



Light scalar from lattice strong dynamics

David Schaich (U. Bern)

WE-Heraeus-Seminar *Understanding the LHC*
Physikzentrum Bad Honnef, 14 February 2017

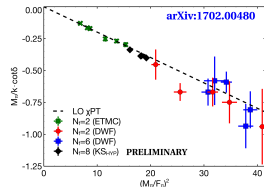
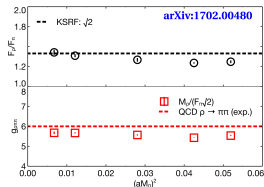
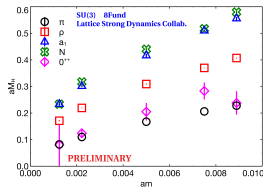
PRD 93:114514 (2016) [[arXiv:1601.04027](#)]

ProcSci LATTICE2016:242 (2017) [[arXiv:1702.00480](#)]

and work in progress with the Lattice Strong Dynamics Collaboration

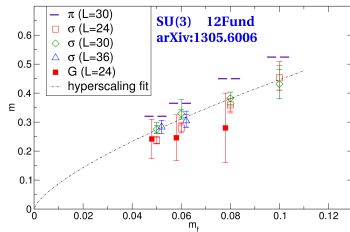
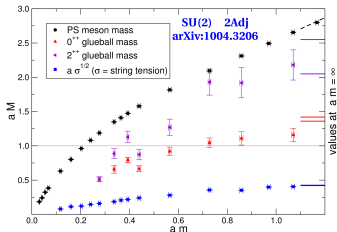
Overview

- Several groups are using lattice gauge theory to explore strongly coupled systems with non-QCDlike dynamics
- These studies find remarkably light scalars in many IR-conformal and near-conformal systems
- Using 8-flavor SU(3) gauge theory as a representative example we are studying more quantities to constrain the low-energy EFT

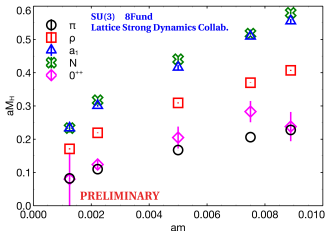
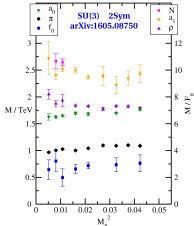
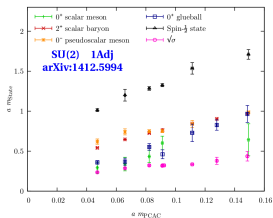


Light scalars from beyond-QCD lattice calculations

Not so shocking in mass-deformed IR-conformal theories



More surprising in systems apparently exhibiting spontaneous chiral symmetry breaking



Lattice Strong Dynamics Collaboration



Argonne **Xiao-Yong Jin**, James Osborn

Bern DS

Boston Rich Brower, Claudio Rebbi, **Evan Weinberg**

Colorado Anna Hasenfratz, Ethan Neil

Edinburgh Oliver Witzel

Livermore Pavlos Vranas

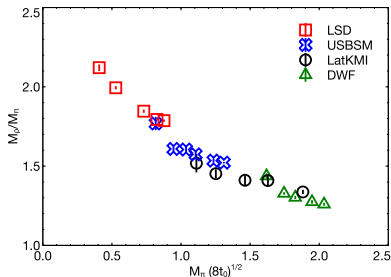
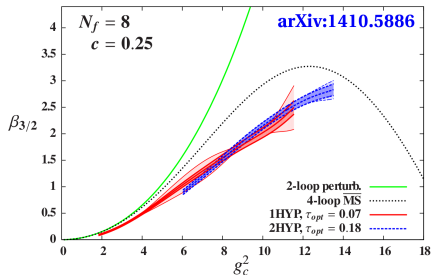
RBRC **Enrico Rinaldi**

UC Davis Joseph Kiskis

Yale Thomas Appelquist, **George Fleming**, **Andrew Gasbarro**

Exploring the range of possible phenomena
in strongly coupled field theories

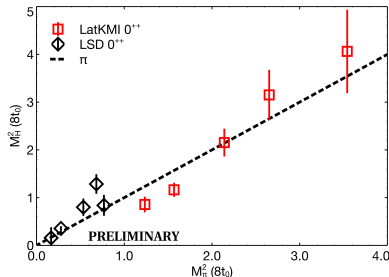
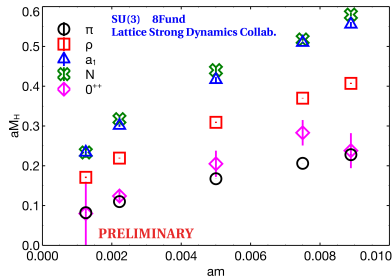
Infrared dynamics of 8-flavor SU(3) gauge theory



- β function is monotonic up to fairly strong couplings
No sign of approach towards conformal IR fixed point [$\beta(g_\star^2) = 0$]
- Ratio M_V/M_P increases monotonically as masses decrease
as expected for spontaneous chiral symmetry breaking ($S_\chi\text{SB}$)
Mass-deformed conformal hyperscaling predicts constant ratio

Want to strengthen conclusion by matching to low-energy EFT,
but must go beyond QCD-like χPT to include light scalar. . .

Light scalar in 8-flavor spectrum



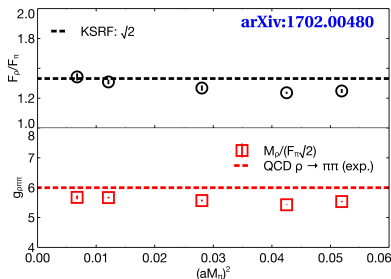
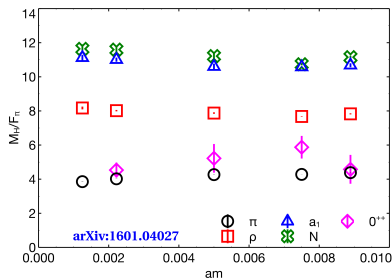
Flavor-singlet scalar degenerate with pseudo-Goldstones
down to lightest fermion masses we can reach on $64^3 \times 128$ lattices

Both M_S and M_P are less than half the vector mass M_V ,
and the hierarchy is growing as we approach the chiral limit

This is very different from QCD

Controlled chiral extrapolations need EFT that includes scalar. . .

2TeV vector resonance with width $\Gamma_V \simeq 450$ GeV



$M_V/F_P \simeq 8 \implies M_V \simeq 2$ TeV if we assume roughly constant ratio
 [NB: $S_\chi SB$ implies $M_P/F_P \rightarrow 0$ in chiral limit!]

We measure $F_V \approx F_P \sqrt{2}$ (KSRF relation, suggesting vector domin.)

Applying second KSRF relation $g_{VPP} \approx M_V/(F_P \sqrt{2})$

gives vector width $\Gamma_V = \frac{g_{VPP}^2 M_V}{48\pi} \simeq 450$ GeV — hard to see at LHC

Work in progress: Constraining EFT

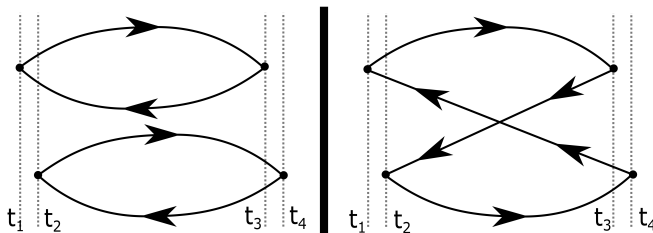
There are many candidate EFTs that include PNGBs + light scalar

(linear σ model; Goldberger–Gristein–Skiba; Soto–Talavera–Tarrus; Matsuzaki–Yamawaki;
Golterman–Shamir; Hansen–Langaebler–Sannino; Appelquist–Ingoldby–Piai)

Need lattice computations of more observables to test EFTs

We are now computing $2 \rightarrow 2$ elastic scattering of PNGBs & scalar,
as well as scalar form factor of PNGB

Example: Scalar exchange in $\pi\pi$ scattering vs. π scalar form factor



Work in progress: Constraining EFT

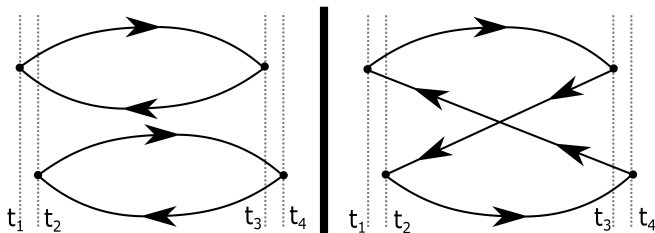
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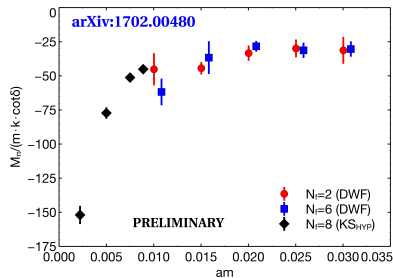
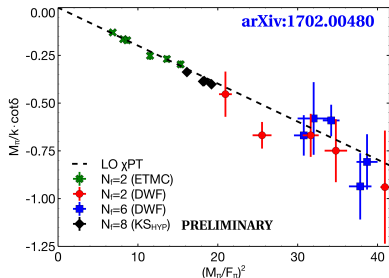
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Example: Scalar exchange in $\pi\pi$ scattering vs. π scalar form factor

Subsequent step: Analog of πK scattering in mass-split system



Initial $2 \rightarrow 2$ elastic scattering results



First looking at analog of QCD $I = 2 \pi\pi$ scattering
(simplest case with no fermion-line-disconnected diagrams)

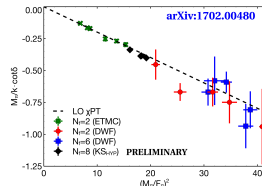
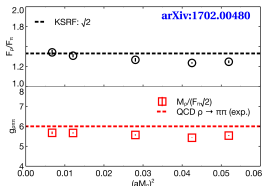
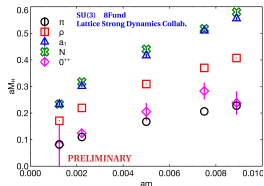
Simplest observable is scattering length $a_{PP} \approx 1/(k \cot \delta)$

$M_P a_{PP}$ vs. M_P^2/F_P^2 curiously close to leading-order χ PT prediction

Dividing by fermion mass m reveals expected tension with χ PT
which predicts $M_P a_{PP}/m = \text{const.}$ at LO and involves 8 LECs at NLO

Recapitulation and outlook

- 8-flavor SU(3) gauge theory is a representative system with a light scalar likely related to near-conformal dynamics
- Growing hierarchy between scalar and broad $\sim 2\text{TeV}$ vector
- Chiral extrapolations need an EFT beyond QCD-like χPT
- We are now studying elastic scattering to test potential EFTs



Thank you!

Thank you!

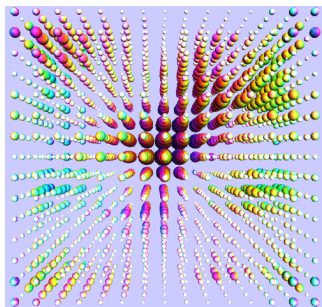
Lattice Strong Dynamics Collaboration

In particular: George Fleming, Andrew Gasbarro, Xiao-Yong Jin,
Enrico Rinaldi, Evan Weinberg

Funding and computing resources



Backup: Essence of numerical lattice calculations



(Image credit: Claudio Rebbi)

Evaluate observables from functional integral
via importance sampling Monte Carlo

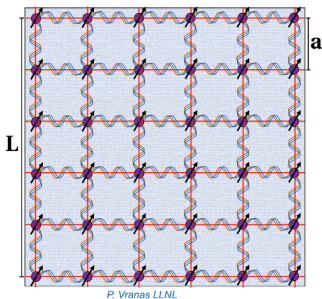
$$\begin{aligned}\langle \mathcal{O} \rangle &= \frac{1}{Z} \int \mathcal{D}\Phi \, \mathcal{O}(\Phi) \, e^{-S[\Phi]} \\ &\longrightarrow \frac{1}{N} \sum_{i=1}^N \mathcal{O}(\Phi_i) \text{ with uncert. } \propto \sqrt{\frac{1}{N}}\end{aligned}$$

Φ are field configurations in discretized euclidean spacetime

$S[\Phi]$ is the lattice action, which should be real and non-negative
so that $\frac{1}{Z} e^{-S}$ can be treated as a probability distribution

The hybrid Monte Carlo algorithm samples Φ with probability $\propto e^{-S}$

Backup: More features of lattice calculations



Spacing between lattice sites (“ a ”)
introduces UV cutoff scale $1/a$

Lattice cutoff preserves hypercubic subgroup
of full Poincaré symmetry

Remove cutoff by taking continuum limit:
 $a \rightarrow 0$ with $L/a \rightarrow \infty$

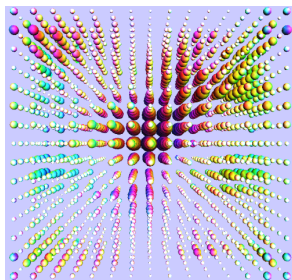
The lattice action S is defined by the bare lagrangian
at the UV cutoff set by the lattice spacing

After generating and saving an ensemble $\{\Phi_n\}$ distributed $\propto e^{-S}$
it is usually quick and easy to measure many observables $\langle \mathcal{O} \rangle$

Changing the action (generally) requires generating a new ensemble

Backup: Hybrid Monte Carlo (HMC) algorithm

Goal: Sample field configurations Φ_i with probability $\frac{1}{Z} e^{-S[\Phi_i]}$



(Image credit: Claudio Rebbi)

HMC is a Markov process, based on
Metropolis–Rosenbluth–Teller (MRT)

Fermions \longrightarrow extensive action computation,
so best to update entire system at once

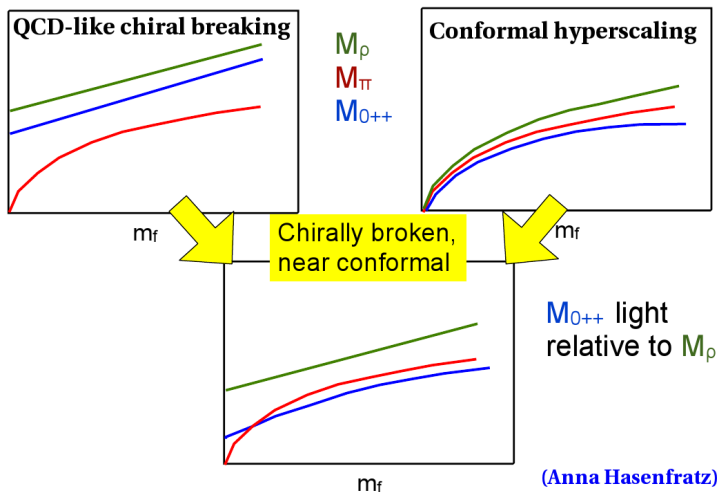
Use fictitious molecular dynamics evolution

- 1 Introduce a fictitious fifth dimension (“MD time” τ)
and stochastic canonical momenta for field variables
- 2 Run inexact MD evolution along a trajectory in τ
to generate a new four-dimensional field configuration
- 3 Apply MRT accept/reject test to MD discretization error

Backup: Qualitative picture of lattice results

Light scalar likely related to near-conformal dynamics

(unconfirmed interpretation as PNGB of approx. scale symmetry)

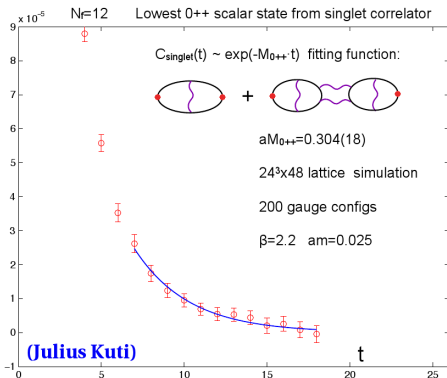


Backup: Technical challenge for scalar on lattice

Only the new strong sector is included in the lattice calculation

⇒ The flavor-singlet scalar mixes with the vacuum

Leads to noisy data and relatively large uncertainties

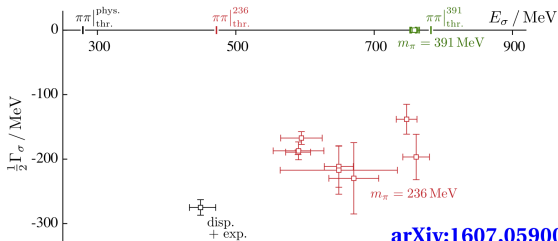
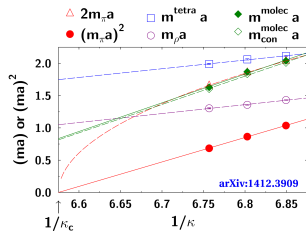


Fermion propagator computation
is relatively expensive

“Disconnected diagrams” formally
need propagators at all L^4 sites

In practice estimate stochastically
to control computational costs

Backup: Isosinglet scalar in QCD spectrum



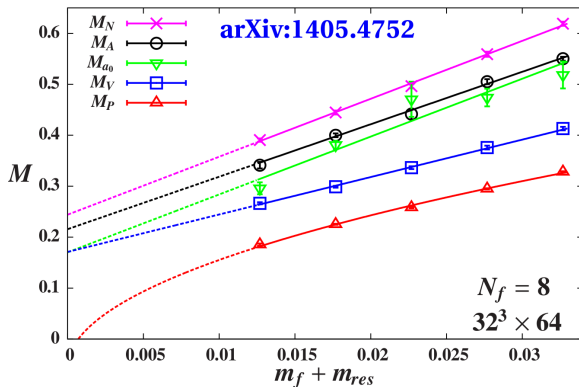
In lattice QCD, the isosinglet scalar is much heavier than the pion

Generally $M_S \gtrsim 2M_P$, and for heavy quarks $M_S > M_V$

For a large range of quark masses m

it mixes significantly with two-pion scattering states

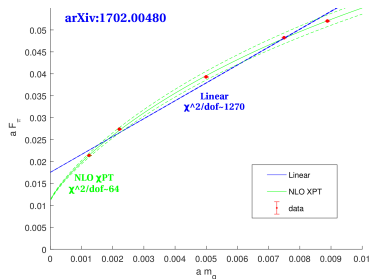
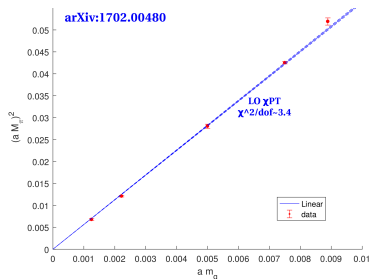
Backup: Non-singlet scalar for $N_F = 8$



In earlier work with domain wall fermions at heavier fermion masses
the non-singlet scalar is heavier than the vector, $M_{a_0} > M_V$

Staggered analyses in progress, but more complicated

Backup: Chiral perturbation theory (χ PT) fits



In addition to omitting the light scalar

χ PT also suffers from large expansion parameter

$$5.8 \leq \frac{2N_F B m}{16\pi^2 F^2} \leq 41.3 \quad \text{for} \quad 0.00125 \leq m \leq 0.00889$$

Big ($\sim 50\sigma$) shift in F from linear extrapolation vs. NLO χ PT

Fit quality is not good, especially for NLO joint fit with $\chi^2/\text{d.o.f.} > 10^4$

Backup: NLO chiral perturbation theory formulas

$$M_P^2 = 2Bm \left[1 + \frac{2N_F Bm}{16\pi^2 F^2} \left\{ 128\pi^2 \left(2L_6^r - L_4^r + \frac{2L_8^r - L_5^r}{N_F} \right) + \frac{\log(2Bm/\mu^2)}{N_F^2} \right\} \right]$$

$$F_P = F \left[1 + \frac{2N_F Bm}{16\pi^2 F^2} \left\{ 64\pi^2 \left(L_4^r + \frac{L_5^r}{N_F} \right) - \frac{1}{2} \log(2Bm/\mu^2) \right\} \right]$$

$$M_{PaPP} = \frac{-2Bm}{16\pi F^2} \left[1 + \frac{2N_F Bm}{16\pi^2 F^2} \left\{ -256\pi^2 \left(\left[1 - \frac{2}{N_F} \right] (L_4^r - L_6^r) + \frac{L_0^r + 2L_1^r + 2L_2^r + L_3^r}{N_F} \right) - 2 \frac{N_F - 1}{N_F^3} + \frac{2 - N_F + 2N_F^2 + N_F^3}{N_F^3} \log(2Bm/\mu^2) \right\} \right]$$

Backup: $2 \rightarrow 2$ elastic scattering on the lattice

Measure both E_{PP} and $M_P \rightarrow k = \sqrt{(E_{PP}/2)^2 - M_P^2}$

s-wave scattering phase shift: $\cot \delta_0(k) = \frac{1}{\pi k L} S\left(\frac{k^2 L^2}{4\pi}\right)$

with regularized ζ function $S(\eta) = \sum_{j \neq 0}^{\Lambda} \frac{1}{j^2 - \eta} - 4\pi\Lambda$

Effective range expansion:

$$k \cot \delta_0(k) = \frac{1}{a_{PP}} + \frac{1}{2} M_P^2 r_{PP} \left(\frac{k^2}{M_P^2} \right) + \mathcal{O} \left(\frac{k^4}{M_P^4} \right)$$