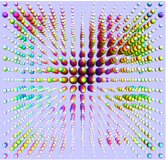
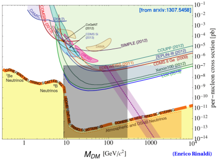
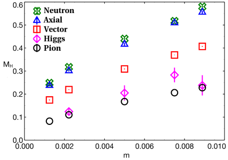
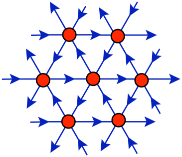
Physics Out Of The Box
The impact of lattice gauge theory









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University of Glasgow, 18 April 2016

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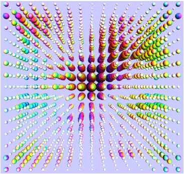
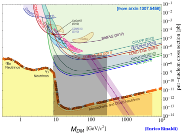
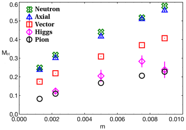
Overview

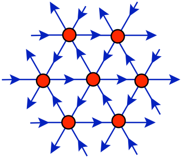
> Lattice gauge theory is a broadly applicable tool
> to study strongly coupled quantum field theories





- A high-level summary of lattice gauge theory
- Applications (recent results & future plans)
  1. Composite dark matter
  2. Composite Higgs bosons
  3. Supersymmetry and gauge–gravity duality (time permitting)
- Outlook





Lattice gauge theory in a nutshell: QFT

Lattice gauge theory is a broadly applicable tool
to study strongly coupled quantum field theories (QFTs)

"QFT = quantum mechanics + special relativity"

Picture relativistic quantum fields filling four-dimensional spacetime

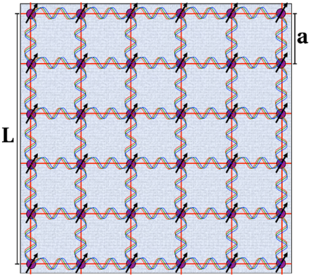
The QFT / StatMech Correspondence

Generating functional (path integral)	Canonical partition function
$\mathcal{Z} = \int \mathcal{D}\Phi \;\; e^{-S[\Phi] \;/\; \hbar}$	$\int \mathcal{D}q \, \mathcal{D}p \;\; e^{-H(q,p) \;/\; k_B T}$
Action $S[\Phi] = \int d^4x \; \mathcal{L}[\Phi(x)]$	Hamiltonian H
$\hbar \longrightarrow$ quantum fluctuations	$k_B T \longrightarrow$ thermal fluctuations

Lattice gauge theory in a nutshell: Discretization

A QFT observable is formally $\langle \mathcal{O} \rangle = \frac{1}{\mathcal{Z}} \int \mathcal{D}\Phi \ \ \mathcal{O}(\Phi) \ \ e^{-S[\Phi] \ / \ \hbar}$

...but this is an infinite-dimensional integral



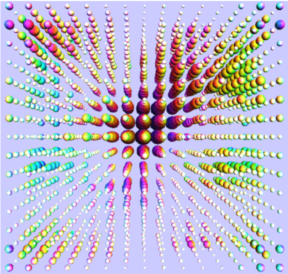
Solve the theory by formulating it
in a finite, discrete spacetime $\longrightarrow$ **the lattice**

Spacing between lattice sites ("a")
introduces symmetry-preserving UV cutoff

Remove cutoff by taking continuum limit:
$a \to 0$ with $L/a \to \infty$

Finite-dimensional integral $\Longrightarrow$ we can compute $\langle \mathcal{O} \rangle$ numerically

Numerical lattice gauge theory calculations



(Image credit: Claudio Rebbi)

Approximate integral using a finite ensemble of field configurations $\{\Phi_i\}$:

$$\langle \mathcal{O} \rangle = \frac{1}{\mathcal{Z}} \int \mathcal{D}\Phi \;\; \mathcal{O}(\Phi) \;\; e^{-S[\Phi]/\hbar} \longrightarrow \frac{1}{N} \sum_{i=1}^{N} \mathcal{O}(\Phi_i)$$

Algorithms choose each configuration Φ_i with probability $\frac{1}{\mathcal{Z}} e^{-S[\Phi_i] \,/\, \hbar}$ to find those that make the most important contributions

Generating ensembles $\{\Phi_i\}$ often dominates computational costs

These saved data can be "mined" to investigate many observables

Application: Dark matter

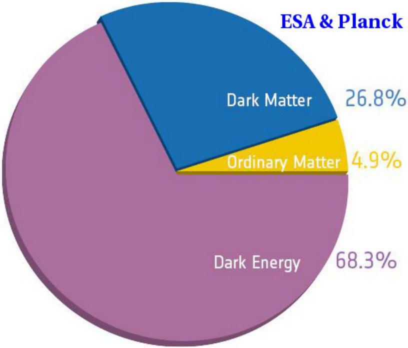
Multiple lines of evidence lead to a consistent conclusion:

Most matter in the universe is "dark"

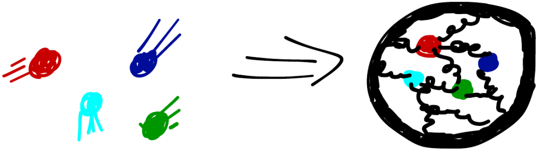
Determining the fundamental nature of this dark matter
is an outstanding challenge for particle physics

$$\frac{\Omega_{\text{dark}}}{\Omega_{\text{ordinary}}} \approx 5 \quad \ldots \text{not } 10^5 \text{ or } 10^{-5}$$

Non-gravitational interactions
explain this observation
but have not yet been seen
in ongoing experiments



Composite dark matter



A simple way to explain current observations

- Charged fermions F interact shortly after the hot big bang
  $\longrightarrow$ Non-gravitational interactions "on" in the early universe to explain the observed dark matter abundance
- Then F **confine** to form stable neutral composite particles the same way quarks form protons, neutrons, etc.
  $\longrightarrow$ Non-observation of present-day experimental signals

Lattice calculations are required to obtain predictions for experiments

A lower bound for composite dark matter

Investigate the most general constraint that can be imposed

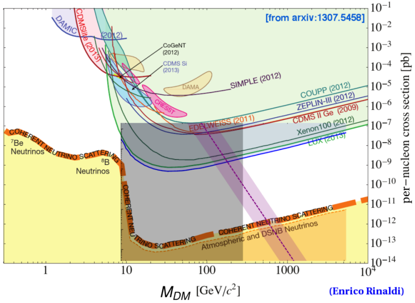
The polarizability of composite dark matter places a lower bound on the rate of observable interactions in underground detectors

We used lattice gauge theory to compute this polarizability (Lattice Strong Dynamics Collaboration, PRL 115:171803, 2015)

Composite dark matter with mass $M_{DM} \lesssim 200$ GeV is ruled out

Signals above $M_{DM} \gtrsim 700$ GeV would be challenging to detect

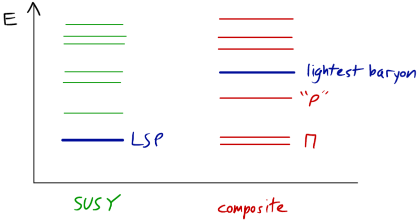
Shaded region is complementary constraint from particle colliders

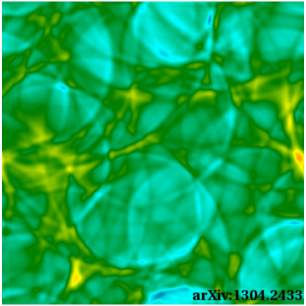


Future plans: Colliders and gravitational waves

Other composite dark-sector states can be discovered at colliders

Additional lattice input required to predict production and decays





Confinement transition in early universe may produce gravitational waves

First-order transition $\longrightarrow$ colliding bubbles

Lattice calculations needed to predict latent heat of transition

Application: Composite Higgs bosons

Another outstanding challenge for particle physics is determining the fundamental nature of the Higgs boson

This is a major focus of ongoing experiments at Run 2 of the Large Hadron Collider

One compelling possibility is **new strong dynamics** that produces a composite Higgs boson

This would solve the 'hierarchy' (or 'fine-tuning') problem, protecting Higgs physics from extreme sensitivity to quantum effects

Lattice gauge theory has a crucial role to play in exploring and understanding new strong dynamics

Composite Higgs beyond the lamppost

Known strong dynamics vs. new strong dynamics

The strong nuclear force is well studied
—Arises from the fundamental theory
of quantum chromodynamics (QCD)
describing quarks and gluons

—Has been used as a ‘lamppost’
to consider new strong dynamics

QCD-like new strong dynamics predicts a Higgs
completely unlike the one discovered in 2012

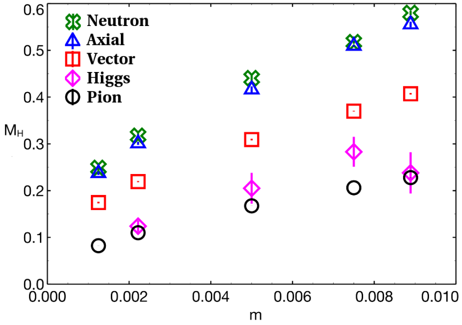
Experimentally viable new strong dynamics
must behave differently than QCD

Lattice calculations can illuminate the range of possibilities

Recent result: Light composite Higgs

In arXiv:1601.04027 we computed composite particle masses in a strongly coupled system different than QCD (exhibiting less dependence on length scale)

Resulting composite Higgs is much lighter than expected from QCD, as required by experiment

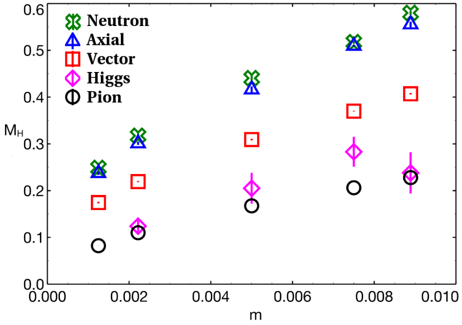


Need to extrapolate fermion mass $m \to 0$

(Can't directly access $m = 0$ in lattice calculations)

Large separation between Higgs and resonances

Future plan: Interactions of light Higgs



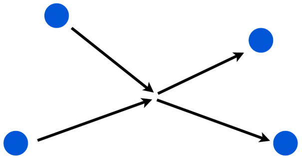
Need to extrapolate
fermion mass $m \to 0$

(Can't directly access $m = 0$
in lattice calculations)

Large separation between
Higgs and resonances

Can gain insight into extrapolation
from interactions of light Higgs and pions

Goal to establish closer connections
between lattice and collider phenomenology



Application: Lattice supersymmetry

Supersymmetry is extremely interesting, especially non-perturbatively

- Widely studied potential roles in new physics at the LHC
- More generally, symmetries simplify systems
  $\longrightarrow$ Insight into strongly coupled dynamics and dualities

Many different methods have been brought to bear:

- Perturbation theory at weak coupling $\lambda \ll 1$,
  "dual" to strong coupling in some systems
- "AdS / CFT" dualities with gravitational systems
- Conformal field theory techniques ("the bootstrap")

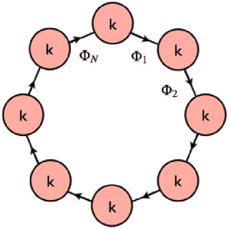
Only lattice gauge theory
provides non-perturbative predictions from first principles

A brief history of lattice supersymmetry

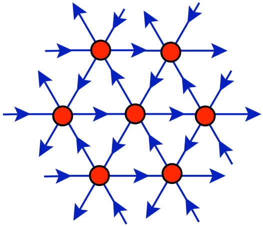
Supersymmetries are "square roots" of infinitesimal translations
which do not exist in discrete space-time

Recent work overcomes this obstacle for certain systems,
including maximally supersymmetric Yang–Mills ("$\mathcal{N} = 4$" SYM)

Preserve **subset** of supersymmetries $\Longrightarrow$ recover rest in continuum



For details see
Catterall, Kaplan & Ünsal
arXiv:0903.4881



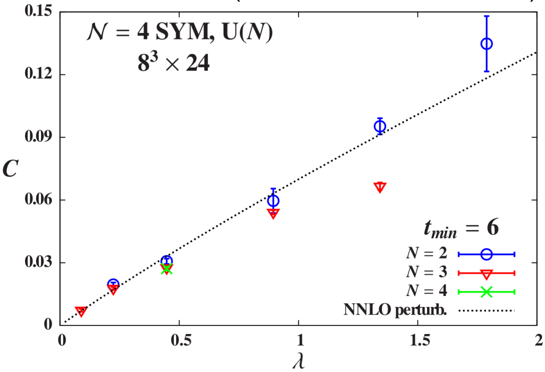
Recent result: Coulomb potential of $\mathcal{N} = 4$ SYM

(PRD 90:065013, 2014)

Measure potential between
two static probes on lattice

Fit to Coulomb form
$V(r) = C/r$

Study coefficient C
as function of coupling λ



Continuum perturbation theory predicts $C(\lambda) = \lambda/(4\pi) + \mathcal{O}(\lambda^2)$,
consistent with two-color ($N = 2$) results for $\lambda \lesssim N$

AdS / CFT predicts $C(\lambda) \propto \sqrt{\lambda}$ for large N and $\lambda \ll N$,
while $N = 3$ results start to bend down for $\lambda \gtrsim 1$

Future plan: Supersymmetric QCD

Supersymmetric Yang–Mills involves only analog of the gluon

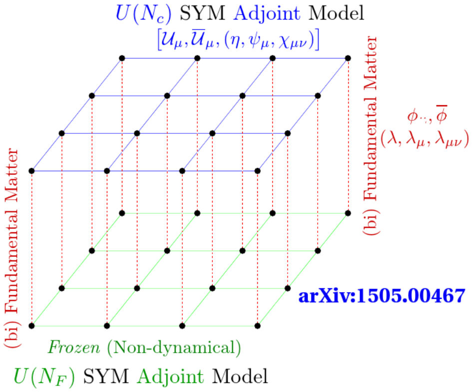
Analogs of quarks needed in order to study certain dualities, spontaneous supersymmetry breaking and more

Difficult to add while preserving supersymmetries on lattice

Quarks arise from links between two Yang–Mills lattice systems

Produces supersymmetric QCD in $d-1$ dimensions

Numerical investigations only just beginning



Outlook: An exciting time for lattice gauge theory

Lattice gauge theory is a broadly applicable tool
to study strongly coupled quantum field theories

- Predict properties of composite dark matter
  in underground detectors, particle colliders and the early universe
- Search for realistic composite Higgs boson
  from new strong dynamics that differs from QCD
- First large-scale lattice studies of supersymmetric systems
  beginning to explore regimes inaccessible to other methods

Thank you!

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Thank you!

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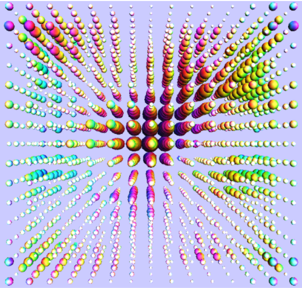
Backup: Hybrid Monte Carlo (HMC) algorithm

Recall goal: Sample field configurations Φ_i with probability $\frac{1}{\mathcal{Z}} e^{-S[\Phi_i]}$



HMC is a Markov process, based on Metropolis–Rosenbluth–Teller (MRT)

Fermions $\longrightarrow$ extensive action computation, so best to update entire system at once

Use fictitious molecular dynamics evolution

1. Introduce a fictitious fifth dimension ("MD time" τ) and stochastic canonical momenta for all field variables
2. Run inexact MD evolution along a trajectory in τ to generate new four-dimensional field configuration
3. Apply MRT accept/reject test to MD discretization error

For decades lattice gauge theory
has helped to drive advances in high-performance computing

IBM Blue Gene/Q @Livermore

USQCD cluster @Fermilab

Results shown above are from
state-of-the-art lattice calculations

$\mathcal{O}$(100M core-hours) invested overall

Many thanks to DOE, NSF
and computing centers!

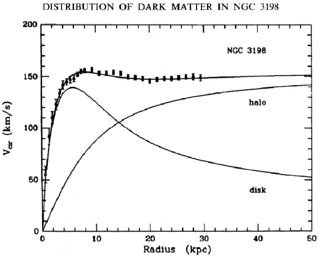
Cray *Blue Waters* @NCSA

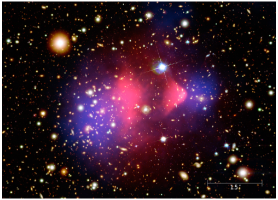
Backup: Lattice Strong Dynamics Collaboration

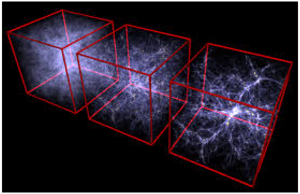
Argonne	Xiao-Yong Jin, James Osborn
Boston	Rich Brower, Claudio Rebbi, Evan Weinberg
Brookhaven	Meifeng Lin
Colorado	Anna Hasenfratz, Ethan Neil
Edinburgh	Oliver Witzel
Livermore	Evan Berkowitz, Enrico Rinaldi, Pavlos Vranas
Oregon	Graham Kribs
RBRC	Ethan Neil, Sergey Syritsyn
Syracuse	DS
UC Davis	Joseph Kiskis
Yale	Thomas Appelquist, George Fleming, Andy Gasbarro

Exploring the range of possible phenomena
in strongly coupled gauge theories

Backup: Evidence for dark matter

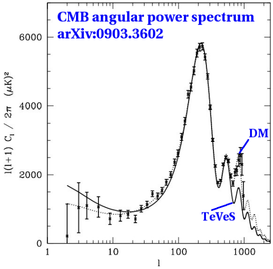






Multiple consistent lines of evidence spanning many scales

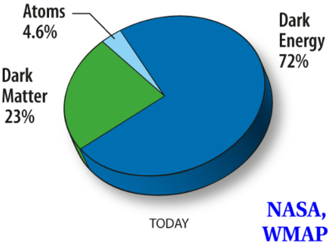
- Rotation curves of galaxies & clusters
- Gravitational lensing
- Structure formation
- Cosmological backgrounds



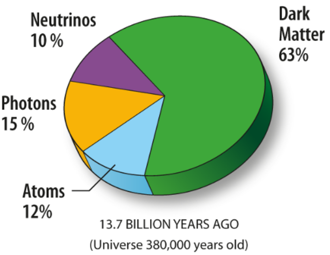
All of these are gravitational effects

Backup: Dark matter density in cosmological history

$$\frac{\Omega_{\text{dark}}}{\Omega_{\text{ordinary}}} \approx 5 \text{ now}$$

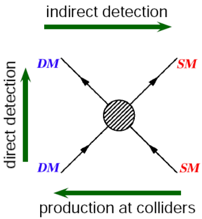


$$\frac{\Omega_{\text{dark}}}{\Omega_{\text{ordinary}}} \approx 5 \text{ at recombination}$$



Simply because both are **matter** and evolve in the same way

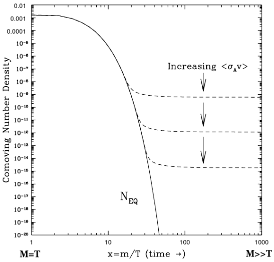
Backup: Thermal freeze-out for relic density



$T \gtrsim M_{DM}$: DM $\longleftrightarrow$ SM
Thermal equilibrium

$T \lesssim M_{DM}$: DM $\longrightarrow$ SM
Rapid depletion of Ω_{DM}

Hubble expansion $\longrightarrow$ dilution
leads to freeze-out



Requires coupling between ordinary matter and dark matter

Mass and coupling of pure thermal relic are related: $\dfrac{M_{DM}}{100\text{ GeV}} \sim 200\alpha$

(The "WIMP miracle" is $\alpha \sim \alpha_{EW} \sim 0.01 \Longrightarrow M_{DM} \sim 200$ GeV $\sim v$)

Thermal relic suppressed by **strong** coupling, easy for composite DM

Backup: Two roads to natural asymmetric dark matter

Basic idea: Dark matter relic density related to baryon asymmetry

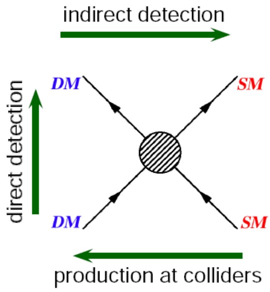
$$\Omega_D \approx 5\Omega_B$$
$$\Longrightarrow M_D n_D \approx 5 M_B n_B$$

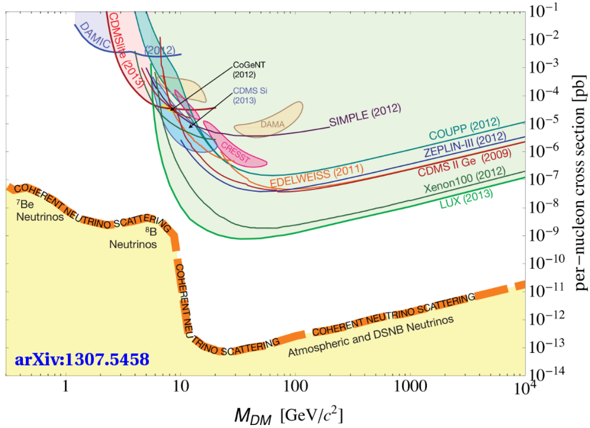
- $n_D \sim n_B \quad \Longrightarrow \quad M_D \sim 5M_B \approx 5$ GeV
  High-dimensional interactions relate baryon# and DM# violation
- $M_D \gg M_B \quad \Longrightarrow \quad n_B \gg n_D \sim \exp\left[-M_D/T_s\right]$
  Sphaleron transitions above $T_s \sim 200$ GeV distribute asymmetries

Both require coupling between ordinary matter and dark matter

Backup: Non-gravitational searches for dark matter

No clear signals in non-gravitational searches for dark matter
(at colliders, in cosmic rays, and underground)





Backup: Composite dark matter interactions

Photon exchange via electromagnetic form factors

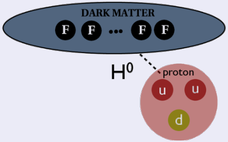
Interactions suppressed by powers of confinement scale $\Lambda \sim M_{DM}$

- **Dimension 5:** Magnetic moment $\longrightarrow \left(\overline{\psi}\sigma^{\mu\nu}\psi\right) F_{\mu\nu}/\Lambda$
- **Dimension 6:** Charge radius $\longrightarrow \left(\overline{\psi}\psi\right) v_\mu \partial_\nu F_{\mu\nu}/\Lambda^2$
- **Dimension 7:** Polarizability $\longrightarrow \left(\overline{\psi}\psi\right) F^{\mu\nu} F_{\mu\nu}/\Lambda^3$

Higgs boson exchange via scalar form factors

Effective Higgs interaction of composite DM
needed for correct Big Bang nucleosynthesis

Higgs couples through $\langle B | m_\psi \overline{\psi}\psi | B \rangle$ (σ terms)



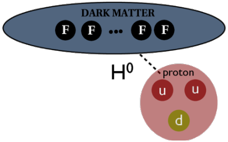
All form factors arise non-perturbatively $\Longrightarrow$ lattice calculations

Backup: Effective Higgs interaction

Exchange of Higgs boson with $M_H = 125$ GeV

may dominate spin-independent direct detection cross section

$$\sigma_H^{(SI)} \propto \left| \frac{\mu_{B,N}}{M_H^2} \; y_\psi \langle B|\overline{\psi}\psi|B\rangle \; y_q \langle N|\overline{q}q|N\rangle \right|^2$$



For quarks $y_q = \frac{m_q}{v} \Longrightarrow y_q \langle N|\overline{q}q|N\rangle \propto \frac{M_N}{v} \frac{\langle N|m_q\overline{q}q|N\rangle}{M_N}$

For dark constituent fermions ψ

there is an additional model parameter, $y_q = \alpha \frac{m_\psi}{v}$

In both cases the scalar form factor is most easily determined

using the Feynman–Hellmann theorem $\frac{\langle B|m_\psi\overline{\psi}\psi|B\rangle}{M_B} = \frac{m_\psi}{M_B}\frac{\partial M_B}{\partial m_\psi}$

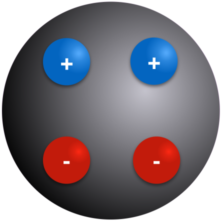
Backup: Stealth dark matter interactions

Composite dark matter with four F

Scalar particle $\longrightarrow$ no magnetic moment

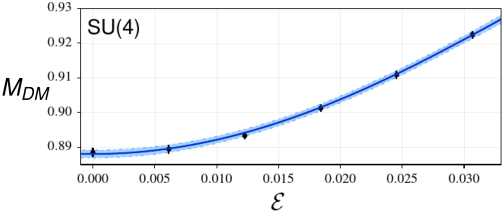
+/- charge symmetry $\longrightarrow$ no charge radius

Higgs exchange can be negligibly small



Polarizability places lower bound on direct-detection cross section

Compute on lattice as dependence of M_{DM} on external field $\mathcal{E}$



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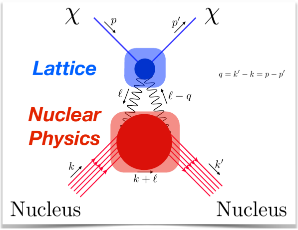
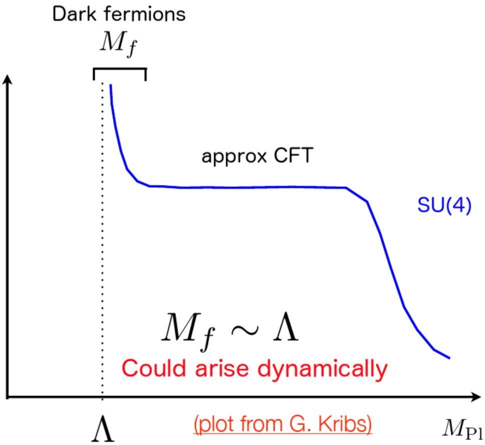


Backup: Stealth dark matter mass scales

Lattice calculations have focused on $m_\psi \simeq \Lambda_D$,
the regime where analytic estimates are least reliable

This mass scale has
some theoretical motivation

In addition,
collider constraints tighten
as mass decreases



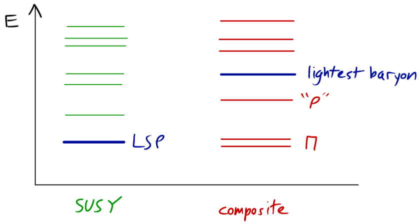
(plot from G. Kribs)

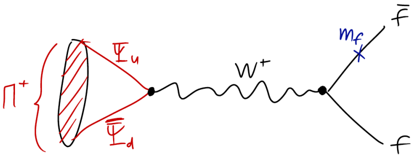
Backup: Stealth dark matter collider detection

Spectrum significantly different from MSSM-inspired models

Very little missing $\not{E}_T$ at colliders

Main constraints from much lighter **charged** "Π" states





Rapid Π decays with $\Gamma \propto m_f^2$

Best current constraints recast stau searches at LEP

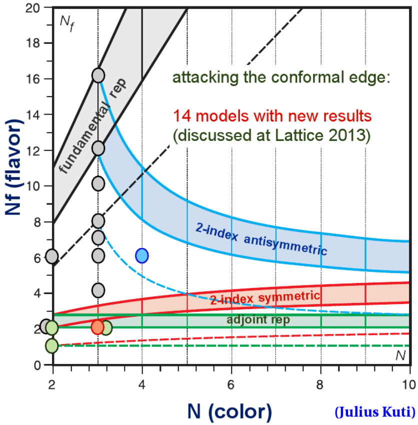
LHC can also search for $t\bar{b} + \bar{t}b$ from $\Pi^+\Pi^-$ Drell–Yan production

Backup: Strategy for composite Higgs studies

Systematically depart from familiar ground of lattice QCD

($N = 3$ with $N_F = 2$ light flavors in fundamental rep)

Explore the range of possible phenomena in strongly coupled theories



—Add more light flavors

$\longrightarrow N_F = 8$ fundamental

—Enlarge fermion rep

$\longrightarrow N_F = 2$ two-index symmetric

—Explore $N = 2$ and 4

$\longrightarrow$ (pseudo)real reps for cosets

$SU(n)/Sp(n)$ and $SU(n)/SO(n)$

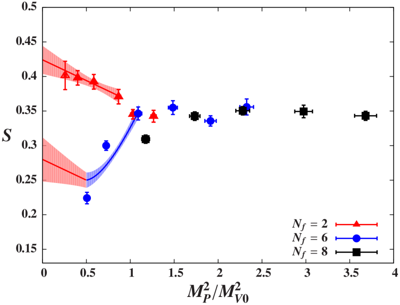
Backup: The S parameter on the lattice

$$\mathcal{L}_\chi \supset \frac{\alpha_1}{2} g_1 g_2 B_{\mu\nu} \mathrm{Tr}\left[U\tau_3 U^\dagger W^{\mu\nu}\right] \longrightarrow \gamma, Z$$ 

γ, Z

Lattice vacuum polarization calculation provides $S = -16\pi^2\alpha_1$

One subtlety is that nonzero masses needed
to keep correlation lengths insensitive to finite lattice volume



$S = 0.42(2)$ for $N_F = 2$
matches scaled-up QCD

Moving away from QCD with larger N_F
produces significant reductions

Extrapolation to correct zero-mass limit
becomes more challenging

Backup: Vacuum polarization is just current correlator

$$S = 4\pi N_D \lim_{Q^2 \to 0} \frac{d}{dQ^2} \Pi_{V-A}(Q^2) - \Delta S_{SM}(M_H)$$

$$\Pi^{\mu\nu}_{V-A}(Q) = Z \sum_x e^{iQ\cdot(x+\widehat{\mu}/2)} \mathrm{Tr}\left[\left\langle \mathcal{V}^{\mu a}(x) V^{\nu b}(0)\right\rangle - \left\langle \mathcal{A}^{\mu a}(x) A^{\nu b}(0)\right\rangle\right]$$

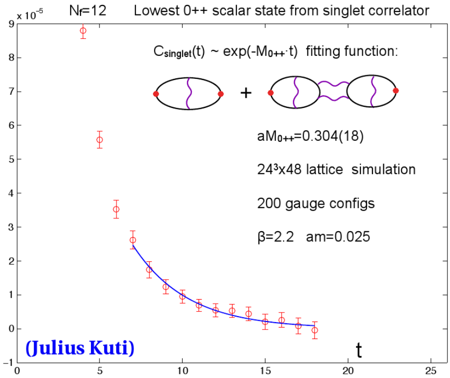
$$\Pi^{\mu\nu}(Q) = \left(\delta^{\mu\nu} - \frac{\widehat{Q}^\mu \widehat{Q}^\nu}{\widehat{Q}^2}\right)\Pi(Q^2) - \frac{\widehat{Q}^\mu \widehat{Q}^\nu}{\widehat{Q}^2}\Pi^L(Q^2) \qquad \widehat{Q} = 2\sin(Q/2)$$

- Renormalization constant Z evaluated non-perturbatively
  Chiral symmetry of domain wall fermions $\Longrightarrow Z = Z_A = Z_V$
  $Z = 0.85$ [2f]; 0.73 [6f]; 0.70 [8f]
- Conserved currents $\mathcal{V}$ and $\mathcal{A}$ ensure that lattice artifacts cancel

Backup: Technical lattice challenge for Higgs state

Only the new strong sector is included in the lattice calculation
$\Longrightarrow$ The Higgs is a singlet that mixes with the vacuum

Leads to noisy data and relatively large uncertainties in Higgs mass



Fermion propagator computation is relatively expensive

"Disconnected diagrams" formally need propagators at all L^4 sites

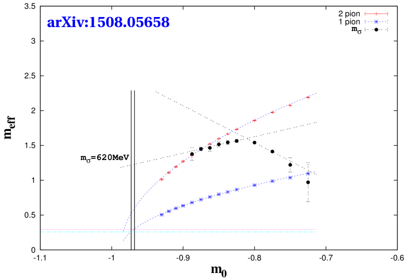
In practice estimate stochastically to control computational costs

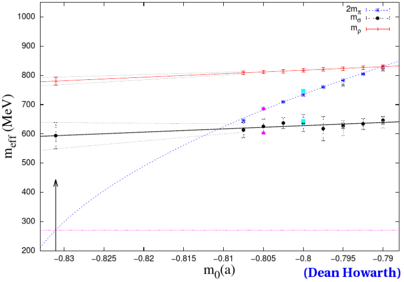
Backup: Composite Higgs in QCD spectrum

In lattice QCD, the Higgs is much heavier than the pion

For a large range of fermion masses m
it mixes significantly with two-pion scattering states

In that case lattice calculations may measure mainly $2M_\pi$





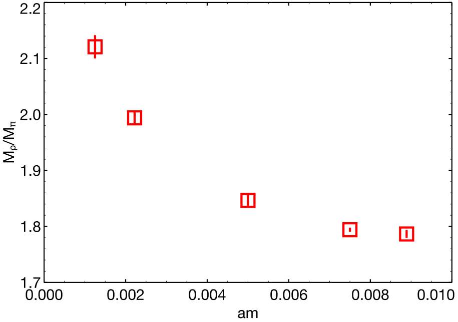
At very large masses the signal may become lost in the noise

Backup: More on composite Higgs spectrum

Higgs much lighter than in QCD-like systems

Degenerate with pions at accessible fermion masses $m > 0$

$\longrightarrow$ Next resonance already more than twice as heavy at these m,
and ratio is growing rapidly towards physical $m \to 0$ limit



Recent work using
lattice volumes up to $64^3 \times 128$

Scale setting suggests
resonance masses $\sim$2–3 TeV

Large separation between
Higgs and resonances

Backup: Status of light composite Higgs from lattice

Without reliable chiral extrapolation we can only estimate

$M_H \sim$ few hundred GeV, with large error bars

Much lighter than scaled-up QCD, still somewhat far from 125 GeV

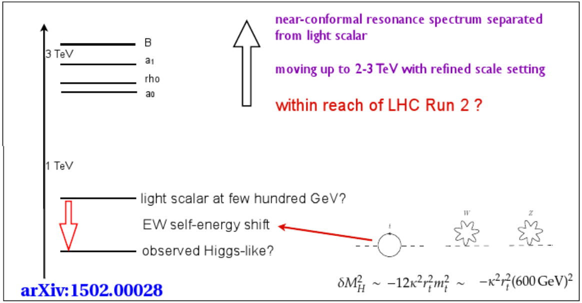
Of course, we **shouldn't** get exactly 125 GeV

since we haven't yet incorporated electroweak & top corrections

These reduce M_H,

but not yet consensus

on size of effect…



Backup: Failure of Leibnitz rule in discrete space-time

Given that $\left\{Q_\alpha, \overline{Q}_{\dot{\alpha}}\right\} = 2\sigma^\mu_{\alpha\dot{\alpha}} P_\mu = 2i\sigma^\mu_{\alpha\dot{\alpha}} \partial_\mu$ is problematic,

why not try $\left\{Q_\alpha, \overline{Q}_{\dot{\alpha}}\right\} = 2i\sigma^\mu_{\alpha\dot{\alpha}} \nabla_\mu$ for a discrete translation?

Here $\nabla_\mu \phi(x) = \dfrac{1}{a}\left[\phi(x + a\widehat{\mu}) - \phi(x)\right] = \partial_\mu \phi(x) + \dfrac{a}{2}\partial_\mu^2 \phi(x) + \mathcal{O}(a^2)$

Essential difference between ∂_μ and ∇_μ on the lattice, $a > 0$

$$\begin{aligned}
\nabla_\mu \left[\phi(x)\chi(x)\right] &= a^{-1}\left[\phi(x + a\widehat{\mu})\chi(x + a\widehat{\mu}) - \phi(x)\chi(x)\right] \\
&= \left[\nabla_\mu \phi(x)\right]\chi(x) + \phi(x)\nabla_\mu \chi(x) + a\left[\nabla_\mu \phi(x)\right]\nabla_\mu \chi(x)
\end{aligned}$$

We only recover the Leibnitz rule $\partial_\mu(fg) = (\partial_\mu f)g + f\partial_\mu g$ when $a \to 0$

$\Longrightarrow$ "Discrete supersymmetry" breaks down on the lattice

(Dondi & Nicolai, "Lattice Supersymmetry", 1977)

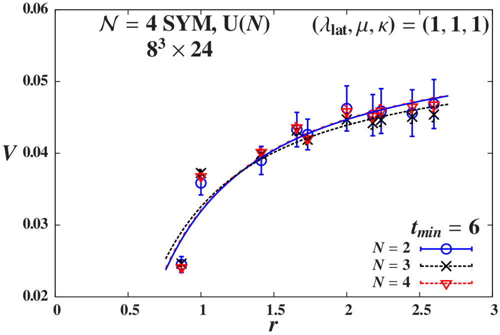
Backup: $\mathcal{N} = 4$ SYM — the fruit fly of QFT

Maximal supersymmetries make $\mathcal{N} = 4$ SYM
arguably the simplest non-trivial field theory in four dimensions

- SU(N) gauge theory with four fermions Ψ^{I} and six scalars Φ^{IJ},
  all massless and in adjoint rep.
- Action consists of kinetic, Yukawa and four-scalar terms
  with coefficients related by symmetries
- Supersymmetric: 16 supercharges Q^{I}_{α} and $\overline{Q}^{\mathrm{I}}_{\dot\alpha}$ with $\mathrm{I} = 1, \cdots, 4$
  Fields and Q's transform under global SU(4) $\simeq$ SO(6) symmetry
- Conformal: β function is zero for any 't Hooft coupling $\lambda = g^2 N$
  (It is the conformal field theory of the first AdS / CFT duality)

Backup: Coulomb potential from lattice $\mathcal{N} = 4$ SYM

Lattice Wilson loops $\longrightarrow$ potential $V(r)$ between two static probes



Fits to confining $V(r) = A - C/r + \sigma r$

produce vanishing string tension $\sigma = 0$ for all couplings λ

Fits to Coulombic $V(r) = A - C/r$ predict Coulomb coefficient $C(\lambda)$

Backup: Konishi operator in lattice $\mathcal{N} = 4$ SYM

$\mathcal{N} = 4$ SYM is conformal for any coupling λ

$\longrightarrow$ power-law decay for all correlation functions $C(r) \propto r^{-2\Delta}$

The scaling dimension Δ_K of the simple Konishi operator has attracted much recent attention

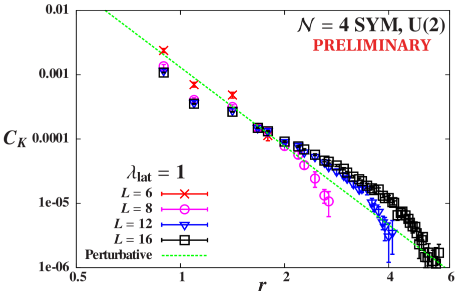
$\mathcal{O}_K(x) = \sum_{\mathrm{I}} \mathrm{Tr}\left[\Phi^{\mathrm{I}}(x)\Phi^{\mathrm{I}}(x)\right]$ (symmetric sum over six scalars)

$C_K(r) = \mathcal{O}_K(x+r)\mathcal{O}_K(x) \propto r^{-2\Delta_K}$

Lattice tools to find Δ_K:

—Finite-size scaling

—Monte Carlo RG



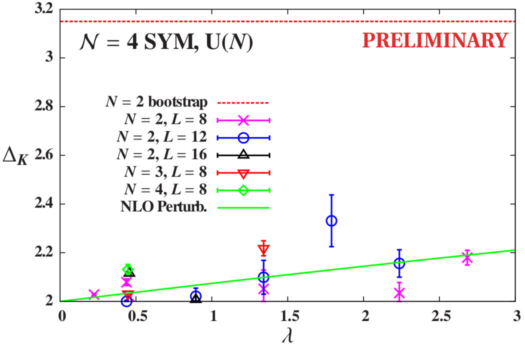
Backup: Konishi scaling dimension from the lattice

Recent studies of Konishi scaling dimension Δ_K
in perturbation theory + S duality, AdS / CFT holography, bootstrap

The first lattice calculation of Δ_K is now underway

Plot shows initial results from Monte Carlo RG (only statistical errors)

Rough agreement between $N = 2, 3, 4$



So far results follow perturbation theory, far from bootstrap bounds

Currently refining analyses, running larger volumes at stronger λ

Backup: Potential sign problem of $\mathcal{N} = 4$ SYM

Integrating over a single Kähler–Dirac fermion Ψ in adjoint rep.,

$$\langle\mathcal{O}\rangle = \frac{1}{\mathcal{Z}}\int[d\mathcal{U}][d\overline{\mathcal{U}}]\ \mathcal{O}\ e^{-S_B[\mathcal{U},\overline{\mathcal{U}}]}\ \text{pf}\,\mathcal{D}[\mathcal{U},\overline{\mathcal{U}}]$$

Pfaffian can be complex for lattice $\mathcal{N} = 4$ SYM, $\text{pf}\,\mathcal{D} = |\text{pf}\,\mathcal{D}|e^{i\alpha}$

Complicates interpretation of $\left\{e^{-S_B}\,\text{pf}\,\mathcal{D}\right\}$ as Boltzmann weight

We carry out phase-quenched RHMC, $\text{pf}\,\mathcal{D} \longrightarrow |\text{pf}\,\mathcal{D}|$

In principle need to reweight phase-quenched (pq) observables:

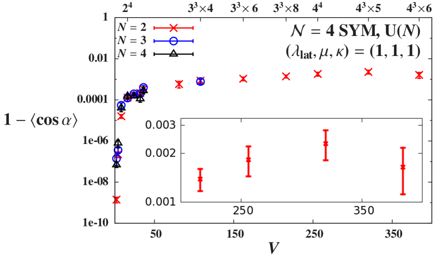
$$\langle\mathcal{O}\rangle = \frac{\left\langle\mathcal{O}e^{i\alpha}\right\rangle_{pq}}{\left\langle e^{i\alpha}\right\rangle_{pq}} \qquad \text{with} \left\langle\mathcal{O}e^{i\alpha}\right\rangle_{pq} = \frac{1}{\mathcal{Z}_{pq}}\int[d\mathcal{U}][d\overline{\mathcal{U}}]\,\mathcal{O}e^{i\alpha}\,e^{-S_B}\,|\text{pf}\,\mathcal{D}|$$

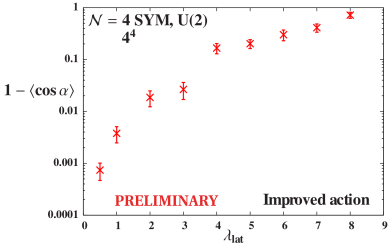
$\Longrightarrow$ Monitor $\left\langle e^{i\alpha}\right\rangle_{pq}$ as function of volume, coupling, N

Backup: Pfaffian dependence on volume and coupling

Left: $1 - \langle\cos(\alpha)\rangle_{pq} \ll 1$ independent of volume and N at $\lambda_{\text{lat}} = 1$

Right: New 4^4 results at $4 \leq \lambda_{\text{lat}} \leq 8$ show much larger fluctuations





Currently measuring Pfaffian phase for more volumes and N

Extremely expensive analysis despite new parallel algorithm:

$\mathcal{O}(n^3)$ scaling $\longrightarrow \sim$50 hours for single 4^4 measurement

Backup: Two puzzles posed by the sign problem

- With periodic temporal boundary conditions for the fermions
  we have an obvious sign problem, $\langle e^{i\alpha} \rangle_{pq}$ consistent with zero
- With anti-periodic BCs and all else the same $e^{i\alpha} \approx 1$,
  phase reweighting has negligible effect

Why such sensitivity to the BCs?

Also, other observables
are nearly identical
for these two ensembles

Why doesn't the sign problem
affect other observables?