGRA, and provide lattice predictions for the unknown coefficients a_i for $i \geq 1$. The two colored curves without error bands are results from earlier studies with smaller N and L.

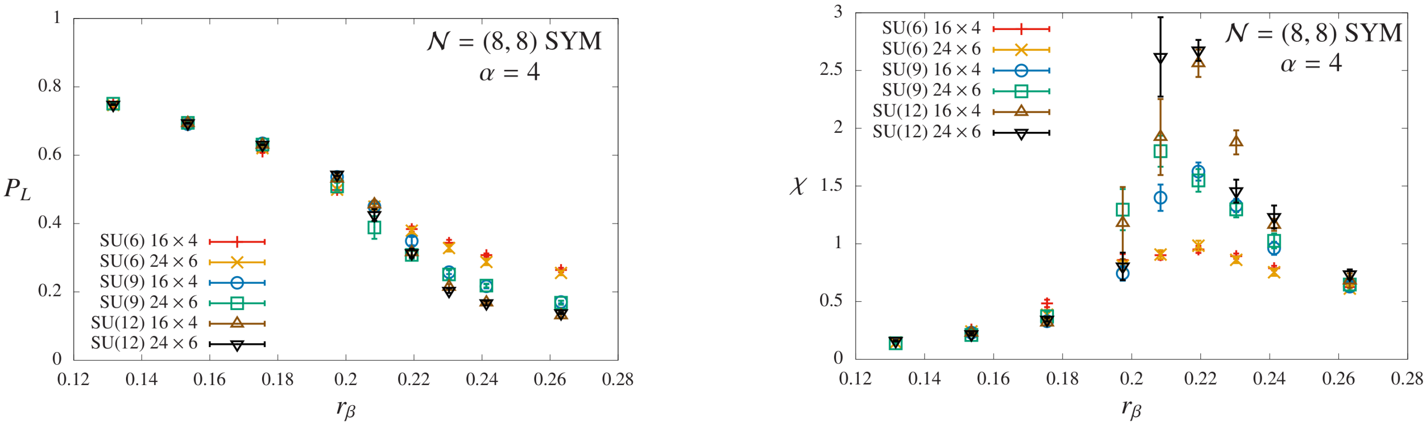

Figure 3: ...

Figure 4: ...