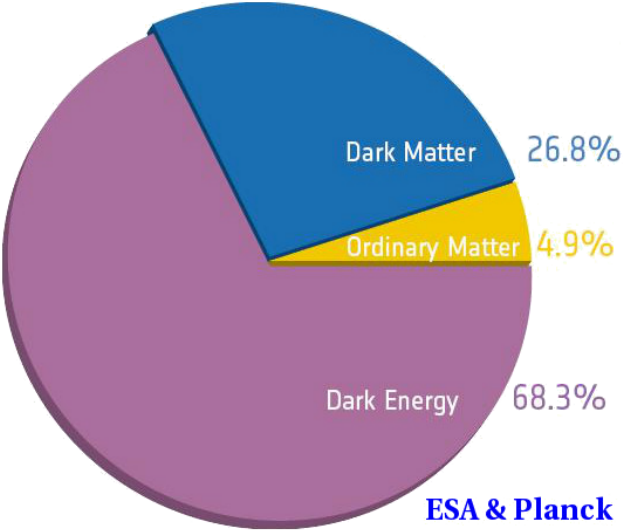
Composite dark matter and the role of lattice field theory

David Schaich (Syracuse)

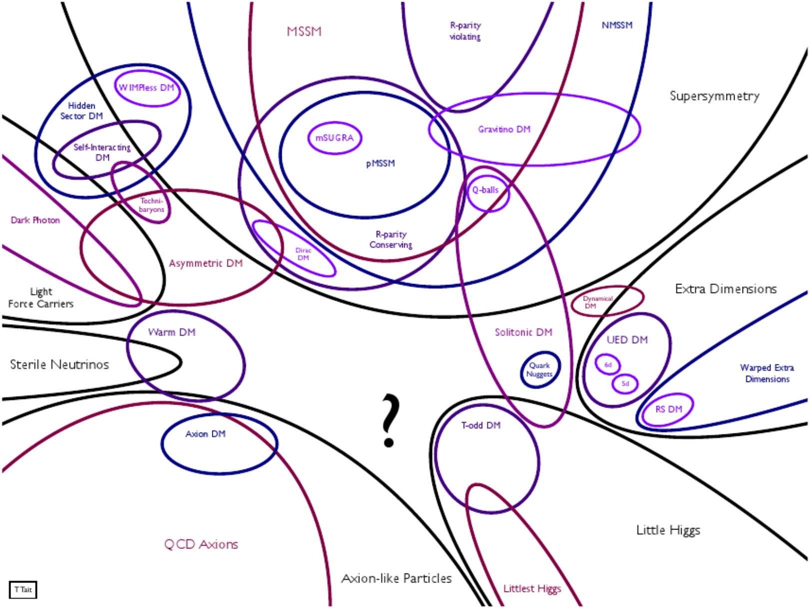
Rensselaer Colloquium
17 February 2016

PRL 115:171803 (2015, Editors' Suggestion) [arXiv:1503.04205]
PRD 92:075030 (2015, Editors' Suggestion) [arXiv:1503.04203]
and work in progress with the Lattice Strong Dynamics Collaboration

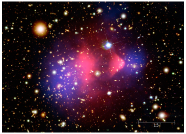
Dark matter — we see it out there...



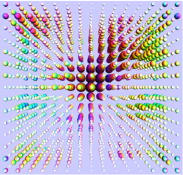
...we don’t yet know what it is



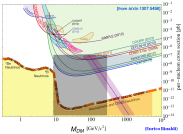
Plan for this talk



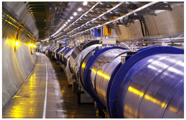
- A brief review of dark matter
  and motivations for compositeness



- Lattice field theory for strong interactions

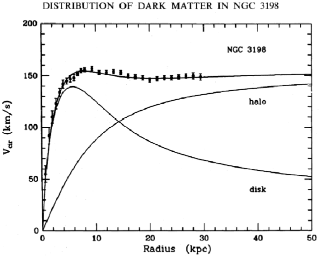


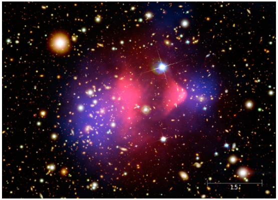
- Predictions for composite dark matter form factors
  $\longrightarrow$ direct detection rates and constraints

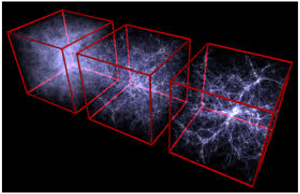


- Complementary collider searches
  and future prospects

Evidence for dark matter

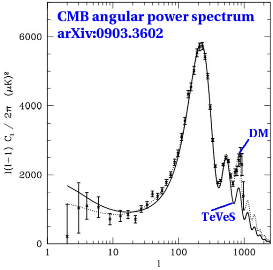




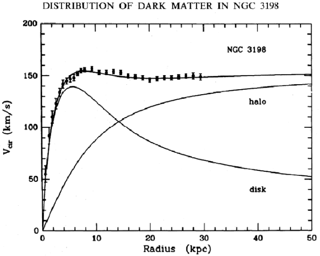


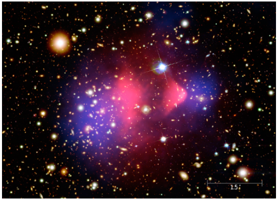
Multiple consistent lines of evidence spanning many scales

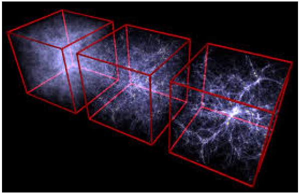
- Rotation curves of galaxies & clusters
- Gravitational lensing
- Structure formation
- Cosmological backgrounds



Evidence for dark matter







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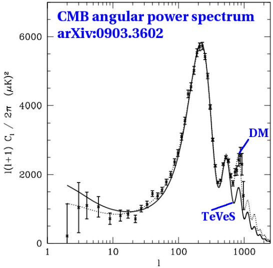


All of these are gravitational effects

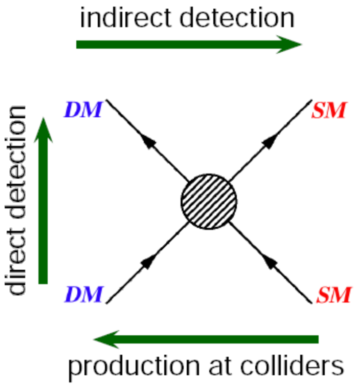
Non-gravitational searches for dark matter

Non-gravitational interactions with known particles and fields ("SM")
give rise to three generic processes:

Direct scattering
in large underground detectors

Indirect annihilation into cosmic rays

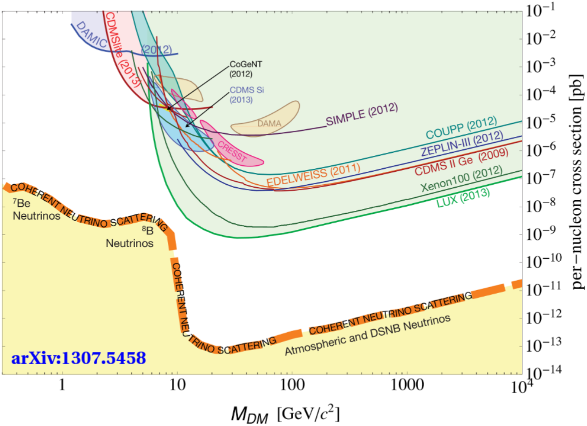
Collider production at high energies



So far there are no unambiguous signals

Non-gravitational searches for dark matter

So far there are no unambiguous signals
at colliders, in cosmic rays, or underground



Even so, there are reasons to expect non-gravitational interactions

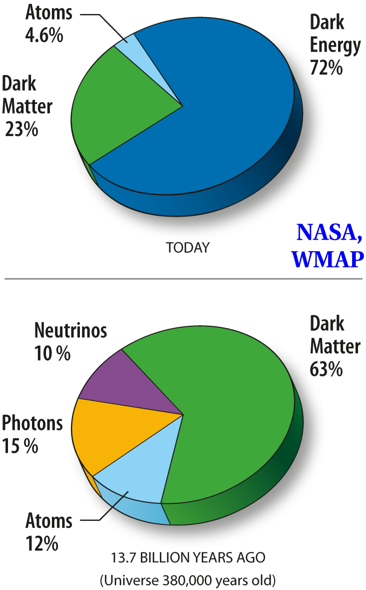
Motivation for non-gravitational interactions

Since the early universe,

$$\frac{\Omega_{DM}}{\Omega_{SM}} \approx 5 \quad \ldots \text{not } 10^5 \text{ or } 10^{-5}$$

All known explanations rely on
non-gravitational interactions

Examples:
—thermal relic
—primordial asymmetry



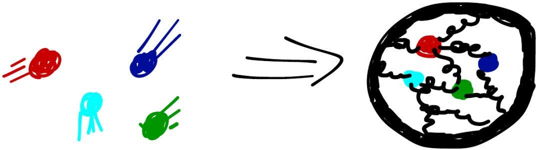
Composite dark matter

A simple idea

- Non-gravitational interactions "on" in the early universe
  $\longrightarrow$ Observed dark matter relic abundance
- Non-gravitational interactions "off" since then
  $\longrightarrow$ Non-observation of direct-detection (etc.) signals

A simple realization: Composite dark matter

- Charged fermions F interact shortly after the hot big bang
- Then F **confine** to form stable neutral composite particles

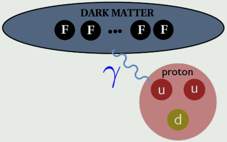


Even neutral composites interact

Direct-detection signals arise from **form factors** of composite particle

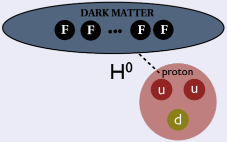
Electromagnetic form factors must produce photon-exchange interactions suppressed by powers of confinement scale $\Lambda \sim M_{DM}$

- Magnetic moment $\longrightarrow 1/\Lambda$
- Charge radius $\longrightarrow 1/\Lambda^2$
- Polarizability $\longrightarrow 1/\Lambda^3$



The scalar form factor can produce a Higgs-exchange interaction

Depends on coupling of F to Higgs. . .



Confinement and form factors are intrinsically non-perturbative

Lattice field theory in a nutshell: QFT

Lattice field theory provides quantitative non-perturbative predictions for strongly interacting quantum field theories (QFTs)

QFT merges quantum mechanics and special relativity
$\longrightarrow$ four-dimensional spacetime filled by relativistic quantum fields

The QFT / StatMech Correspondence

Generating functional (path integral)	Canonical partition function
$\mathcal{Z} = \int \mathcal{D}\Phi \;\; e^{-S[\Phi]\;/\;\hbar}$	$\int \mathcal{D}q\,\mathcal{D}p \;\; e^{-H(q,p)\;/\;k_B T}$
$\Phi \longrightarrow$ field configurations	$(q, p) \longrightarrow$ phase space
Action $S[\Phi] = \int d^4x\; \mathcal{L}[\Phi(x)]$	Hamiltonian H
$\hbar(=1) \longrightarrow$ quantum fluctuations	$k_B T \longrightarrow$ thermal fluctuations

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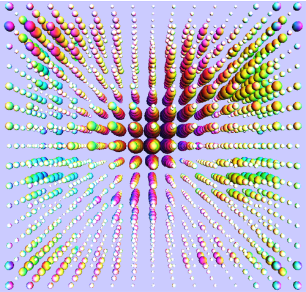
Lattice field theory in a nutshell: Regularization

Any QFT observable is formally $\langle \mathcal{O} \rangle = \dfrac{1}{\mathcal{Z}} \displaystyle\int \mathcal{D}\Phi \;\; \mathcal{O}(\Phi) \;\; e^{-S[\Phi]}$

…but this is an infinite-dimensional integral

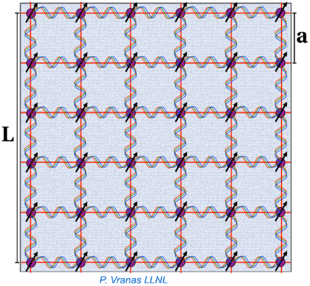

(Image credit: Claudio Rebbi)

Regularize the theory by formulating it in a finite, discrete spacetime $\longrightarrow$ **the lattice**

Lattice field theory in a nutshell: Numerics

Any QFT observable is formally $\langle\mathcal{O}\rangle = \frac{1}{\mathcal{Z}}\int\mathcal{D}\Phi\ \ \mathcal{O}(\Phi)\ \ e^{-S[\Phi]}$

…but this is an infinite-dimensional integral



Regularize the theory by formulating it in a finite, discrete spacetime $\longrightarrow$ **the lattice**

Spacing between lattice sites ("a") introduces UV cutoff scale $1/a$

Remove cutoff by taking continuum limit: $a \to 0$ with $L/a \to \infty$

Finite-dimensional integral $\Longrightarrow$ we can compute $\langle\mathcal{O}\rangle$ numerically using importance sampling Monte Carlo algorithms

Lattice Strong Dynamics Collaboration

Argonne	Xiao-Yong Jin, James Osborn
Boston	Rich Brower, Claudio Rebbi, Evan Weinberg
Colorado	Ethan Neil
Edinburgh	Oliver Witzel
Livermore	Evan Berkowitz, Enrico Rinaldi, Mike Buchoff, Pavlos Vranas
Oregon	Graham Kribs
StonyBrook	Sergey Syritsyn
Syracuse	DS
UC Davis	Joseph Kiskis
Yale	Thomas Appelquist, George Fleming, Andy Gasbarro

Exploring the range of possible phenomena in strongly coupled field theories

Composite dark matter on the lattice

- Magnetic moment and charge radius — arXiv:1301.1693
- Effective Higgs interaction — arXiv:1402.6656, arXiv:1503.04203
- Polarizability — arXiv:1503.04205
- Form factors for collider searches — underway

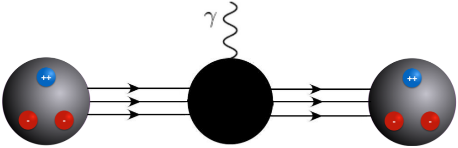
IBM Blue Gene/Q @Livermore

USQCD cluster @Fermilab

Results to be shown are from state-of-the-art lattice calculations
Many thanks to DOE, through Livermore and USQCD!

Magnetic moment and charge radius on the lattice

The observable to compute on the lattice is the composite dark matter interacting with a photon



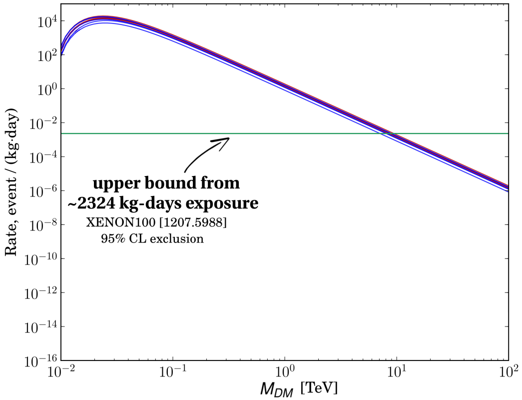
With $q = p' - p$ the non-perturbative vertex function is schematically

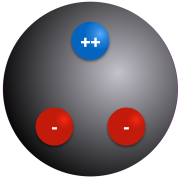
$$\left\langle DM(p') \left| \Gamma_\mu(q^2) \right| DM(p) \right\rangle \sim F_1(q^2)\gamma_\mu + F_2(q^2)\frac{i\sigma_{\mu\nu}q^\nu}{2M_{DM}}$$

- The electric charge is $F_1(0) = 0$
- The magnetic moment is $F_2(0)$
- The charge radius is $-6 \left.\frac{dF_1(q^2)}{dq^2}\right|_{q^2=0} + \frac{3F_2(0)}{2M_{DM}^2}$

Magnetic moment and charge radius results

Lattice calculations of magnetic moment and charge radius
$\longrightarrow$ predict direct detection event rate for given dark matter mass M_{DM}



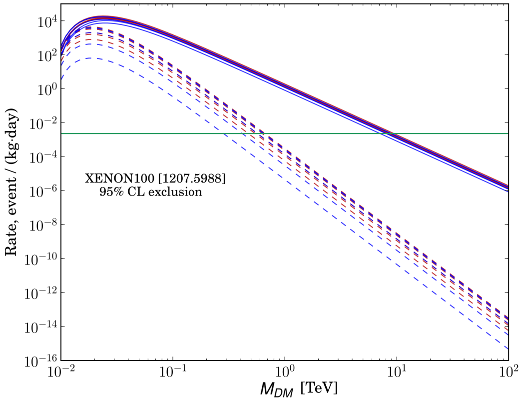


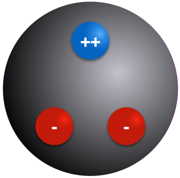
$M_{DM} \lesssim 10$ TeV excluded
by XENON100 results
(PRL 109:181301, 2012)

(Each curve considers different balance of Λ vs. mass of F in M_{DM})

Magnetic moment vs. charge radius independently

Charge radius contributions (dashed) are suppressed $\sim 1/M_{DM}^2$





XENON100 sensitivity
to charge radius
weaker by $\sim 20\times$

Symmetries can forbid both magnetic moment and charge radius
but interaction via polarizability is unavoidable

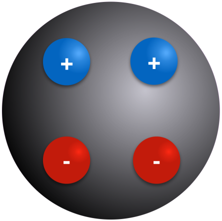
Stealth dark matter — symmetries and polarizability

Composite dark matter with four F

Scalar particle $\longrightarrow$ no magnetic moment

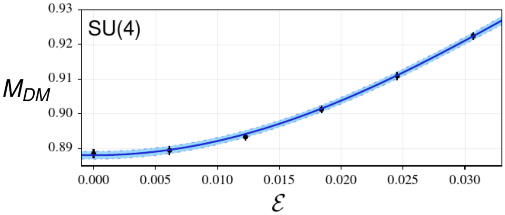
+/- charge symmetry $\longrightarrow$ no charge radius

Higgs exchange can be negligibly small

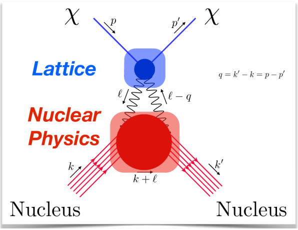


Polarizability places lower bound on direct-detection cross section

Compute on lattice as dependence of M_{DM} on external field $\mathcal{E}$



$\rightsquigarrow$

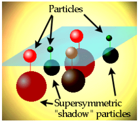


Stealth dark matter

Direct detection cross section

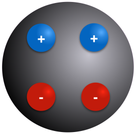
Fourth Dirac neutrino

$\sigma \sim 10^{-2}$ pb



"Focus point" neutralino

$10^{-6} \lesssim \sigma \lesssim 10^{-5}$ pb



Stealth dark matter

$$\sigma \sim \left(\frac{200 \text{ GeV}}{M_{DM}}\right)^6 \times 10^{-9} \text{ pb}$$

Radar cross section

Boeing 747

$\sigma \sim 10^2$ m^2

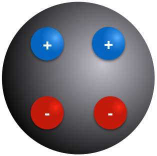
Falcon

$\sigma \sim 10^{-2}$ m^2

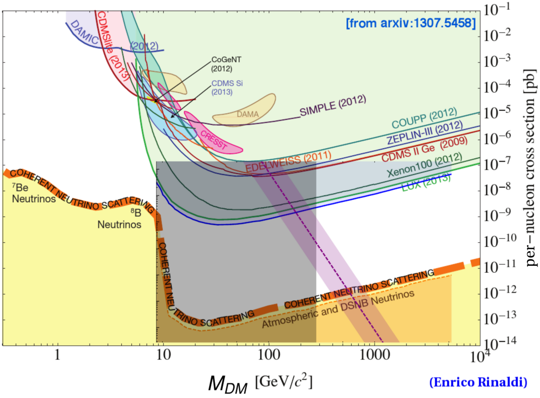
Stealth F-22

$\sigma < 10^{-3}$ m^2

Direct detection of stealth dark matter



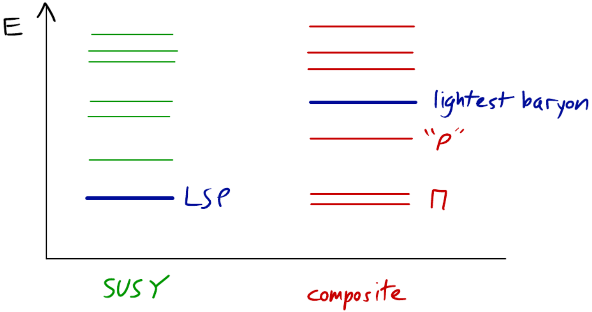
Cross section specific to Xenon



- Signal below coherent neutrino background for $M_{DM} \gtrsim 700$ GeV
- Uncertainties dominated by nuclear matrix element
- Shaded region is complementary constraint from collider searches

Stealth dark matter at colliders

The dark matter is the only stable composite particle,
not the lightest composite particle formed from F

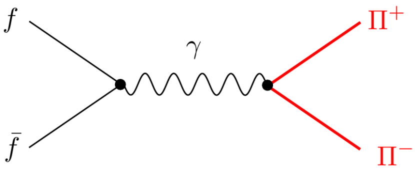


$\Longrightarrow$ Collider searches for typical "missing energy" signals
are not well suited for stealth dark matter

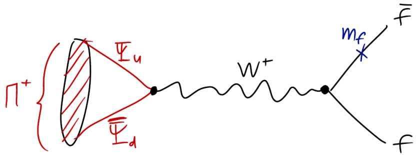
$\Longrightarrow$ Main constraints are from much lighter **charged** "Π" states

Stealth dark matter collider detection

Production



Decay



Constraint shown earlier combined two ingredients:

- LEP searches for supersymmetric tau-partners (same signal as Π)
- Lattice calculations of mass ratio M_{DM}/M_{Π}

Neglected two composite effects we are now computing on the lattice:

- Form factor $F_1(q^2)$ of Π particles, at $q^2 = (2M_{\Pi})^2$
- Decay constant F_{Π} of Π particles

Recapitulation and outlook

- **Composite dark matter** is elegant, viable and well motivated
- **Lattice field theory** is needed to obtain quantitative predictions
- **Direct detection** proceeds through electromagnetic form factors, with a lower bound from the polarizability
- **Collider searches** could be more powerful than direct detection once relevant form factors are computed
- **Many more** characteristic features omitted from this talk
  (Higgs exchange; indirect detection; relic abundance;
  gravitational waves from early-universe confinement transition)

Thank you!

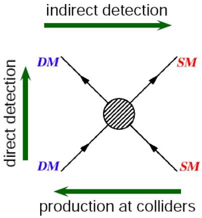
Thank you!

Collaborators

Tom Appelquist, Evan Berkowitz, Rich Brower, Mike Buchoff, George Fleming, Xiao-Yong Jin, Joe Kiskis, Graham Kribs, Ethan Neil, James Osborn, Claudio Rebbi, Enrico Rinaldi, Sergey Syritsyn, Pavlos Vranas, Evan Weinberg, Oliver Witzel

Funding and computing resources

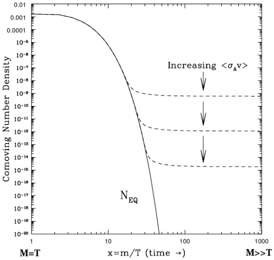
Backup: Thermal freeze-out for relic density



$T \gtrsim M_{DM}$: DM $\longleftrightarrow$ SM
Thermal equilibrium

$T \lesssim M_{DM}$: DM $\longrightarrow$ SM
Rapid depletion of Ω_{DM}

Hubble expansion $\longrightarrow$ dilution
leads to freeze-out



Requires coupling between standard model and dark matter

Mass and coupling of pure thermal relic are related: $\dfrac{M_{DM}}{100\text{ GeV}} \sim 200\alpha$

(The "WIMP miracle" is $\alpha \sim \alpha_{EW} \sim 0.01 \Longrightarrow M_{DM} \sim 200$ GeV $\sim v$)

Strong coupling $\alpha \sim 16 \Longrightarrow M_{DM} \sim 100$ TeV if pure thermal relic

Backup: Two roads to natural asymmetric dark matter

Basic idea: Dark matter relic density related to baryon asymmetry

$$\Omega_{DM} \approx 5\Omega_B$$
$$\Longrightarrow M_{DM} n_{DM} \approx 5 M_B n_B$$

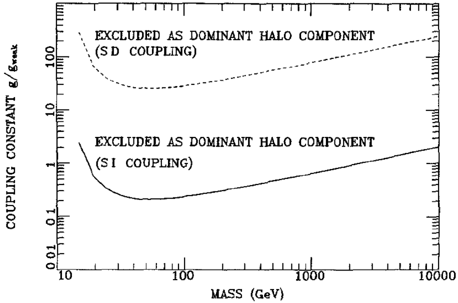
- $n_{DM} \sim n_B \quad \Longrightarrow \quad M_{DM} \sim 5 M_B \approx 5$ GeV
  High-dimensional interactions relate baryon# and DM# violation
- $M_{DM} \gg M_B \quad \Longrightarrow \quad n_B \gg n_{DM} \sim \exp\left[-M_{DM}/T_s\right]$
  Sphaleron transitions above $T_s \sim 200$ GeV distribute asymmetries
  Expect $M_{DM} \lesssim 1$ TeV

Both require coupling between standard model and dark matter

Backup: Weakly interacting composite dark matter

SU(N) composite dark matter "baryons" are bosons if N is even,
Dirac fermions if N is odd

If the dark matter has a net weak charge
$\Longrightarrow$ Unsuppressed tree-level Z-exchange interaction with nuclei
$\Longrightarrow$ Spin-independent cross section $\sigma \sim 10^{-2}$ pb
$\Longrightarrow$ Ruled out decades ago (Example: Ahlen et al., 1987)



Neutralinos are Majorana fermions, so evade this bound

Backup: Importance sampling Monte Carlo

$$\langle \mathcal{O} \rangle = \frac{1}{\mathcal{Z}} \int \mathcal{D}\Phi \;\; \mathcal{O}(\Phi) \;\; e^{-S[\Phi]}$$

Importance sampling Monte Carlo

Approximate integral with a finite ensemble of field configurations $\{\Phi_i\}$

Algorithms choose each configuration Φ_i with probability $\frac{1}{\mathcal{Z}} e^{-S[\Phi_i]}$

to find those that make the most important contributions

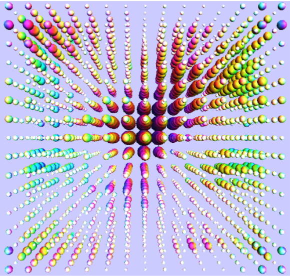
Then $\langle \mathcal{O} \rangle = \frac{1}{N} \sum_{i=1}^{N} \mathcal{O}(\Phi_i)$ with statistical uncertainty $\propto \sqrt{\frac{1}{N}}$

Generating ensembles $\{\Phi_i\}$ often dominates computational costs

These saved data can be reused to investigate many observables

Backup: Hybrid Monte Carlo (HMC) algorithm

Goal: Sample field configurations Φ_i with probability $\frac{1}{\mathcal{Z}} e^{-S[\Phi_i]}$



(Image credit: Claudio Rebbi)

HMC is a Markov process, based on Metropolis–Rosenbluth–Teller (MRT)

Fermions $\longrightarrow$ extensive action computation, so best to update entire system at once

Use fictitious molecular dynamics evolution

1. Introduce a fictitious fifth dimension ("MD time" τ) and stochastic canonical momenta for all field variables
2. Run inexact MD evolution along a trajectory in τ to generate new four-dimensional field configuration
3. Apply MRT accept/reject test to MD discretization error

Backup: Form factor calculations on the lattice

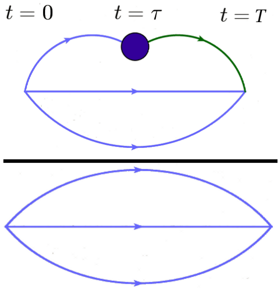
With $q = p' - p$ and $Q^2 = -q^2 > 0$,

$$\left\langle DM(p') \left| \overline{\psi}\gamma^\mu\psi \right| DM(p) \right\rangle = \overline{U}(p') \left[F_1^\psi(Q^2)\gamma^\mu + F_2^\psi(Q^2) \frac{i\sigma^{\mu\nu}q_\nu}{2M_{DM}} \right] U(p)$$

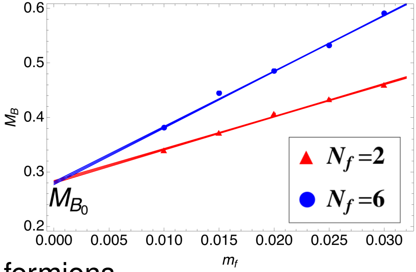
$$\kappa \equiv F_2(0) \qquad \left\langle r^2 \right\rangle = \int d^3r \left[r^2\ \rho(r) \right] \equiv -6 \left. \frac{dF_1(Q^2)}{dQ^2} \right|_{Q^2=0} + \frac{3\kappa}{2M_{DM}^2}$$

$$\begin{aligned} R_{\mathcal{O}}\left(\tau, T, p, p'\right) & \\ \longrightarrow & \left\langle B(p') \left| \mathcal{O} \right| B(p) \right\rangle \\ & + \mathcal{O}\left(e^{-\Delta\tau}\right) + \mathcal{O}\left(e^{-\Delta T}\right) \\ & + \mathcal{O}\left(e^{-\Delta(T-\tau)}\right) \end{aligned}$$

$R(\tau, T, p, p') \sim$



Backup: Three-fermion composite dark matter



Initial explorations re-analyze existing lattice ensembles

- SU(3) gauge group (like QCD)
- $32^3 \times 64$ lattices with domain wall fermions
- **Compare** $N_F = 2$ or 6 degenerate flavors, with fixed confinement scale $\Lambda \sim M_{B_0}$
- **Scan** range of fermion masses m_F
  Unlike QCD fermions are relatively heavy, $0.55 \lesssim M_\Pi / M_V \lesssim 0.75$

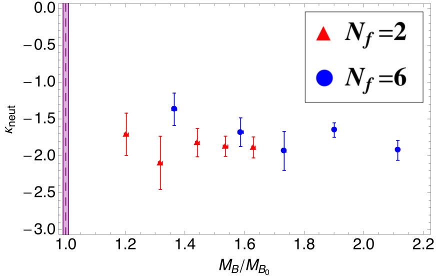
Also unlike QCD, fermions are all $SU(2)_L$ singlets

$Q = Y$, assign half of fermions $Q_P = 2/3$, other half $Q_M = -1/3$
DM candidate is electroweak-neutral "dark baryon" $B = \mathrm{PMM}$

Backup: Form factor results

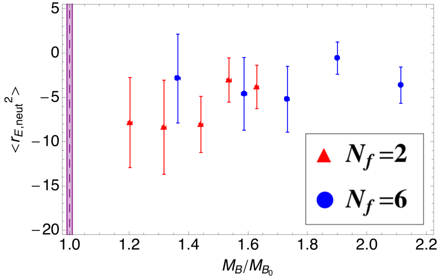
Magnetic moment κ

$\left(\overline{\psi}\sigma^{\mu\nu}\psi\right) F_{\mu\nu}/\Lambda$ (dim-5)



Charge radius $\langle r^2 \rangle$

$\left(\overline{\psi}\psi\right) v_\mu \partial_\nu F^{\mu\nu}/\Lambda^2$ (dim-6)



Results show little dependence on N_F or on m_F

κ comparable to neutron's $\kappa_N = -1.91$,

$\langle r^2 \rangle$ smaller than $\langle r^2 \rangle_N \approx -38$, due to our larger M_Π/M_V

Insert into the usual calculations to predict scattering rates. . .

Backup: Event rate calculations and lattice input

$$\text{Rate} = \frac{M_{detector}}{M_T} \frac{\rho_{DM}}{M_{DM}} \int_{E_{min}}^{E_{max}} dE_R \mathcal{A}cc(E_R) \left\langle v_{DM} \frac{d\sigma}{dE_R} \right\rangle_f$$

$$\frac{d\sigma}{dE_R} = \frac{\overline{|\mathcal{M}_{SI}|^2} + \overline{|\mathcal{M}_{SD}|^2}}{16\pi \left(M_{DM} + M_T\right)^2 E_R^{max}} \qquad E_R^{max} = \frac{2M_{DM}^2 M_T v_{col}^2}{\left(M_{DM} + M_T\right)^2}$$

From magnetic moment κ and charge radius $\left\langle r^2 \right\rangle$:

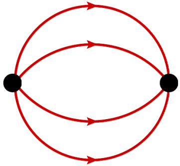
$$\frac{\overline{|\mathcal{M}_{SI}|^2}}{e^4 \left[ZF_c(Q)\right]^2} = \left(\frac{M_T}{M_{DM}}\right)^2 \left[\frac{4}{9} M_{DM}^4 \left\langle r^2 \right\rangle^2 + \frac{\kappa^2 \left(M_T + M_{DM}\right)^2 \left(E_R^{max} - E_R\right)}{M_T E_R}\right]$$

$$\overline{|\mathcal{M}_{SD}|^2} = e^4 \frac{2}{3} \left(\frac{J+1}{J}\right) \left[\left(A \frac{\mu_{T,DM}}{\mu_{n,DM}}\right) F_s(Q)\right]^2 \kappa^2$$

From polarizability C_F — note dependence on Z^4:

$$\sigma_{SI} = \frac{Z^4}{A^2} \frac{144\pi \alpha_{em}^4 \mu_{n,DM}^2}{M_{DM}^6 R^2} C_F^2 \qquad \text{per nucleon}$$

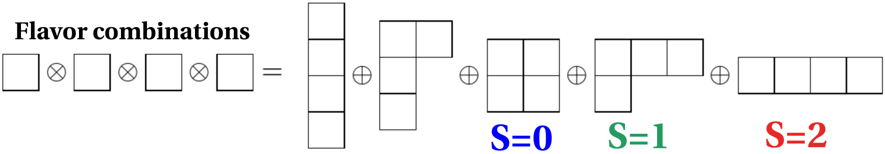
Backup: Four-fermion composite dark matter



Generate quenched SU(4) lattice ensembles

Lattice volumes up to $64^3 \times 128$,
several lattice spacings to check systematic effects

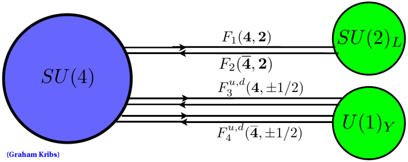
Again consider relatively heavy fermions $\longrightarrow$ $0.5 \lesssim M_{PS}/M_V \lesssim 0.9$



Dark matter candidate is spin-zero baryon $\longrightarrow$ no magnetic moment

Interested in models with at least two flavors to anti-symmetrize
Those with custodial SU(2) global symmetry $\longrightarrow$ no charge radius

Backup: Stealth dark matter model details



Field	$SU(N_D)$	$(SU(2)_L,\ Y)$	Q
$F_1 = \begin{pmatrix} F_1^u \\ F_1^d \end{pmatrix}$	$\mathbf{N}$	$(\mathbf{2},\ 0)$	$\begin{pmatrix} +1/2 \\ -1/2 \end{pmatrix}$
$F_2 = \begin{pmatrix} F_2^u \\ F_2^d \end{pmatrix}$	$\bar{\mathbf{N}}$	$(\mathbf{2},\ 0)$	$\begin{pmatrix} +1/2 \\ -1/2 \end{pmatrix}$
F_3^u	$\mathbf{N}$	$(\mathbf{1}, +1/2)$	$+1/2$
F_3^d	$\mathbf{N}$	$(\mathbf{1}, -1/2)$	$-1/2$
F_4^u	$\bar{\mathbf{N}}$	$(\mathbf{1}, +1/2)$	$+1/2$
F_4^d	$\bar{\mathbf{N}}$	$(\mathbf{1}, -1/2)$	$-1/2$

Mass terms $\sim m_V\left(F_1 F_2 + F_3 F_4\right) + y\left(F_1 \cdot H F_4 + F_2 \cdot H^\dagger F_3\right) + \text{h.c.}$

Both vector-like masses m_V and Higgs couplings y are **required**

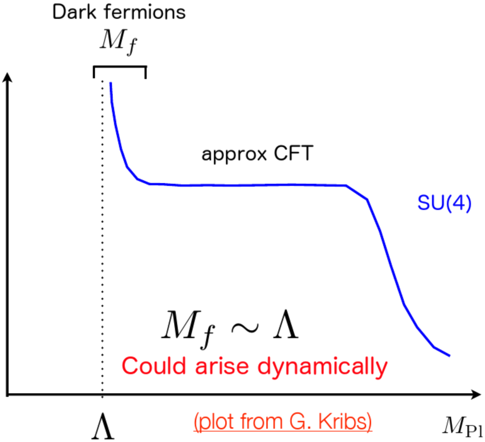
- Higgs couplings ensure rapid meson decay in early universe
- Vector-like masses avoid bounds on direct detection via Higgs exchange

Backup: Stealth dark matter mass scales

Lattice calculations have focused on $m_F \simeq \Lambda$,
the regime where analytic estimates are least reliable

This mass scale has
some theoretical motivation

In addition,
collider constraints tighten
as m_F decreases
($\longrightarrow$ larger M_{DM}/M_{Π})

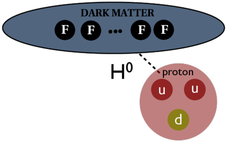


(plot from G. Kribs)

Backup: Effective Higgs interaction

Exchange of Higgs boson with $M_H = 125$ GeV
may dominate spin-independent direct detection cross section

$$\sigma_H^{(SI)} \propto \left| \frac{\mu_{B,N}}{M_H^2} \; y_\psi \left\langle B \left| \overline{\psi}\psi \right| B \right\rangle \; y_q \left\langle N \left| \overline{q}q \right| N \right\rangle \right|^2$$



For quarks $y_q = \dfrac{m_q}{v} \Longrightarrow y_q \left\langle N \left| \overline{q}q \right| N \right\rangle \propto \dfrac{M_N}{v} \dfrac{\left\langle N \left| m_q \overline{q}q \right| N \right\rangle}{M_N}$

For dark fermions ψ there is an additional model parameter,

$$y_\psi = \alpha \frac{m_\psi}{v} \quad \text{with} \quad \alpha \equiv \frac{v}{m_\psi} \frac{\partial m_\psi(h)}{\partial h}\bigg|_{h=v} = \frac{yv}{yv + m_V}$$

$\alpha \to 1$ when $m_\psi(h) \propto h$ ($m_V = 0$ as for quarks)

$\alpha \to 0$ when $m_\psi(h) = m_V$ (no effective Higgs interaction)

Backup: Effective Higgs interaction

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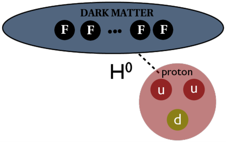
$$\sigma_H^{(SI)} \propto \left| \frac{\mu_{B,N}}{M_H^2} \; y_\psi \left\langle B \left| \overline{\psi}\psi \right| B \right\rangle \; y_q \left\langle N \left| \overline{q}q \right| N \right\rangle \right|^2$$



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In both cases the scalar form factor is most easily determined

using the Feynman–Hellmann theorem $\dfrac{\left\langle B \left| m_\psi \overline{\psi}\psi \right| B \right\rangle}{M_B} = \dfrac{m_\psi}{M_B} \dfrac{\partial M_B}{\partial m_\psi}$

Backup: Lattice results for Higgs exchange

$$\sigma_H^{(SI)} \propto \left| y_\psi \left\langle B \left| \overline{\psi}\psi \right| B \right\rangle \right|^2$$

Matrix element $\propto \dfrac{\partial M_{DM}}{\partial m_\psi}$

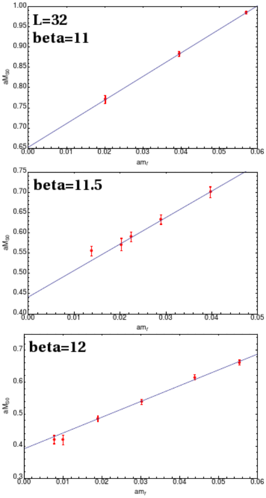
(Feynman–Hellmann)

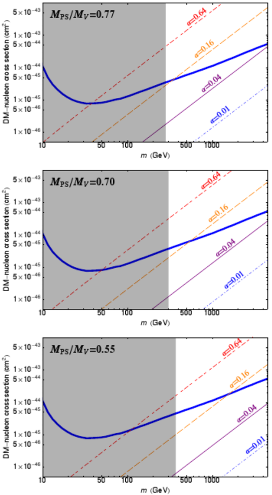
We find

$0.15 \lesssim \dfrac{m_\psi}{M_{DM}} \dfrac{\partial M_{DM}}{\partial m_\psi} \lesssim 0.34$

Compare with QCD

$0.04 \lesssim \dfrac{m_q}{M_N} \dfrac{\partial M_N}{\partial m_q} \lesssim 0.08$

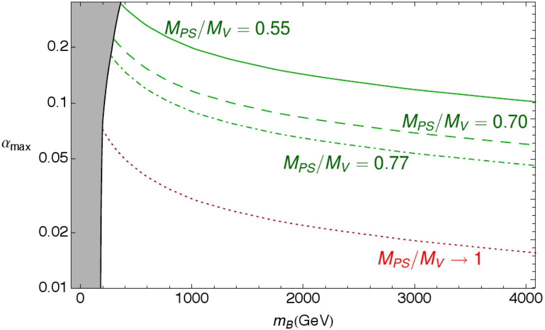




Predicts maximum α allowed by LUX (PRL 112:091303, 2014)

Backup: Bounds on effective Higgs coupling

Higgs-exchange cross section results predict
maximum effective Higgs coupling allowed by LUX



Maximum coupling α
depends on M_Π/M_V
and dark matter mass

LEP bound more significant
for smaller m_F & M_Π

Bottom line: Effective Higgs interaction tightly constrained,
$\alpha \lesssim 0.3$ means fermion masses must be mainly vector-like

Backup: Feynman–Hellmann theorem

- $m_\psi \overline{\psi}\psi$ is the only term in the hamiltonian that depends on m_ψ
$$\longrightarrow \left\langle B \left| \frac{\partial H}{\partial m_\psi} \right| B \right\rangle = \left\langle B \left| \overline{\psi}\psi \right| B \right\rangle$$

- $\langle B | H | B \rangle = M_B$ is just the baryon mass, so

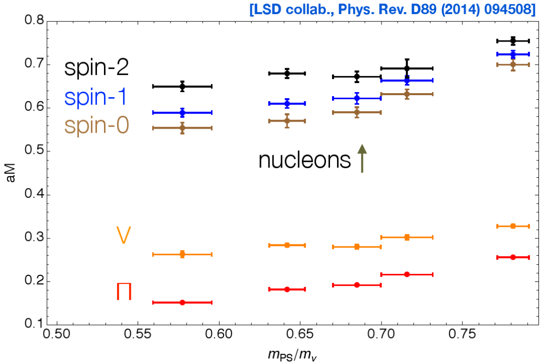
$$\begin{aligned}
\frac{\partial M_B}{\partial m_\psi} &= \frac{\partial}{\partial m_\psi} \langle B | H | B \rangle \\
&= \left\langle \frac{\partial B}{\partial m_\psi} \left| H \right| B \right\rangle + \left\langle B \left| H \right| \frac{\partial B}{\partial m_\psi} \right\rangle + \left\langle B \left| \frac{\partial H}{\partial m_\psi} \right| B \right\rangle \\
&= M_B \langle \frac{\partial B}{\partial m_\psi} | B \rangle + M_B \langle B | \frac{\partial B}{\partial m_\psi} \rangle + \left\langle B \left| \frac{\partial H}{\partial m_\psi} \right| B \right\rangle \\
&= M_B \frac{\partial}{\partial m_\psi} \langle B | B \rangle + \left\langle B \left| \overline{\psi}\psi \right| B \right\rangle \\
&= \left\langle B \left| \overline{\psi}\psi \right| B \right\rangle
\end{aligned}$$

Backup: Indirect detection

Two possible sources of cosmic rays for indirect detection:

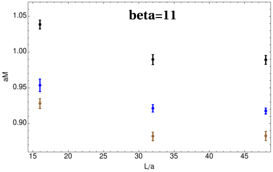
- DM–antiDM annihilation into (many) lighter Π that then decay
- γ-rays from splitting between baryons with spin $S = 0$, 1 and 2

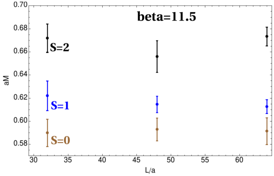
Predictions require lattice results for composite mass spectrum

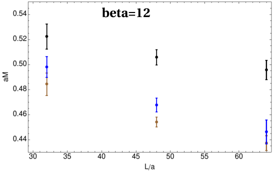


Backup: Volume and discretization effects

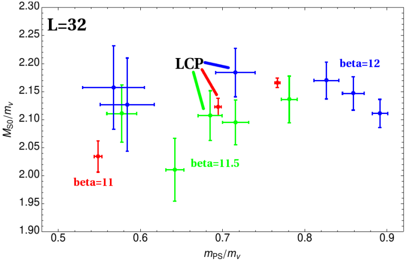
Baryon masses vs. L at fixed coupling β and fermion mass m_ψ:

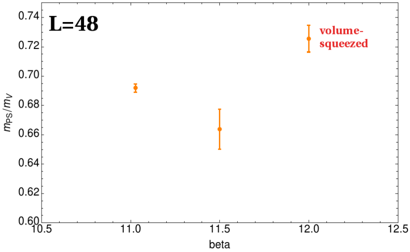






Edinburgh-style plot of $\frac{M_{S0}}{M_V}$ vs. $\frac{M_{PS}}{M_V}$ and line of constant physics (LCP):





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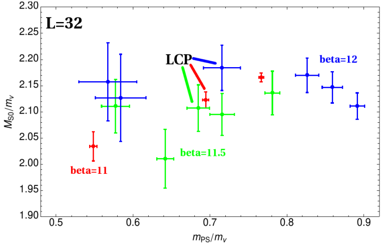
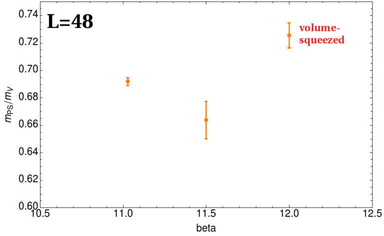
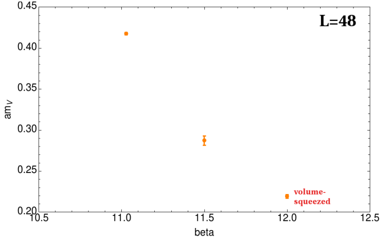
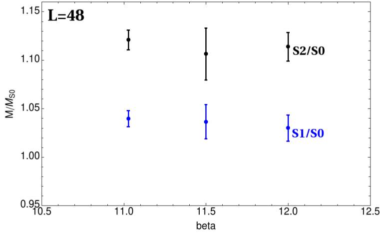




Lattice spacing and discretization effects for $\frac{M_{S2,S1}}{M_{S0}}$ on LCP:





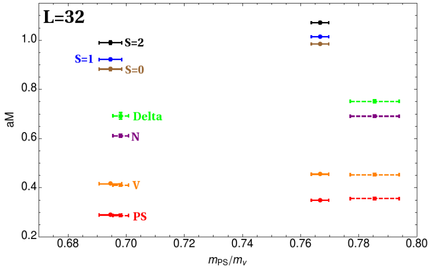
Backup: Large-N predictions for SU(4) baryons

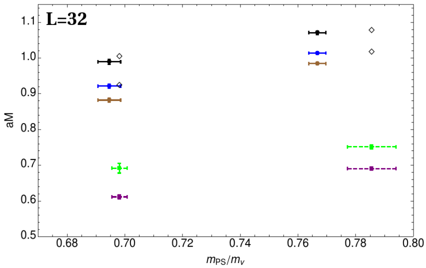
Rotor spectrum for spin-J large-N baryons:

$$M(N,J) = NM_0 + C + B\frac{J(J+1)}{N} + \mathcal{O}\left(\frac{1}{N^2}\right)$$

—Match SU(3) and SU(4) pseudoscalar and vector meson masses

—Fit M_0, C and B with nucleon, Δ and spin-0 baryon masses
$\longrightarrow$ predictions for $S = 1, 2$ baryons (diamonds)



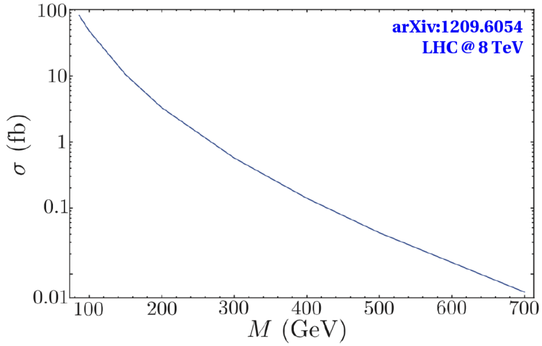


Backup: Stealth dark matter at the LHC

Collider searches at the LHC

require integrating over parton distributions in the proton

Resulting production cross section (still setting $F_1(4M_\Pi^2) = 1$):



LHC can also search for $\Pi^+\Pi^- \longrightarrow t\overline{b} + \overline{t}b$ in addition to $\tau^+\tau^- + \not{E}_T$

Should eventually extend bounds well beyond LEP's $M_\Pi \gtrsim 90$ GeV