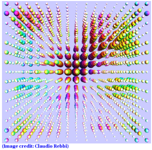
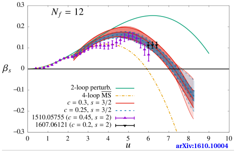
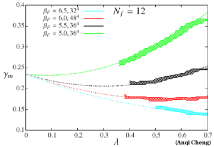
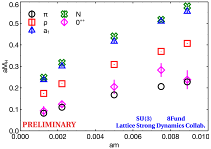
Lattice gauge theory at the electroweak scale









David Schaich (U. Bern)

Strong dynamics at the electroweak scale
Montpellier, 6 December 2017

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Overview and plan

Lattice gauge theory is a broadly applicable tool
to study strongly coupled systems

Especially important
when QCD-based intuition may be unreliable

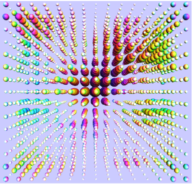
A high-level summary of lattice gauge theory

β functions and anomalous dimensions

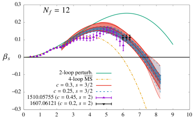
Light scalar from near-conformal dynamics

More possible topics for discussion

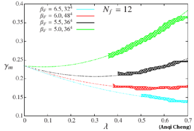
- Electroweak S parameter
- Composite dark matter
- Multi-rep. composite Higgs UV completions
- . . .



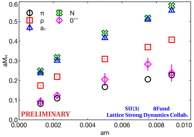
(Image credit: Claudio Rebbi)



arXiv:1610.10004

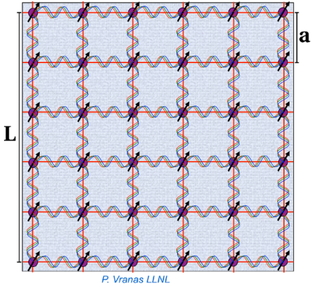


(Anqi Cheng)



The essence of lattice gauge theory

Lattice discretization is a non-perturbative regularization of QFT



Formulate theory on finite, discrete euclidean space-time $\longrightarrow$ **the lattice**

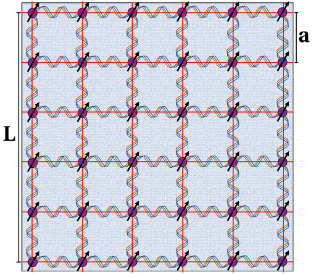
Spacing between lattice sites ("a") $\longrightarrow$ UV cutoff scale $1/a$

Removing cutoff: $a \to 0$ (with $L/a \to \infty$)

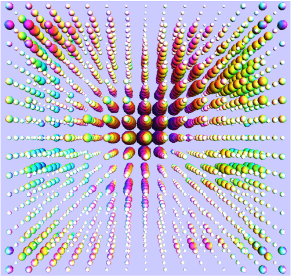
Finite number of degrees of freedom $\left(\sim 10^9\right)$ $\longrightarrow$ numerically compute observables via importance sampling

$$\langle\mathcal{O}\rangle = \frac{1}{\mathcal{Z}} \int \mathcal{D}U \;\; \mathcal{O}(U) \;\; e^{-S[U]} \quad \longrightarrow \quad \frac{1}{N} \sum_{k=1}^{N} \mathcal{O}(U_k)$$

Features of lattice gauge theory



P. Vranas LLNL



(Image credit: Claudio Rebbi)

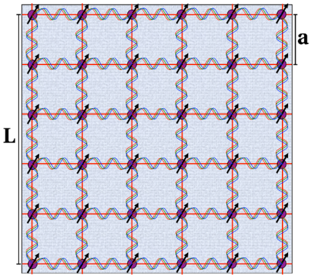
Fully non-perturbative predictions from first principles (lagrangian)

Fully gauge invariant—no gauge fixing required

Applies directly in four dimensions

Euclidean SO(4) rotations & translations ($\longrightarrow$ Poincaré symmetry) recovered automatically in the $a \to 0$ continuum limit

Limitations of lattice gauge theory



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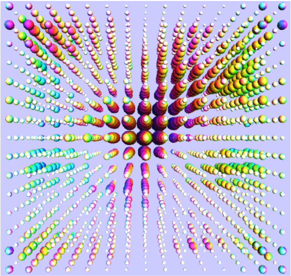


(Image credit: Claudio Rebbi)

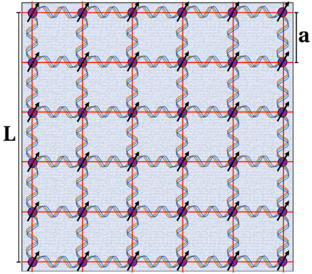
Need UV completion, (usually) include only strong sector

Finite volume (usually) needs to contain all correlation lengths
$\longrightarrow$ unphysically large masses extrapolated to chiral limit via EFT

Chiral symmetry of lattice fermion operator complicated

Obstructions to chiral gauge theories, real-time dynamics, susy

Limitations of lattice gauge theory



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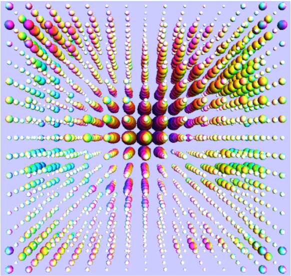

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Lattice fermion discretizations

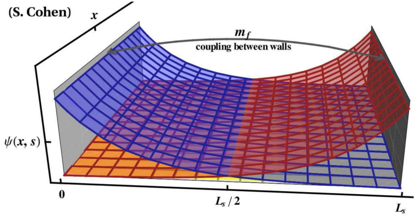
Tension between chiral symmetry vs. ‘doubling’ of lattice fermions

Naive $\longrightarrow$ $16F$ continuum fermions from F lattice fields,
large $\mathrm{U}(4F)_V \times \mathrm{U}(4F)_A$ chiral symmetry

Staggered $\longrightarrow$ $4F$ continuum fermions, $\mathrm{U}(F)_V \times \mathrm{U}(F)_A$ chiral symm.

Wilson $\longrightarrow$ F continuum fermions, no chiral symmetry

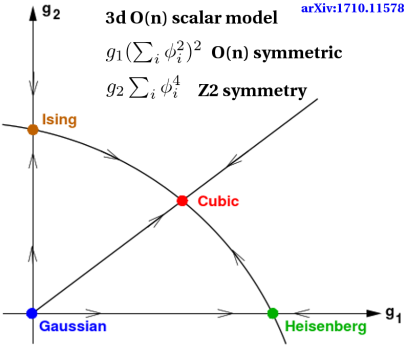
Domain wall $\longrightarrow$ F continuum fermions,
lattice “remnant” $\mathrm{SU}(F)_V \times \mathrm{SU}(F)_A$ chiral symmetry



Symmetries of lattice fermions

Different lattice symmetries for fixed N_F continuum fermions

Domain wall	**Staggered**	**Wilson**
$SU(N_F)_V \times SU(N_F)_A$	$U(N_F/4)_V \times U(N_F/4)_A$	None



All $\longrightarrow$ same UV continuum limit
('lattice universality')

Possibility
different lattice symmetries
$\longrightarrow$ different IR dynamics?

Example of 3d $O(n)$ scalar model

Lattice gauge theory beyond QCD

Lattice calculations especially important for non-QCD strong dynamics

Exploratory investigations of representative systems
$\longrightarrow$ elucidate generic dynamical phenomena, connect with EFT

arXiv:1309.1206 arXiv:1510.05018 arXiv:1701.07782

Executive Summary

- Use the Higgs boson as a new tool for discovery
- Pursue the physics associated with neutrino mass
- Identify the new physics of dark matter
- Understand cosmic acceleration: dark energy and inflation
- Explore the unknown: new particles, interactions, and physical principles.

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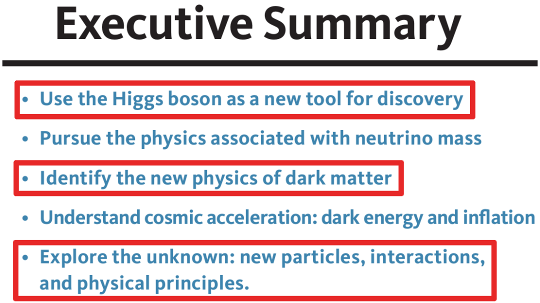

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Non-QCD strong dynamics

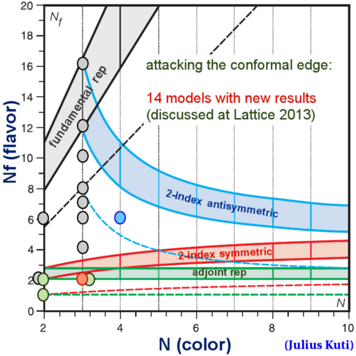
Two main directions (not mutually exclusive)

Near-conformal dynamics from many fermionic d.o.f.

$\longrightarrow$ large number of fundamental fermions or a few in a larger rep

Different symmetries from different gauge group or reps

$\longrightarrow$ (pseudo)real reps for cosets SU(n)/Sp(n) or SU(n)/SO(n)



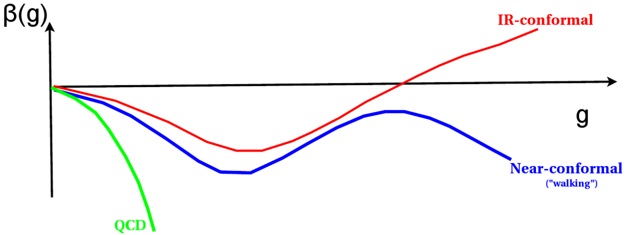
Today focus on near-conformality

Study a few representative systems,
look for similarities/difference vs. QCD

Start with non-perturbative β function

β function motivation

$$\beta = \frac{dg^2}{d \log \mu^2} \longrightarrow \text{scale dependence of running coupling}$$



Perturbative $\quad \beta(g^2) = -\frac{g^4(\mu^2)}{16\pi^2}\left[b_1 + b_2 \frac{g^2(\mu^2)}{16\pi^2}\right] + \mathcal{O}\left(g^8\right)$

Asymptotic freedom in UV $\longrightarrow$ $b_1 = \frac{1}{3}\left[11 C_2(G) - 4 N_F T(R)\right] > 0$

$b_2 < 0$ might give non-trivial conformal fixed point in IR

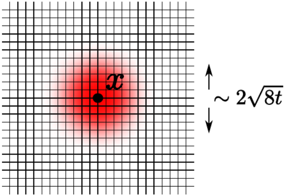
Banks & Zaks make argument rigorous for $b_1 \approx 0$

Lattice g^2 for non-perturbative β function

First step: Define measurable g^2 with scale given by lattice size L

Use Yang–Mills gradient flow
(integrating infinitesimal smoothing operation)

Local observables measured after "flow time" t
depend on original fields within $r \simeq \sqrt{8t}$



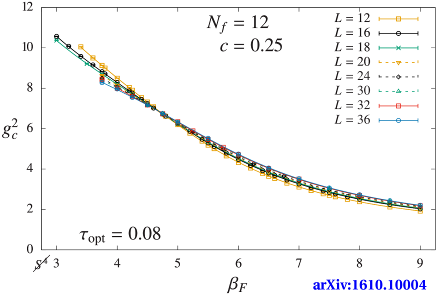
Flowed energy density $E(t) = -\frac{1}{2}\mathrm{Tr}\left[G_{\mu\nu}(t)G^{\mu\nu}(t)\right]$
perturbatively gives $g^2_{\overline{\mathrm{MS}}}(\mu) \propto t^2 E(t)$ with $\mu = 1/\sqrt{8t}$

Tie to lattice size by defining $g_c^2(L; a)$ at fixed $c = L/\sqrt{8t}$
(scheme dependent as expected)

Step scaling for non-perturbative β function

Next step: Scale change $L \longrightarrow sL$ gives discrete β function

$$\beta_s(g_c^2; L) = \frac{g_c^2(sL; a) - g_c^2(L; a)}{\log(s^2)} \quad \xrightarrow{s\to 1} \quad -\beta\left(g^2(\mu^2)\right)$$



$N_F = 12$ staggered fermions,
bare coupling $\beta_F \simeq 12/g_0^2$

With $s = 3/2$ have

$L = 12 \to 18$	$16 \to 24$
$20 \to 30$	$24 \to 36$

$s = 2$ and $4/3$ also accessible

g_c^2 for all L cross around $g_c^2 \approx 7 \longrightarrow \beta_s(g_c^2; L) = 0$
Does β_s remain zero as $L \to \infty$?

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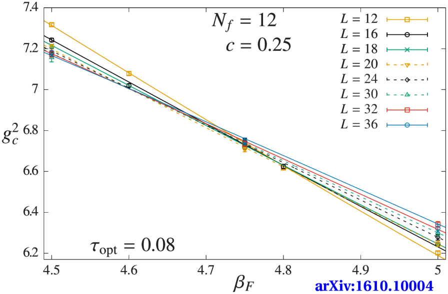

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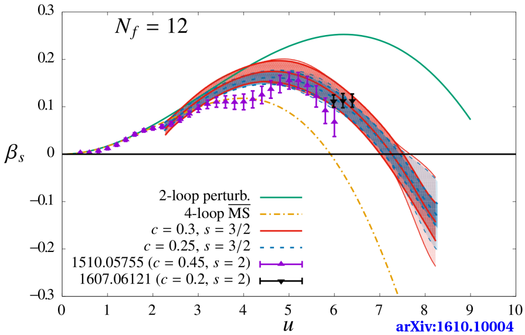
Continuum extrapolation

Final step: Extrapolate $(a/L) \to 0$ to obtain continuum $\beta_s(g_c^2)$



$N_F = 12$ staggered results seemed broadly consistent

Even for different schemes and scale changes s

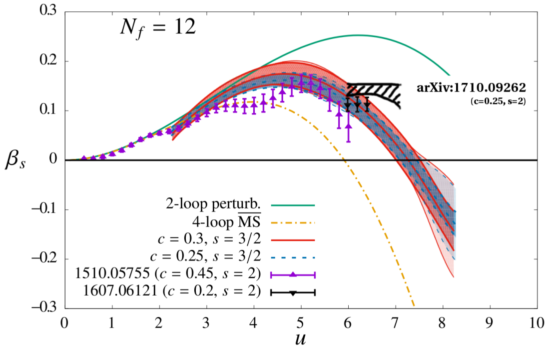
Slope at fixed point $g_\star^2 \approx 7.3$
$\longrightarrow \gamma_g^\star = -0.26(2)$
(scheme independent)

Simple $(a/L)^2 \to 0$ extrapolations fine near gaussian UV fixed point

May need $g_c^2(L; a) - g_\star^2 \propto L^{\gamma_g^\star}$ finite-size scaling near IR fixed point…

Current status of staggered $N_F = 12$ β function

Developing tension between two independent staggered analyses
$\longrightarrow$ not yet consensus about $N_F = 12$ fixed point



Same lattice symmetries
$\longrightarrow$ same fixed point

Despite details of
lattice action, analyses

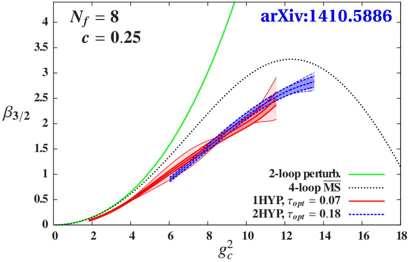
Main difference is
larger $sL \leq 56$ vs. 36

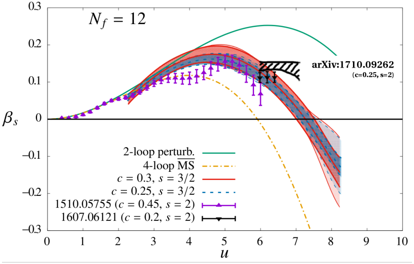
Tension related to $(a/L)^2 \to 0$ extrapolations vs. finite-size scaling?

β function wrap-up: Challenge I

β function becomes very small as N_F increases

Order of magnitude decrease for $N_F = 8$ (left) vs. $N_F = 12$ (right)



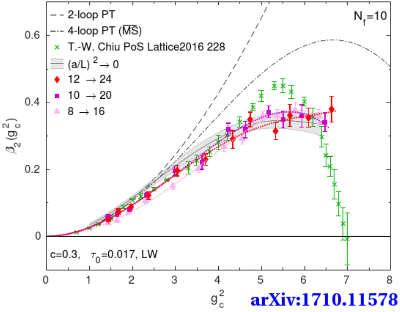


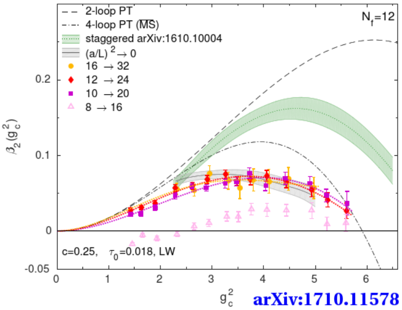
Hard to distinguish slow running vs. no running on finite lattices

β function wrap-up: Challenge II

Different symmetries of lattice fermions

$\longrightarrow$ IR fixed points in different universality classes?





Recently reported tensions between staggered vs. domain wall results

$\longrightarrow$ currently developing story

Anomalous dimension motivation

At IR fixed point, universal anomalous dimensions $\gamma^\star$
$\longrightarrow$ scheme-independent critical exponents characterizing CFT

Large γ wanted for fermion mass generation by new strong dynamics
(hopefully discussed in previous talk)

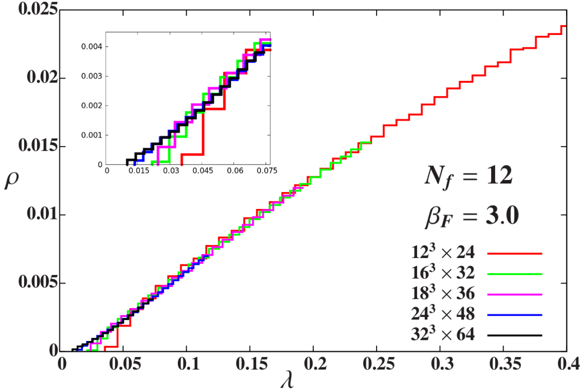
Near-conformality $\longrightarrow$ scheme and scale dependence negligible?

Plan: Focus on staggered $N_F = 12$ IRFP

- Already saw $\gamma_g^\star \approx -0.26$ from slope of β function
- Extract mass anomalous dimension $\gamma_m^\star$ from Dirac eigenmodes
- Extract $\gamma_m^\star$ and $\gamma_g^\star$ from spectrum finite-size scaling
- Prospects for baryon anomalous dim. for partial compositeness

$\gamma_m^\star$ from Dirac eigenvalue mode number $\nu(\lambda)$

$\mathcal{L} \supset \overline{\psi}\,(\not{D} + m)\,\psi \quad \longrightarrow \quad \not{D}$ eigenvalues sensitive to $\gamma_m^\star = 3 - \mathrm{d}\left[\overline{\psi}\psi\right]$



Histogram of eigenvalues
$\longrightarrow$ spectral density $\rho(\lambda)$

Integral is **mode number**

$$\nu(\lambda) = 2V \int_0^\lambda \rho(\omega) d\omega$$

Conformal FP: $\rho(\lambda) \propto \lambda^\alpha$
$\longrightarrow \nu(\lambda) \propto \lambda^{1+\alpha}$

Mode number RG invariant $\longrightarrow 1 + \gamma_m^\star = \dfrac{4}{1+\alpha}$ (Del Debbio & Zwicky)

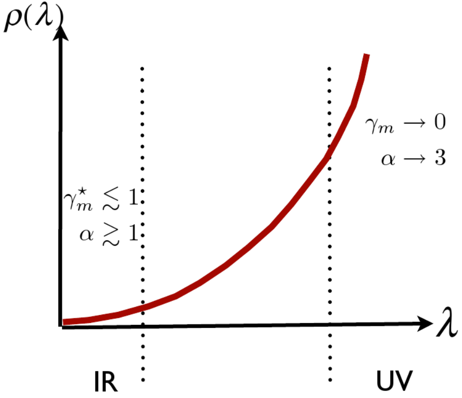
Scale-dependent $\gamma_{\text{eff}}(\lambda)$ from eigenmodes

λ defines energy scale $\longrightarrow$ $\nu(\lambda)$ gives effective $\gamma_{\text{eff}}(\lambda)$ at that scale

UV: Asymp. freedom $\Rightarrow \gamma_{\text{eff}}(\lambda) \to 0$
or $\alpha(\lambda) \to 3$

IRFP $\Longrightarrow$ $\gamma_{\text{eff}}(\lambda) \xrightarrow{\lambda \to 0} \gamma_m^\star$

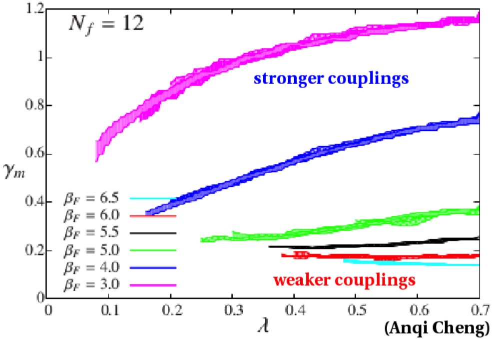
$\langle\overline{\psi}\psi\rangle \propto \rho(0) \neq 0 \implies \alpha \to 0,$
breakdown of $\rho(\lambda) \propto \lambda^\alpha$



Monitor $\gamma_{\text{eff}}(\lambda)$ evolution from perturbative UV to strongly coupled IR

$\gamma_{\text{eff}}(\lambda)$ from eigenmodes for $N_F = 12$

Fit $\nu(\lambda) \propto \lambda^{1+\alpha}$ in small range of λ $\longrightarrow$ $1 + \gamma_{\text{eff}}(\lambda) = \dfrac{4}{1+\alpha(\lambda)}$



$\nu(\lambda)$ computed stochastically

Include fit ranges in error bands

Multiple L^4 volumes overlaid,
L-sensitive data dropped

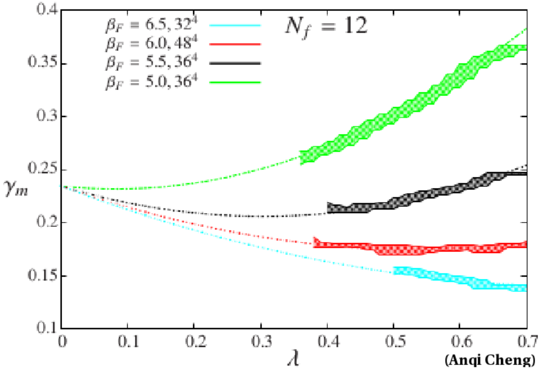
All systems have $\rho(0) = 0$

Strong dependence on irrelevant bare coupling $\beta_F \simeq 12/g_0^2$

γ_{eff} increasing with λ $\sim$ "backward flow" at strong coupling

$\gamma_m^\star(\lambda)$ from eigenmodes for $N_F = 12$

Extrapolate $\lim\limits_{\lambda\to 0} \gamma_{\text{eff}}(\lambda) = \gamma_m^\star$ at conformal IR fixed point



Zoom in on largest volumes, couplings closest to $g_\star^2$ (in this scheme)

Joint quadratic extrapolation $\longrightarrow \gamma_m^\star = 0.24(3)$

Uncertainty dominated by $\lambda \to 0$ extrapolation

Single fit for some range of $\lambda > 0$ would give precise result

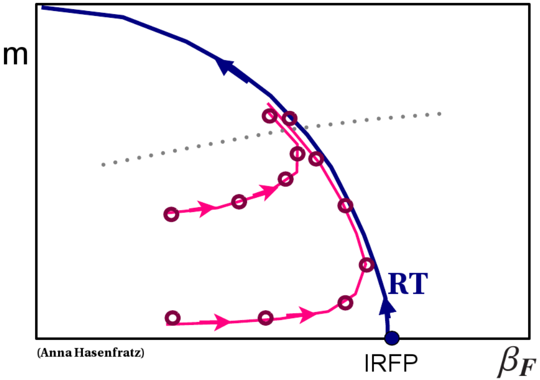
but generally **not** $\gamma_m^\star$ at the $\lambda \to 0$ IR fixed point

Wilson RG picture of finite-size scaling

Fermion mass m is relevant coupling; gauge coupling β_F is irrelevant

Increase m and decrease RG flow (L)

$\longrightarrow$ same point on renormalized trajectory (RT)



Universal flow along RT

Correlation lengths depend on scaling variable

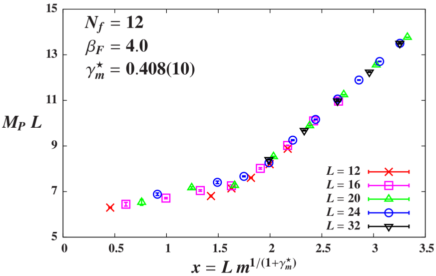
$$x \equiv L\, m^{1/(1+\gamma_m^\star)}$$

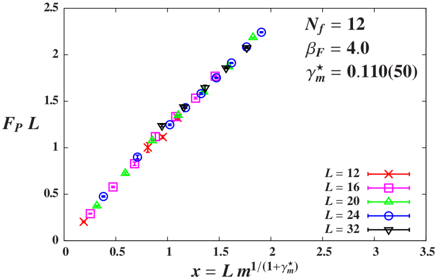
Assuming RG flow quickly reaches RT

Naive finite-size scaling for $N_F = 12$

Correlation lengths depend on scaling variable $x \equiv L\, m^{1/(1+\gamma_m^\star)}$

$\longrightarrow \gamma_m^\star$ from optimizing **curve collapse** of $M_H L = f_H(x)$





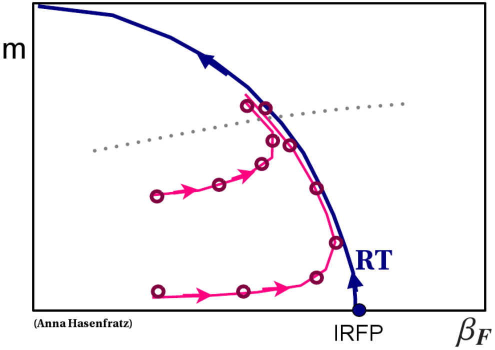
Curve collapse $\longrightarrow$ non-universal $\gamma_m^\star$ from different observables

Conformality requires universal $\gamma^\star$

$\longrightarrow$ corrections to scaling from near-marginal gauge coupling?

Corrections to finite-size scaling

Slowly running gauge coupling $\longrightarrow$ RG flow may not reach RT
$\longrightarrow$ non-universal results from curve collapse



Leading correction to scaling:

$$M_H L = f_H(x, gm^{\omega})$$

where $\omega = -\gamma_g^{\star}/(1+\gamma_m^{\star})$

Two-loop $\overline{\text{MS}}$: small $\omega \approx 0.2$

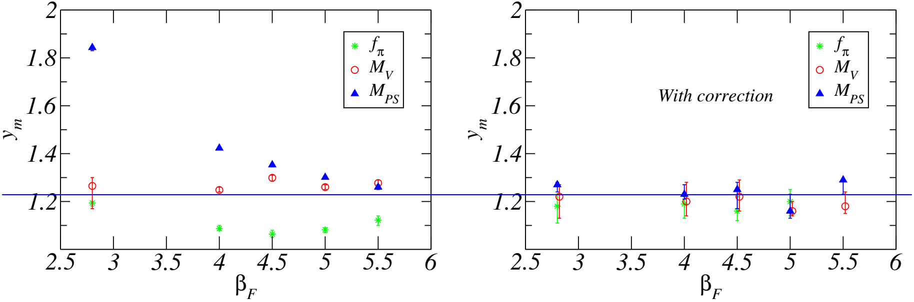
Hard to extract both $\gamma_m^{\star}$ and $\gamma_g^{\star}$ from curve collapse analyses
$\longrightarrow$ simplify $f_H(x, gm^{\omega}) \approx f_H(x)\left[1 + c_g m^{\omega}\right]$

Consistent corrected finite-size scaling for $N_F = 12$

Approximate $M_H L \approx f_H(x)\left[1 + c_g m^{\omega}\right]$

$\longrightarrow$ consistent $\gamma_m^{\star}$ from all observables and β_F

Quality of curve collapse also improves



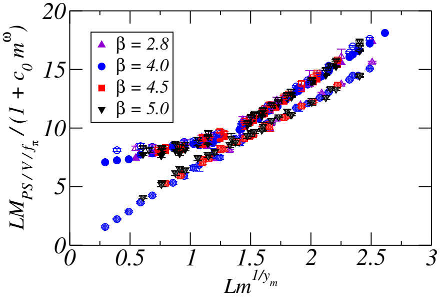
Can attempt combined analyses of multiple data sets. . .

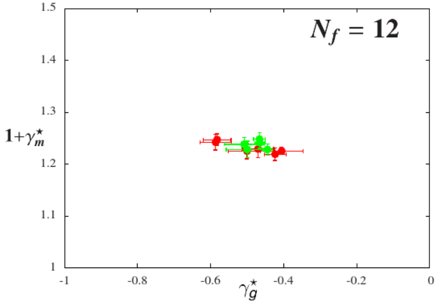
Combined finite-size scaling analyses for $N_F = 12$

Approximate $M_H L \approx f_H(x)\left[1 + c_g m^\omega\right]$

$\longrightarrow$ consistent $\gamma_m^\star$ from all observables and β_F

Combined analyses of multiple data sets better constrain $\gamma_m^\star$ and $\gamma_g^\star$





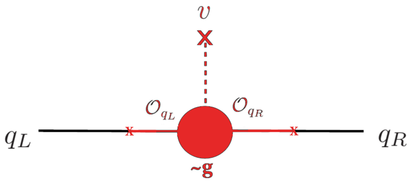
Result from green points: $\gamma_m^\star = 0.235(15)$ and $\gamma_g^\star \simeq -0.5$

Baryon anomalous dim. for partial compositeness

SM fermions q couple linearly to $\mathcal{O}_q \sim \psi\psi\psi$ of new strong dynamics

$$\longrightarrow m_q \sim v \left(\frac{\text{TeV}}{\Lambda_F}\right)^{4-2\gamma_3}$$

with $\gamma_3 = \frac{9}{2} - \text{d}[\psi\psi\psi]$



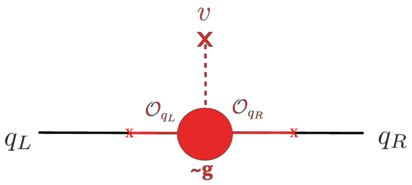
Large mass hierarchy $\longleftrightarrow$ $\mathcal{O}(1)$ anomalous dimensions

Example: With $\Lambda_F = 10^{10}$ TeV, $\mathcal{O}$(MeV) quarks need $\gamma_3 \approx 1.75$
$\mathcal{O}$(GeV) quarks need $\gamma_3 \approx 1.9$

Compute $\gamma_{\mathcal{O}} = -\dfrac{d \log Z_{\mathcal{O}}(\mu)}{d \log \mu}$,

$Z_{\mathcal{O}}(\mu)$ from standard lattice RI/MOM non-perturbative renormalization

Baryon anomalous dim. for partial compositeness



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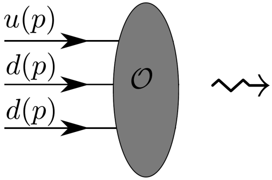
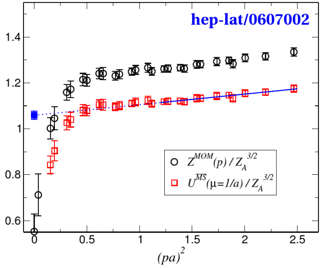
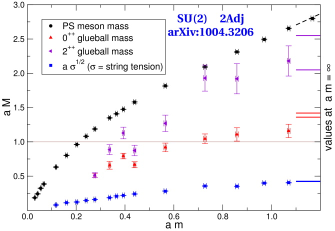


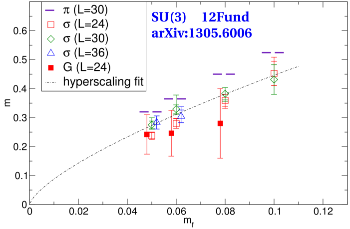


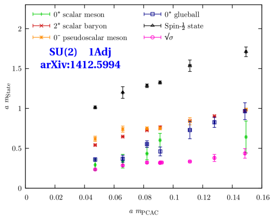
$N_F = 10, 12$ DWF pilot studies starting, re-using β function work

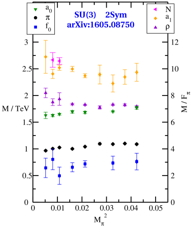
Light scalars from beyond-QCD lattice calculations

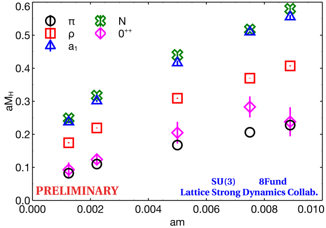
All near-conformal lattice studies so far observe light singlet scalar qualitatively different from QCD



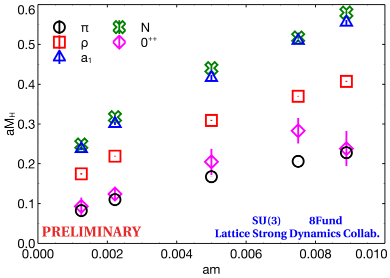


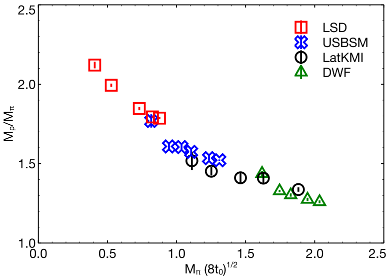






Light scalar in 8-flavor SU(3) spectrum



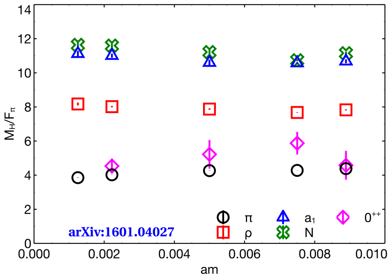


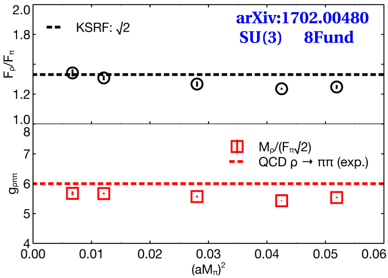
Flavor-singlet scalar degenerate with pseudo-Goldstones
down to lightest masses that fit into $64^3 \times 128$ lattices

Both M_S and M_P less than half the vector mass M_V,
hierarchy growing as we approach the chiral limit
$\longrightarrow$ qualitatively different from QCD

Controlled chiral extrapolations need EFT that includes scalar…

Vector resonance generically QCD-like





Without EFT, roughly constant ratio $M_V/F_P \simeq 8 \rightsquigarrow M_V \simeq 2$ TeV$/\sqrt{\xi}$

[NB: expect $M_P/F_P \to 0$ in chiral limit!]

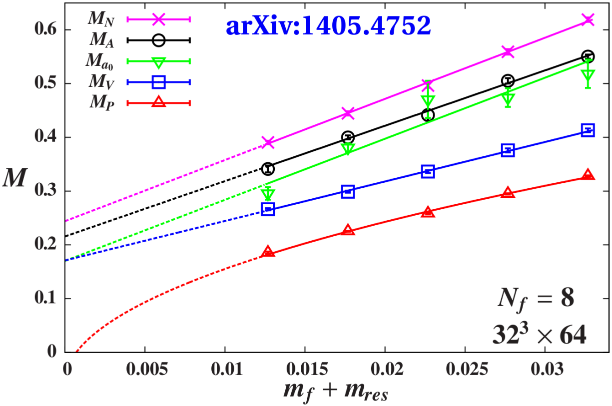
We measure $F_V \approx F_P\sqrt{2}$ (KSRF relation, suggesting vector domin.)

Applying second KSRF relation $g_{VPP} \approx M_V/(F_P\sqrt{2})$

$$\longrightarrow \text{vector width } \Gamma_V \approx \frac{g^2_{VPP} M_V}{48\pi} \simeq 450 \text{ GeV — hard to see at LHC}$$

QCD-like non-singlet scalar a_0 for $N_F = 8$

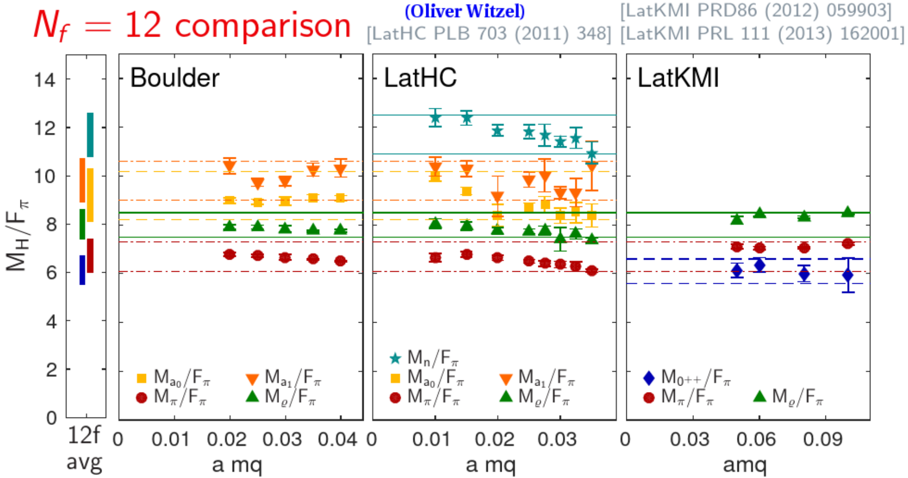
May be relevant for holographic approaches. . .



Earlier work with domain wall fermions farther from chiral limit

$\longrightarrow$ non-singlet scalar a_0 heavier than vector, $M_{a_0} \gtrsim M_V$

QCD-like non-singlet scalar a_0 for $N_F = 12$



Staggered $N_F = 12$ results also show $M_{a_0} \gtrsim M_V$

Analyses complicated by staggered spin–flavor mixing

Work in progress: Constraining EFT

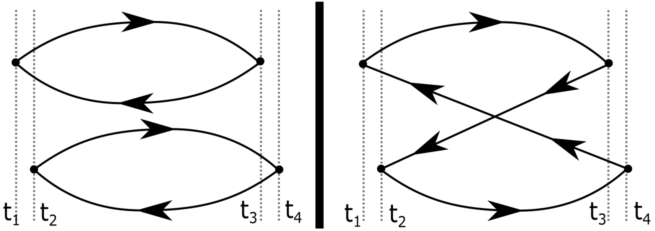
There are many candidate EFTs that include PNGBs + light scalar

(linear σ model; Goldberger–Gristein–Skiba; Soto–Talavera–Tarrus; Matsuzaki–Yamawaki; Golterman–Shamir; Hansen–Langaeble–Sannino; Appelquist–Ingoldby–Piai)

Need lattice computations of more observables to test EFTs

Now computing $2 \to 2$ elastic scattering of PNGBs & scalar, scalar form factor of PNGB

Subsequent step: Analog of πK scattering in mass-split system

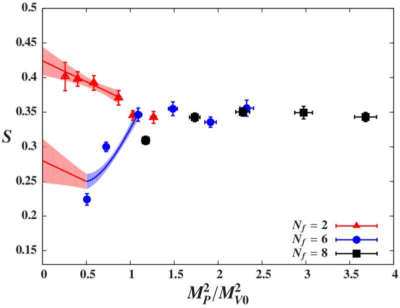


S parameter on the lattice

$$\mathcal{L}_\chi \supset \frac{\alpha_1}{2} g_1 g_2 B_{\mu\nu} \mathrm{Tr}\left[U \tau_3 U^\dagger W^{\mu\nu}\right] \longrightarrow$$

Lattice vacuum polarization calculation provides $S = -16\pi^2 \alpha_1$

Non-zero masses and chiral extrapolation needed
to avoid sensitivity to finite lattice volume



$S = 0.42(2)$ for $N_F = 2$
matches scaled-up QCD

Larger $N_F \longrightarrow$ significant reduction

Extrapolation to correct zero-mass limit
becomes more challenging

Vacuum polarization from current correlator

$$S = 4\pi N_D \lim_{Q^2 \to 0} \frac{d}{dQ^2} \Pi_{V-A}(Q^2) - \Delta S_{SM}(M_H)$$

$$\Pi^{\mu\nu}_{V-A}(Q) = Z \sum_x e^{iQ\cdot(x+\widehat{\mu}/2)} \mathrm{Tr}\left[\left\langle \mathcal{V}^{\mu a}(x) V^{\nu b}(0)\right\rangle - \left\langle \mathcal{A}^{\mu a}(x) A^{\nu b}(0)\right\rangle\right]$$

$$\Pi^{\mu\nu}(Q) = \left(\delta^{\mu\nu} - \frac{\widehat{Q}^\mu \widehat{Q}^\nu}{\widehat{Q}^2}\right) \Pi(Q^2) - \frac{\widehat{Q}^\mu \widehat{Q}^\nu}{\widehat{Q}^2} \Pi^L(Q^2) \qquad \widehat{Q} = 2\sin(Q/2)$$

- Renormalization constant Z evaluated non-perturbatively
  Chiral symmetry of domain wall fermions $\Longrightarrow Z = Z_A = Z_V$
  $Z = 0.85$ [2f]; 0.73 [6f]; 0.70 [8f]

- Conserved currents $\mathcal{V}$ and $\mathcal{A}$ ensure that lattice artifacts cancel

Composite dark matter

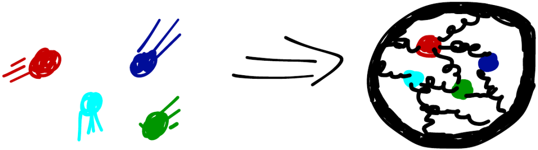
Many possibilities: (arXiv:1604.04627)

dark baryon, dark nuclei, dark pion, dark quarkonium, dark glueball...

Example: Stealth Dark Matter (arXiv:1503.04203, arXiv:1503.04205)

Deconfined charged fermions $\longrightarrow$ relic density

Confined SM-singlet dark baryon $\longrightarrow$ direct detection via form factors



For QCD-like SU(3) baryon, direct detection $\longrightarrow$ $M_{DM} \gtrsim 20$ TeV

due to leading magnetic moment interaction (arXiv:1301.1693)

A lower bound for stealth dark matter

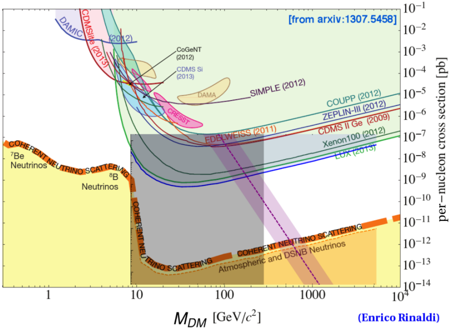
SU(4) bosonic baryons forbid leading magnetic moment
and sub-leading charge radius interactions in non-rel. EFT

EM polarizability is unavoidable — compute it on the lattice
$\longrightarrow$ lower bound on the direct detection rate

Nuclear cross section $\propto Z^4/A^2$,
these results specific to Xenon

Uncertainties dominated
by nuclear matrix element

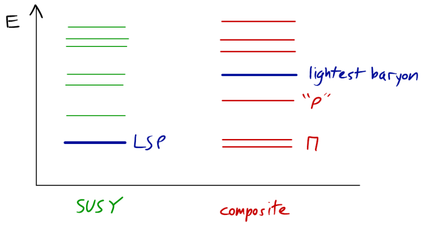
Shaded region is complementary
constraint from particle colliders

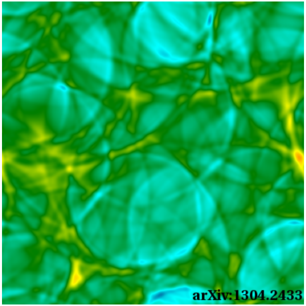


Future plans: Colliders and gravitational waves

Other composite dark-sector states
can be discovered at colliders

Additional lattice input can help
predict production and decays





Confinement transition in early universe
may produce gravitational waves

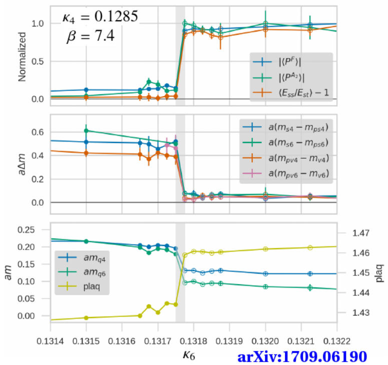
First-order transition $\longrightarrow$ colliding bubbles

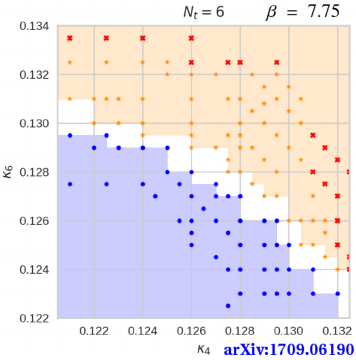
Lattice calculations needed
to predict properties of transition

Multi-rep finite-temperature phase diagram

SU(4) gauge theory with $N_4 = 2$ fund. and $N_6 = 2$ two-index-symm.

Step towards composite Higgs model with $N_4 = 3$ and $N_6 = 2.5$

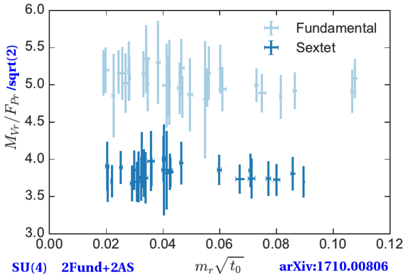


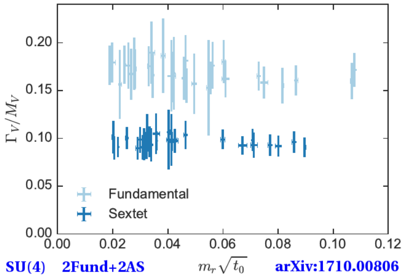


Simultaneous first-order chiral/deconfinement transitions for both reps

Multi-rep mesonic spectrum

Looks broadly consistent with large-N rescalings of QCD





Left: $M_V/F_P \sim 8\sqrt{\frac{3}{4}}\frac{1}{\sqrt{2}} \approx 4.9$

Right: Narrower vector resonance widths expected for larger N

Recap: An exciting time for lattice gauge theory

Lattice gauge theory is a broadly applicable tool
to study strongly coupled systems and BSM physics

Exploring generic features of representative systems beyond QCD

- β functions and anomalous dimensions
- Light scalar from near-conformal dynamics
- Low-energy constants including S parameter
- Composite dark matter and more...

Thank you!

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Recap: An exciting time for lattice gauge theory

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Thank you!

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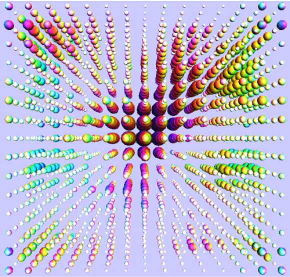
Backup: Hybrid Monte Carlo (HMC) algorithm

Goal: Sample field configurations U with probability $\frac{1}{\mathcal{Z}} e^{-S[U]}$


(Image credit: Claudio Rebbi)

HMC is Markov process based on Metropolis–Rosenbluth–Teller

Fermions $\longrightarrow$ extensive action computation

$\Longrightarrow$ Global updates using fictitious molecular dynamics

1. Introduce fictitious "MD time" τ and stochastic canonical momenta for fields
2. Inexact MD evolution along trajectory in τ $\longrightarrow$ new four-dimensional field configuration
3. Accept/reject test on MD discretization error

Backup: Lattice QCD for BSM

High-precision non-perturbative QCD calculations
reduce uncertainties and help resolve potential new physics

- Hadronic matrix elements & form factors for flavor physics
  Sub-percent precision for easiest observables (arXiv:1607.00299)

- Hadronic contributions to $(g-2)_\mu$ (arXiv:1311.2198)
  Targeting $\sim$0.1% precision for vac. pol., $\sim$10% for light-by-light

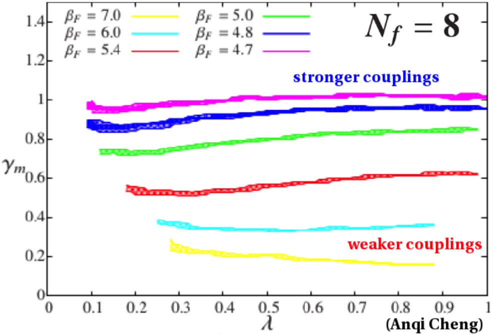
- m_c, m_b and $\alpha_s(m_Z)$ to $\sim$0.1% for Higgs couplings (arXiv:1404.0319)

- High-temp. topological suscept. for axion DM (arXiv:1606.07494)

- Nucleon electric dipole moment, form factors (arXiv:1701.07792)

Backup: $\gamma_{\text{eff}}(\lambda)$ from eigenmodes for $N_F = 8$

Fit $\nu(\lambda) \propto \lambda^{1+\alpha}$ in small range of λ $\longrightarrow$ $1 + \gamma_{\text{eff}}(\lambda) = \dfrac{4}{1 + \alpha(\lambda)}$



$\nu(\lambda)$ computed stochastically

Include fit ranges in error bands

Multiple L^4 volumes overlaid,
L-sensitive data dropped

All systems have $\rho(0) = 0$

Appears to evolve slowly across wide range of scales,
qualitatively different from $N_F = 12$ and QCD-like $N_F = 4$

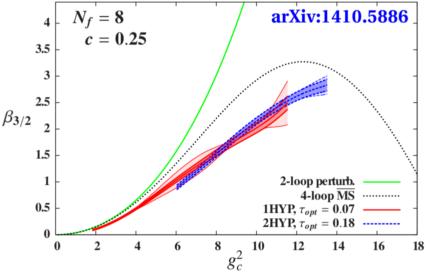
Backup:
Lattice Strong Dynamics Collaboration

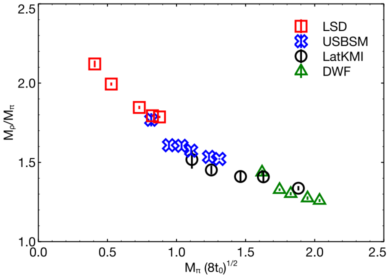
L—S
D

Argonne	Xiao-Yong Jin, James Osborn
Bern	DS
Boston	Rich Brower, Claudio Rebbi, Evan Weinberg
Colorado	Anna Hasenfratz, Ethan Neil, Oliver Witzel
UC Davis	Joseph Kiskis
Livermore	Pavlos Vranas
Oregon	Graham Kribs
RBRC	Enrico Rinaldi
Yale	Thomas Appelquist, George Fleming, Andrew Gasbarro

Exploring the range of possible phenomena
in strongly coupled gauge theories

Backup: 8-flavor SU(3) infrared dynamics





- β function monotonic up to fairly strong $g^2 \sim 14$

  No sign of approach towards conformal IR fixed point $[\beta(g_\star^2) = 0]$

- Ratio M_V/M_P increases monotonically as masses decrease

  as expected for spontaneous chiral symmetry breaking (SχSB)

  Mass-deformed conformal hyperscaling predicts constant ratio

Strengthen conclusion by matching to low-energy EFT

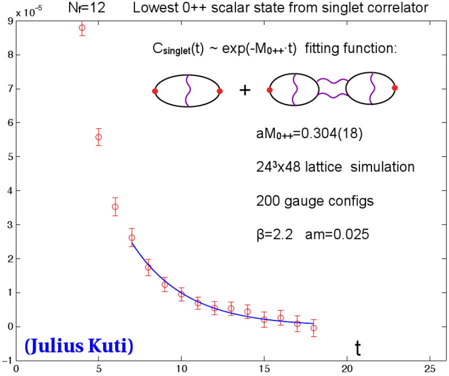
$\longrightarrow$ must go beyond QCD-like χPT to include light scalar...

Backup: Technical challenge for scalar on lattice

Only new strong sector included in the lattice calculations

$\Longrightarrow$ flavor-singlet scalar mixes with the vacuum

Leads to noisy data and relatively large uncertainties

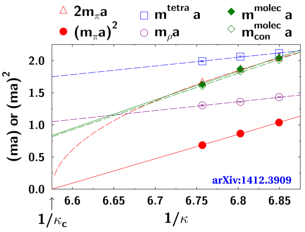


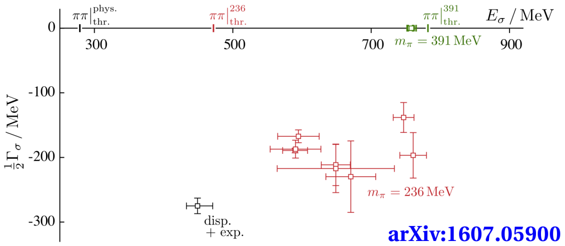
Fermion propagator computation relatively expensive

"Disconnected diagrams" formally need propagators at all L^4 sites

In practice estimate stochastically to control computational costs

Backup: Isosinglet scalar in QCD spectrum





Lattice QCD $\longrightarrow$ isosinglet scalar much heavier than pion

Generally $M_S \gtrsim 2M_P \longrightarrow M_S > M_V$ for heavy quarks

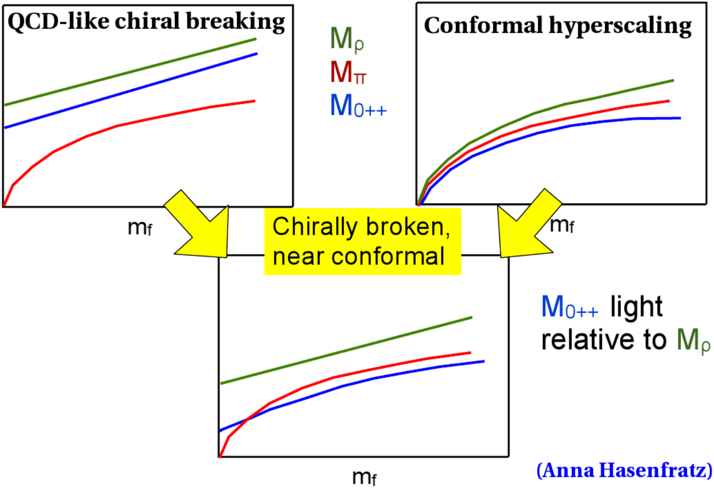
For a large range of quark masses m

it mixes significantly with two-pion scattering states

Backup: Qualitative picture of light scalar

Light scalar likely related to near-conformal dynamics

$\longrightarrow$ possibly dilaton, PNGB of approximate scale symmetry?



M_{0++} light relative to M_ρ

(Anna Hasenfratz)

Backup: $2 \to 2$ elastic scattering on the lattice

Measure both E_{PP} and M_P $\longrightarrow$ $k = \sqrt{(E_{PP}/2)^2 - M_P^2}$

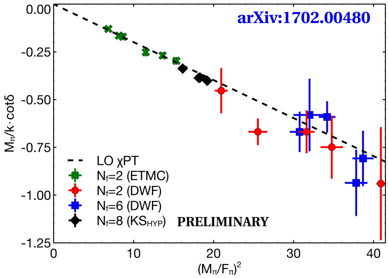
s-wave scattering phase shift: $\cot\delta_0(k) = \frac{1}{\pi k L}\, S\left(\frac{k^2 L^2}{4\pi}\right)$

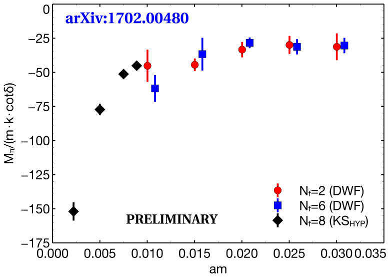
where regularized ζ function $S(\eta) = \sum_{j \neq 0}^{\Lambda} \frac{1}{j^2 - \eta} - 4\pi\Lambda$

Effective range expansion:

$$k \cot\delta_0(k) = \frac{1}{a_{PP}} + \frac{1}{2} M_P^2 r_{PP} \left(\frac{k^2}{M_P^2}\right) + \mathcal{O}\left(\frac{k^4}{M_P^4}\right)$$

Backup: Initial 2 $\rightarrow$ 2 elastic scattering results





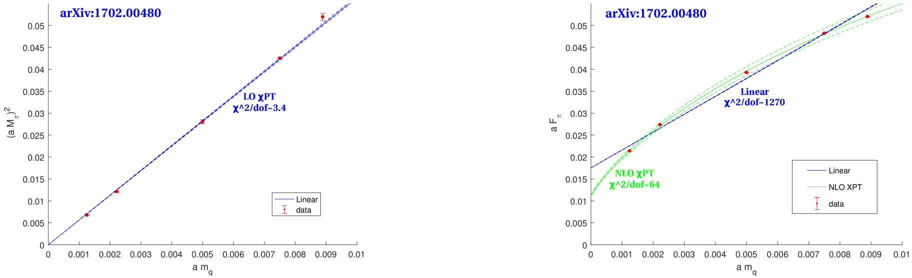
Simplest case: Analog of QCD $I = 2$ $\pi\pi$ scattering
(no fermion-line-disconnected diagrams)

Simplest observable: Scattering length $a_{PP} \approx 1/(k \cot\delta)$

Left: $M_P a_{PP}$ vs. M_P^2/F_P^2 curiously close to leading-order χPT

Right: Divide by fermion mass m $\longrightarrow$ tension with χPT as expected
(predicts constant at LO; involves 8 LECs at NLO)

Backup: 8f chiral perturbation theory (χPT) fits



χPT omits the light scalar **and** suffers from large expansion parameter

$$5.8 \leq \frac{2N_F Bm}{16\pi^2 F^2} \leq 41.3 \qquad \text{for} \qquad 0.00125 \leq m \leq 0.00889$$

$\sim 50\sigma$ shift in F between linear extrapolation vs. NLO χPT

Poor fit quality, especially for NLO joint fit $\left(\chi^2/\text{d.o.f.} > 10^4\right)$

Backup: NLO chiral perturbation theory formulas

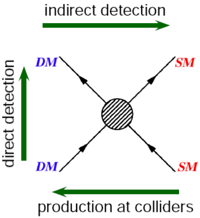
$$M_P^2 = 2Bm\left[1 + \frac{2N_F Bm}{16\pi^2 F^2}\left\{128\pi^2\left(2L_6^r - L_4^r + \frac{2L_8^r - L_5^r}{N_F}\right) + \frac{\log\left(2Bm/\mu^2\right)}{N_F^2}\right\}\right]$$

$$F_P = F\left[1 + \frac{2N_F Bm}{16\pi^2 F^2}\left\{64\pi^2\left(L_4^r + \frac{L_5^r}{N_F}\right) - \frac{1}{2}\log(2Bm/\mu^2)\right\}\right]$$

$$\begin{aligned} M_P a_{PP} = \frac{-2Bm}{16\pi F^2}\left[1 + \frac{2N_F Bm}{16\pi^2 F^2}\left\{-256\pi^2\left(\left[1 - \frac{2}{N_F}\right](L_4^r - L_6^r)\right.\right.\right. \\ \left. + \frac{L_0^r + 2L_1^r + 2L_2^r + L_3^r}{N_F}\right) - 2\frac{N_F - 1}{N_F^3} \\ \left.\left. + \frac{2 - N_F + 2N_F^2 + N_F^3}{N_F^3}\log\left(2Bm/\mu^2\right)\right\}\right] \end{aligned}$$

Backup: Thermal freeze-out for relic density

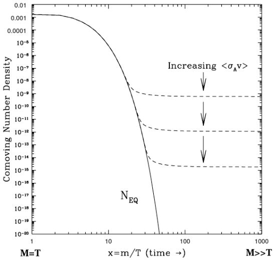
Requires coupling between ordinary matter and dark matter



$T \gtrsim M_{DM}$: DM $\longleftrightarrow$ SM
Thermal equilibrium

$T \lesssim M_{DM}$: DM $\longrightarrow$ SM
Rapid depletion of Ω_{DM}

Hubble expansion
$\longrightarrow$ dilution $\longrightarrow$ freeze-out



$2 \to 2$ scattering relates coupling and mass as $200\alpha \sim \dfrac{M_{DM}}{100\text{ GeV}}$

Strong $\alpha \sim 16$ $\longrightarrow$ 'natural' mass scale $M_{DM} \sim 300$ TeV

Smaller $M_{DM} \gtrsim 1$ TeV possible from $2 \to n$ scattering or asymmetry

Backup: Two roads to natural asymmetric dark matter

Idea: Dark matter relic density related to baryon asymmetry

$$\begin{aligned} \Omega_D &\approx 5\Omega_B \\ \Longrightarrow M_D n_D &\approx 5 M_B n_B \end{aligned}$$

- $n_D \sim n_B \quad \Longrightarrow \quad M_D \sim 5M_B \approx 5$ GeV
  High-dim. interactions relate baryon# and DM# violation
- $M_D \gg M_B \quad \Longrightarrow \quad n_B \gg n_D \sim \exp\left[-M_D/T_s\right] \qquad T_s \sim 200$ GeV
  EW sphaleron processes above T_s distribute asymmetries

Both require coupling between ordinary matter and dark matter

Backup: Composite dark matter interactions

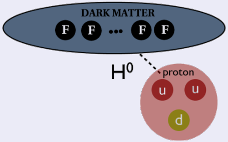
Photon exchange via electromagnetic form factors

Interactions suppressed by powers of confinement scale $\Lambda \sim M_{DM}$

- **Dimension 5:** Magnetic moment $\longrightarrow$ $\left(\overline{\psi}\sigma^{\mu\nu}\psi\right) F_{\mu\nu}/\Lambda$
- **Dimension 6:** Charge radius $\longrightarrow$ $\left(\overline{\psi}\psi\right) v_\mu\partial_\nu F_{\mu\nu}/\Lambda^2$
- **Dimension 7:** Polarizability $\longrightarrow$ $\left(\overline{\psi}\psi\right) F^{\mu\nu}F_{\mu\nu}/\Lambda^3$

Higgs exchange via scalar form factors

Higgs couples through $\langle B \left| m_\psi \overline{\psi}\psi \right| B\rangle$ (σ terms)



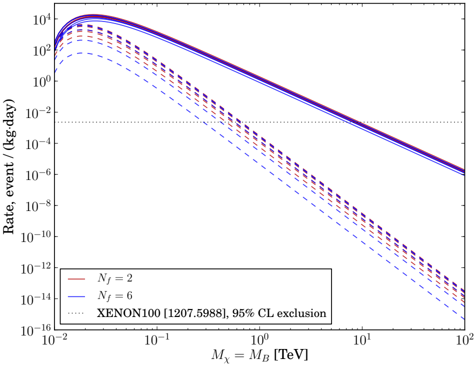
Needed for Big Bang nucleosynthesis
($\longrightarrow$ rapid charged 'meson' decay)

Non-perturbative form factors $\Longrightarrow$ lattice calculations

Backup: SU(3) direct detection constraints

Solid: Predicted event rate for SU(3) model vs. DM mass M_B

Dashed: Sub-leading charge radius contribution
suppressed $\sim 1/M_B^2$ compared to magnetic moment

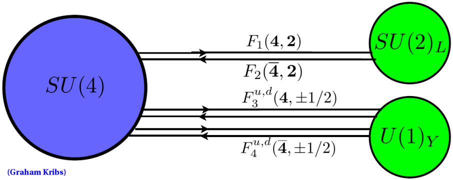


XENON100 (arXiv:1207.5988)
excludes $M_B \lesssim 10$ TeV

More recent LUX, PandaX
$\longrightarrow M_B \gtrsim 25$ TeV

SU(N) with even $N \geq 4$
forbids mag. moment. . .

Backup: Stealth dark matter model details



Field	$SU(N_D)$	$(SU(2)_L,\ Y)$	Q
$F_1 = \begin{pmatrix} F_1^u \\ F_1^d \end{pmatrix}$	$\mathbf{N}$	$(\mathbf{2},\ 0)$	$\begin{pmatrix} +1/2 \\ -1/2 \end{pmatrix}$
$F_2 = \begin{pmatrix} F_2^u \\ F_2^d \end{pmatrix}$	$\bar{\mathbf{N}}$	$(\mathbf{2},\ 0)$	$\begin{pmatrix} +1/2 \\ -1/2 \end{pmatrix}$
F_3^u	$\mathbf{N}$	$(\mathbf{1}, +1/2)$	$+1/2$
F_3^d	$\mathbf{N}$	$(\mathbf{1}, -1/2)$	$-1/2$
F_4^u	$\bar{\mathbf{N}}$	$(\mathbf{1}, +1/2)$	$+1/2$
F_4^d	$\bar{\mathbf{N}}$	$(\mathbf{1}, -1/2)$	$-1/2$

Mass terms $\quad m_V\,(F_1 F_2 + F_3 F_4) + y\left(F_1 \cdot H F_4 + F_2 \cdot H^\dagger F_3\right) + \text{h.c.}$

Vector-like masses evade Higgs-exchange direct detection bounds

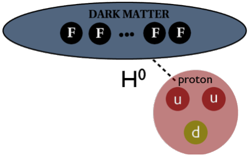
Higgs couplings $\to$ charged meson decay before Big Bang nucleosyn.

Both required

Backup: Effective Higgs interaction

$M_H = 125$ GeV $\longrightarrow$ Higgs exchange can dominate direct detection

$$\sigma_H \propto \left| \frac{\mu_{DM,N}}{M_H^2} \; y_\psi \langle DM \left| \overline{\psi}\psi \right| DM \rangle \; y_q \langle N \left| \overline{q}q \right| N \rangle \right|^2$$



Quark $\quad y_q = \dfrac{m_q}{v}$

Dark $\quad y_\psi = \alpha \dfrac{m_\psi}{v}$ suppressed by $\alpha \equiv \dfrac{v}{m_\psi} \dfrac{\partial m_\psi(h)}{\partial h} \Big|_{h=v} = \dfrac{yv}{yv + m_V}$

Can determine scalar form factors using Feynman–Hellmann theorem

$$\langle DM \left| \overline{\psi}\psi \right| DM \rangle = \frac{\partial M_{DM}}{\partial m_\psi}$$

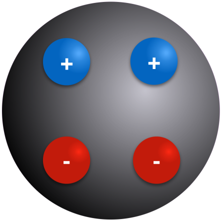
Backup: Stealth dark matter EM form factors

Lightest SU(4) dark baryon

Scalar $\longrightarrow$ no magnetic moment

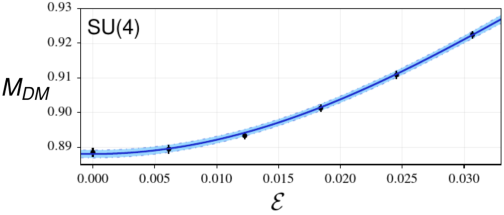
+/- charge symmetry $\longrightarrow$ no charge radius

Small α $\longrightarrow$ Higgs exchange suppressed



Polarizability $\longrightarrow$ lower bound on direct-detection cross section

Compute on lattice as dependence of M_{DM} on external field $\mathcal{E}$



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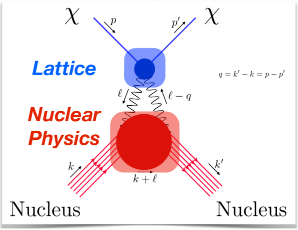
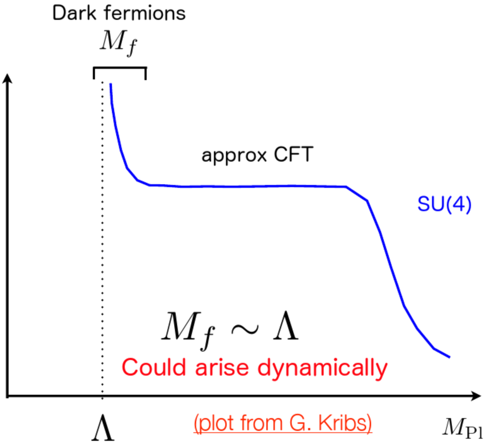


Backup: Stealth dark matter mass scales

Lattice studies focus on $m_\psi \simeq \Lambda_{DM}$ (effective theories least reliable)

$m_\psi \simeq \Lambda_{DM}$ could arise dynamically

Smaller $m_\psi \longrightarrow$ stronger collider constraints



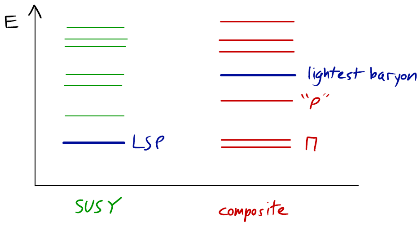
(plot from G. Kribs)

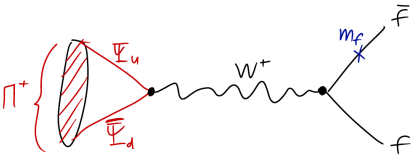
Backup: Stealth dark matter at colliders

Spectrum significantly different from typical susy

Very little missing E_T

Main constraints from much lighter **charged** "Π" states





Rapid Π decays, $\Gamma \propto m_f^2$

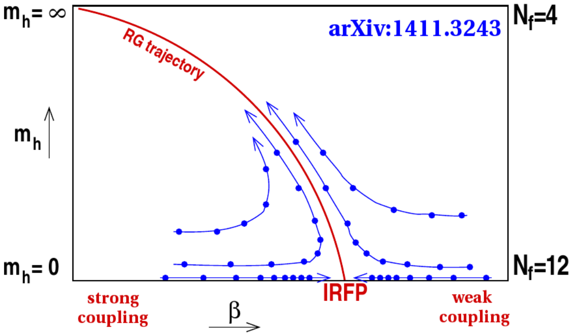
Best current constraints recast LEP stau searches

LHC can search for $t\overline{b} + \overline{t}b$ from $\Pi^+\Pi^-$ Drell–Yan

Backup: Philosophy of mixed-mass approach

$N_F = N_\ell + N_h$ fermions, light $m_\ell \to 0$ at fixed heavy $m_h > 0$
$\longrightarrow$ approximate conformality without extra PNGBs

Smaller $m_h \longrightarrow$ larger range of scales for approximately conformality



Real-space RG flow lines (from UV to IR)

"IRFP" in UV
$\longrightarrow$ work on either side