Physics Out Of The Box

The impact of lattice gauge theory and large-scale computing

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Michigan State HEP Seminar, 3 November 2015

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Plan for this talk

- A high-level introduction to lattice gauge theory
- Selected applications to high-energy physics
  1. Electroweak symmetry breaking through new strong dynamics
  2. Supersymmetric lattice systems and AdS / CFT duality
  3. Composite dark matter searches at colliders and underground (time permitting)
- Outlook (more to come in subsequent research discussion)

Questions encouraged

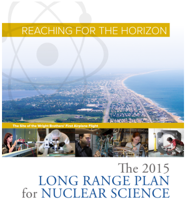
"It is better to uncover a little than to cover a lot" —Viki Weisskopf

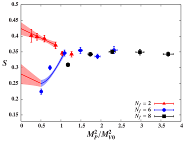
Some things to take away

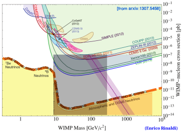
- Computing is a tool
  It doesn't do our physical understanding for us

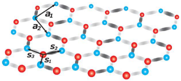
- Lattice gauge theory is a broadly applicable tool
  Used in particle, nuclear, condensed matter physics

- Computing capabilities continue to increase
  $\longrightarrow$ Numerical approaches increasingly valuable









Lattice gauge theory in a nutshell: QFT

Lattice gauge theory is a **regularization** of quantum field theory (QFT)

Let's break this down for non-experts...

QFT merges quantum mechanics and special relativity
$\longrightarrow$ four-dimensional spacetime filled by relativistic quantum fields

The QFT / StatMech Correspondence

Generating functional (path integral)	Canonical partition function
$\mathcal{Z} = \int \mathcal{D}\Phi \;\; e^{-S[\Phi] \,/\, \hbar}$	$\int \mathcal{D}q\, \mathcal{D}p \;\; e^{-H(q,p) \,/\, k_B T}$
Action $S[\Phi] = \int d^4x \; \mathcal{L}[\Phi(x)]$	Hamiltonian H
$\hbar (= 1) \longrightarrow$ quantum fluctuations	$k_B T \longrightarrow$ thermal fluctuations

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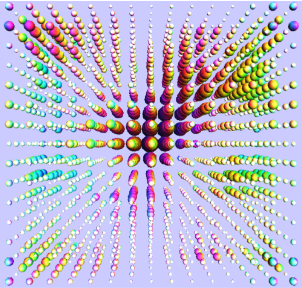
Lattice gauge theory in a nutshell: Regularization

Any given QFT observable is formally $\langle\mathcal{O}\rangle = \dfrac{1}{\mathcal{Z}}\displaystyle\int \mathcal{D}\Phi\ \ \mathcal{O}(\Phi)\ \ e^{-S[\Phi]}$

...but this is an infinite-dimensional integral



Regularize the theory by formulating it in a finite, discrete spacetime $\longrightarrow$ **the lattice**

Provides a fully non-perturbative and gauge-invariant definition of the theory

Spacing between lattice sites ("a") introduces UV cutoff scale $1/a$

Lattice cutoff preserves hypercubic subgroup of full Lorentz symmetry

Remove cutoff by taking continuum limit $a \to 0$ (with $L/a \to \infty$)

Lattice gauge theory in a nutshell: Regularization

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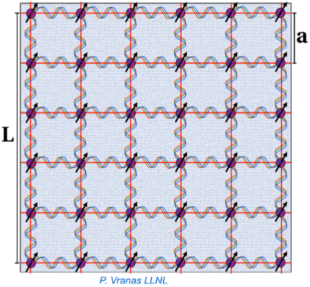

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Lattice gauge theory in a nutshell: Numerics

$$\langle \mathcal{O} \rangle = \frac{1}{\mathcal{Z}} \int \mathcal{D}\Phi \;\; \mathcal{O}(\Phi) \;\; e^{-S[\Phi]}$$

Finite-dimensional integral $\Longrightarrow$ we can compute $\langle \mathcal{O} \rangle$ numerically

Importance sampling Monte Carlo

Approximate integral with a finite ensemble of field configurations $\{\Phi_i\}$

Algorithms choose each configuration Φ_i with probability $\frac{1}{\mathcal{Z}} e^{-S[\Phi_i]}$
to find those that make the most important contributions

Now $\langle \mathcal{O} \rangle = \frac{1}{N} \sum_{i=1}^{N} \mathcal{O}(\Phi_i)$ with statistical uncertainty $\propto \sqrt{\frac{1}{N}}$

Generating ensembles $\{\Phi_i\}$ often dominates computational costs

These saved data can be reused to investigate many observables

For decades lattice gauge theory
has helped to drive advances in high-performance computing

IBM Blue Gene/Q @Livermore

USQCD cluster @Fermilab

Results to be shown are from
state-of-the-art lattice calculations

$\mathcal{O}$(500M core-hours) invested overall

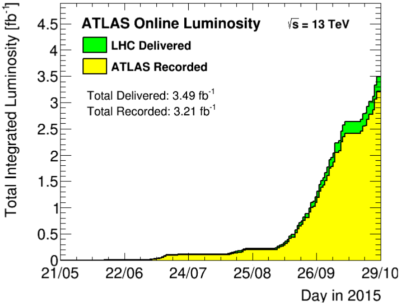
Many thanks to DOE, NSF
and computing centers!

Cray *Blue Waters* @NCSA

Application: Electroweak symmetry breaking

Speaking of big machines...
LHC experiments
are collecting data at $\sqrt{s} = 13$ TeV!

Soon we will see new constraints on
physics beyond the standard model
...and possibly new discoveries!



One compelling possibility is new strong dynamics
that produces a composite Higgs boson

Protects the electroweak scale from sensitivity to quantum effects
(solving the hierarchy / fine-tuning problem)

Lattice gauge theory has a crucial role to play
in exploring and understanding new strong dynamics

Composite Higgs vs. QCD

Electroweak symmetry breaking through new strong dynamics
remains viable but must satisfy stringent experimental constraints

- The composite Higgs boson must have a mass of 125 GeV
and standard-model-like properties
- Electroweak precision observables (e.g., *S* parameter)
must be consistent with the standard model

If the new strong dynamics resembles QCD
these conditions are not satisfied

New strong dynamics different from QCD
can be studied non-perturbatively by lattice calculations

Lattice Strong Dynamics Collaboration

Argonne	Xiao-Yong Jin, James Osborn
Boston	Rich Brower, Claudio Rebbi, Evan Weinberg
Brookhaven	Meifeng Lin
Colorado	Anna Hasenfratz, Ethan Neil
Edinburgh	Oliver Witzel
Livermore	Evan Berkowitz, Enrico Rinaldi, Pavlos Vranas
Oregon	Graham Kribs
RBRC	Ethan Neil, Sergey Syritsyn
Syracuse	DS
UC Davis	Joseph Kiskis
Yale	Thomas Appelquist, George Fleming, Andy Gasbarro

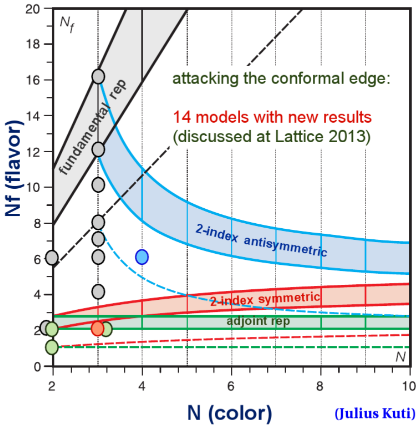
Exploring the range of possible phenomena
in strongly-coupled gauge theories

Strategy for lattice studies of new strong dynamics

Systematically depart from familiar ground of lattice QCD

($N = 3$ with $N_F = 2$ light flavors in fundamental rep)

Identify generic features of non-QCD strong dynamics



—Add more light flavors

$\longrightarrow N_F = 8$ fundamental

—Enlarge fermion rep

$\longrightarrow N_F = 2$ two-index symmetric

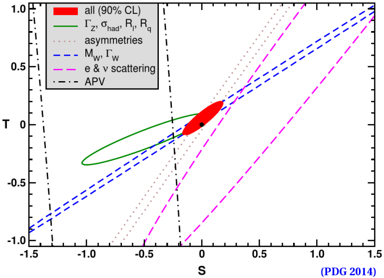
—Explore $N = 2$ and 4

$\longrightarrow$ (pseudo)real reps for cosets

$SU(N)/Sp(N)$ and $SU(N)/SO(N)$

Electroweak precision observable — the *S* parameter

Corrections to vacuum polarization of neutral EW gauge bosons



S remains an important constraint on new strong dynamics

Experiment: $S = 0.03 \pm 0.10$

Scaled-up QCD: $S \approx 0.43$

S is a low-energy constant (LEC) of electroweak chiral lagrangian $\mathcal{L}_\chi$

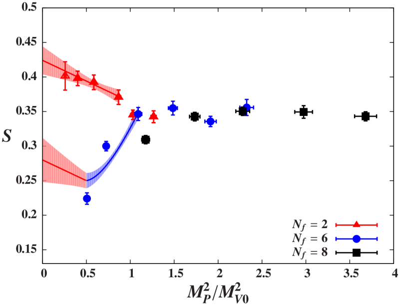
Predicting LECs of low-energy effective theories is a standard application of lattice gauge theory

The S parameter on the lattice

$$\mathcal{L}_\chi \ni \frac{\alpha_1}{2} g_1 g_2 B_{\mu\nu} \mathrm{Tr}\left[U \tau_3 U^\dagger W^{\mu\nu}\right] \longrightarrow$$

Lattice vacuum polarization calculation provides $S = -16\pi^2\alpha_1$

One subtlety is that nonzero masses needed
to keep correlation lengths insensitive to finite lattice volume

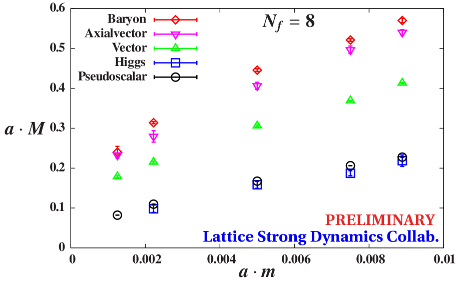


$S = 0.42(2)$ for $N_F = 2$
matches scaled-up QCD

Moving away from QCD with larger N_F
produces significant reductions

Extrapolation to correct zero-mass limit
becomes more challenging

The composite spectrum with eight flavors



Work in progress using lattice volumes up to $64^3 \times 128$

Scale setting suggests resonance masses $\sim$2–3 TeV

Large separation between Higgs and resonances

Higgs degenerate with pseudo-Goldstones in accessible regime

Dramatically different from QCD-like dynamics,
where $M_H \approx 2M_P$ in this regime (dominated by two-pion scattering)

Typical chiral extrapolation integrates out everything except pions, can't reliably be applied to these data

Status of light composite Higgs from lattice

Without reliable chiral extrapolation we can only estimate

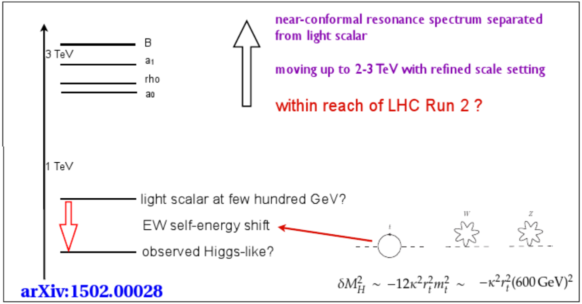
$M_H \sim$ few hundred GeV, with large error bars

Much lighter than scaled-up QCD, still somewhat far from 125 GeV

Of course, we **shouldn't** get exactly 125 GeV

since we haven't yet incorporated electroweak & top corrections

These reduce M_H, but not yet consensus on size of effect...



Application: Lattice supersymmetry

Supersymmetry is extremely interesting, especially non-perturbatively

- Widely-studied potential roles in BSM physics
  Central to ongoing LHC experimental program
  $\longrightarrow$ current results greatly constrain simplest scenarios
- More generally, symmetries simplify systems
  $\longrightarrow$ Insight into confinement, dynamical symmetry breaking,
  conformal field theories (exhibiting scale invariance), etc.
- Dualities: gauge–gauge (Seiberg) & gauge–gravity (AdS/CFT)
  $\longrightarrow$ potential non-perturbative definition of string theory
- AdS/CFT-inspired modelling of quark–gluon plasma,
  finite-density phase diagram, condensed matter systems, etc.

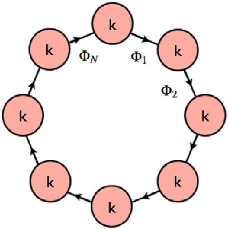
A brief history of lattice supersymmetry

Supersymmetry generalizes Poincaré symmetry,
adding spinorial generators Q and $\overline{Q}$ to translations, rotations, boosts

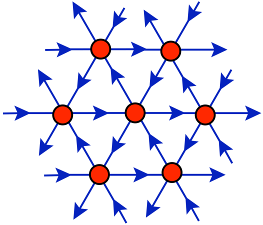
The algebra includes $Q\overline{Q} + \overline{Q}Q = 2\sigma^{\mu} P_{\mu}$,
broken in discrete space-time because no infinitesimal translations

Recent work overcomes this obstacle in certain contexts,
including maximally supersymmetric Yang–Mills ("$\mathcal{N} = 4$" SYM)

Preserve supersymmetry **sub**algebra $\Longrightarrow$ recover rest in continuum



For details see
Catterall, Kaplan & Ünsal
arXiv:0903.4881



$\mathcal{N} = 4$ SYM — the fruit fly of QFT

Maximal supersymmetries make $\mathcal{N} = 4$ SYM
arguably the simplest non-trivial field theory in four dimensions

All fields related by supersymmetries, interactions fixed
(SU(N) gauge field, four fermions and six scalars, all massless)

It is the conformal field theory of the first AdS/CFT duality

Widely studied by many different methods:

- Perturbation theory at weak coupling $\lambda \ll 1$,
  related to strong coupling by S duality under $\frac{4\pi N}{\lambda} \longleftrightarrow \frac{\lambda}{4\pi N}$
- AdS/CFT holography for $N \to \infty$ and $\lambda \to \infty$ but $\lambda \ll N$
- Numerical optimization of conformal bootstrap relations

Only lattice gauge theory can access nonperturbative λ at moderate N

$\mathcal{N} = 4$ SYM — the role of lattice gauge theory

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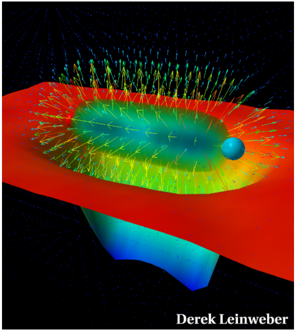
Let's see two results from ongoing large-scale lattice calculations
(work in progress with S. Catterall, P. Damgaard, J. Giedt & T. DeGrand)

1. Coupling dependence of the Coulomb potential
2. Scaling dimension of simplest conformal primary operator

Coulomb potential from lattice $\mathcal{N} = 4$ SYM

Measure potential $V(r)$ between two static probes



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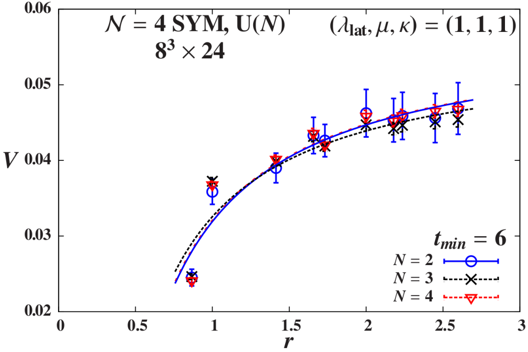


Fits to confining $V(r) = A - C/r + \sigma r$

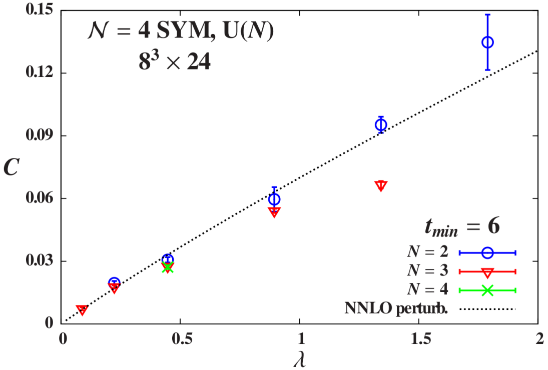
produce vanishing string tension $\sigma = 0$ for all couplings λ

Fits to Coulombic $V(r) = A - C/r$ predict Coulomb coefficient $C(\lambda)$

Coulomb coefficient from lattice $\mathcal{N} = 4$ SYM

Continuum perturbation theory predicts $C(\lambda) = \lambda/(4\pi) + \mathcal{O}(\lambda^2)$

AdS/CFT predicts $C(\lambda) \propto \sqrt{\lambda}$ for $N \to \infty$, $\lambda \to \infty$, $\lambda \ll N$



$N = 2$ results agree with perturbation theory for all $\lambda \lesssim N$

$N = 3$ results bend down for $\lambda \gtrsim 1$ — approaching AdS/CFT?

Konishi operator in lattice $\mathcal{N} = 4$ SYM

$\mathcal{N} = 4$ SYM is conformal at all λ

$\longrightarrow$ power-law decay for all correlation functions $C(r) \propto r^{-2\Delta}$

The scaling dimension Δ_K of the simple Konishi operator has attracted much recent attention

$$\mathcal{O}_K(x) = \sum_{\mathrm{I}} \mathrm{Tr}\left[\Phi^{\mathrm{I}}(x)\Phi^{\mathrm{I}}(x)\right] \qquad \text{(symmetric sum over six scalars)}$$

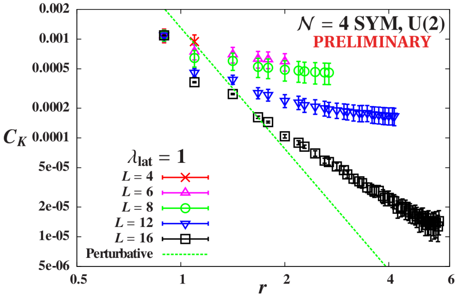
$$C_K(r) = \mathcal{O}_K(x+r)\mathcal{O}_K(x) \propto r^{-2\Delta_K}$$

Obvious sensitivity to volume as desired for conformal system

Lattice tools to find Δ_K:

—Finite-size scaling

—Monte Carlo RG



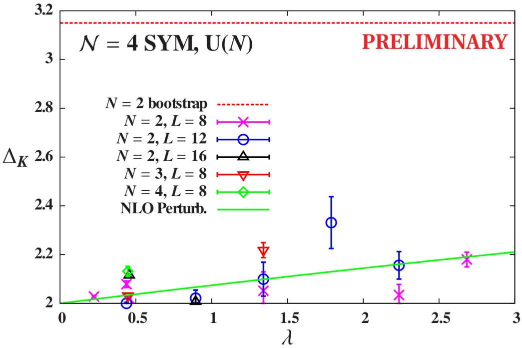
Konishi scaling dimension from lattice $\mathcal{N} = 4$ SYM

Recent predictions for Konishi scaling dimension Δ_K
from perturbation theory + S duality, AdS/CFT, bootstrap

The first lattice calculation of Δ_K is now underway

Initial results from Monte Carlo RG (only statistical errors)

Rough agreement between $N = 2, 3, 4$

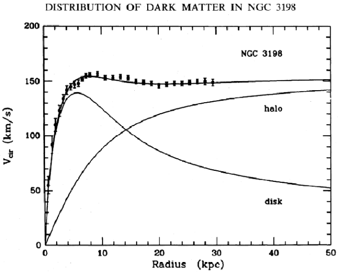


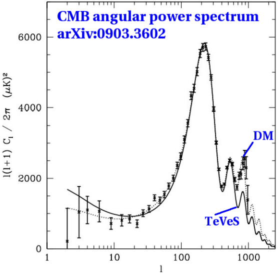
So far results follow perturbation theory, far from bootstrap bounds

Currently refining analyses, running larger volumes at stronger λ

Application: Dark matter

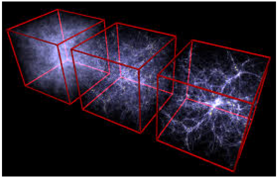
Dark mater is 'known unknown' physics beyond the standard model



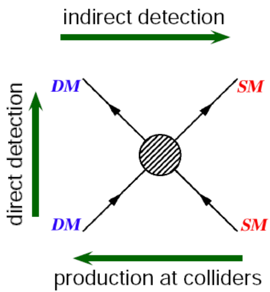


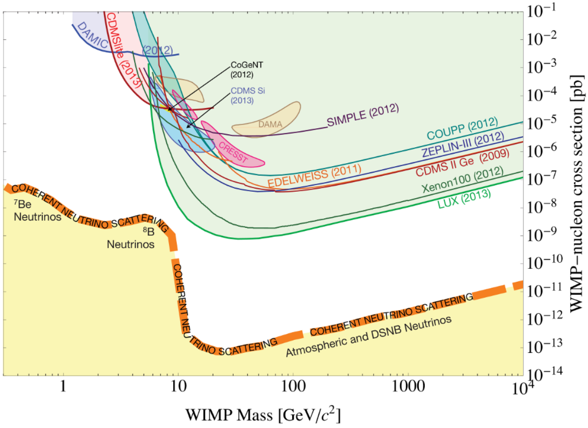
Abundant gravitational evidence on many scales

- Rotation curves of galaxies & clusters
- Gravitational lensing
- Structure formation
- Cosmological backgrounds



No clear signals in non-gravitational searches for dark matter
(at colliders, in cosmic rays, and underground)





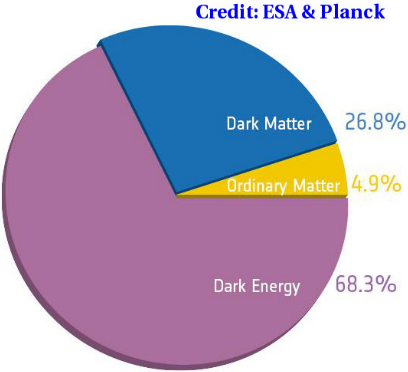
But we expect dark matter to interact with standard model fields. . .

Motivation for non-gravitational interactions

$$\frac{\Omega_{DM}}{\Omega_{SM}} \approx 5 \quad \ldots \text{not } 10^5 \text{ or } 10^{-5}$$

Suggests DM–SM coupling

Common feature of thermal
and asymmetric mechanisms
for relic density generation



If relic density relies on coupling to standard model fields,
such interactions must obey current experimental constraints

Composite dark matter is a natural solution

Charged fermions confined in **electroweak-neutral** "dark baryon"

Direct detection of composite dark matter

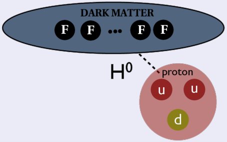
Photon exchange via electromagnetic form factors

Interactions suppressed by powers of confinement scale $\Lambda \sim M_{DM}$

- **Dimension 5:** Magnetic moment $\longrightarrow$ $\left(\overline{\psi}\sigma^{\mu\nu}\psi\right) F_{\mu\nu}/\Lambda$
- **Dimension 6:** Charge radius $\longrightarrow$ $\left(\overline{\psi}\psi\right) v_\mu \partial_\nu F_{\mu\nu}/\Lambda^2$
- **Dimension 7:** Polarizability $\longrightarrow$ $\left(\overline{\psi}\psi\right) F^{\mu\nu} F_{\mu\nu}/\Lambda^3$

Higgs boson exchange via scalar form factor

Effective Higgs interaction of composite DM may be produced by constituent fermions



Scalar form factor gives $\langle B | m_\psi \overline{\psi}\psi | B \rangle$ (σ term)

All form factors arise non-perturbatively $\Longrightarrow$ lattice calculations

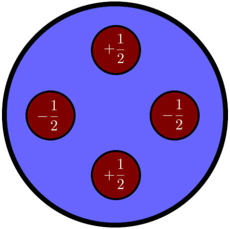
Stealth dark matter

PRL 115:171803

SU(4) gauge theory $\longrightarrow$ scalar dark baryon

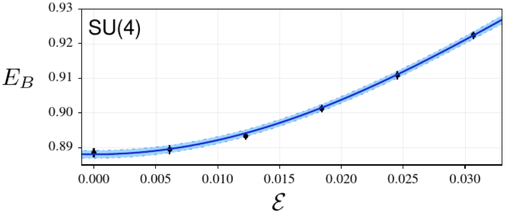
Symmetries forbid magnetic moment
and charge radius

Higgs exchange can also be suppressed



Polarizability places lower bound on direct-detection cross section

Compute on lattice as quadratic response to external field $\mathcal{E}$



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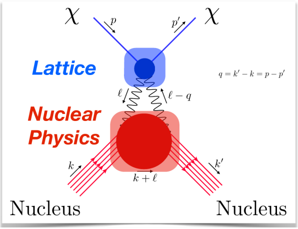


Stealth dark matter direct detection

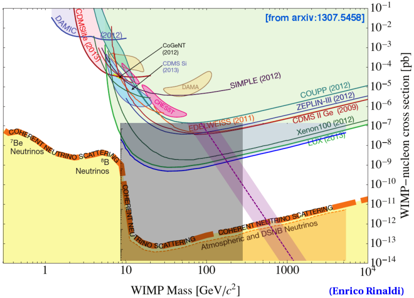
Polarizability places lower bound on direct-detection cross section

Not significantly constrained by existing experiments
and falls below coherent neutrino background for $M_{DM} \gtrsim 1$ TeV

Cross section specific to Xenon

Uncertainties dominated
by nuclear matrix element

(Similar matrix elements
arise in double-beta decay)



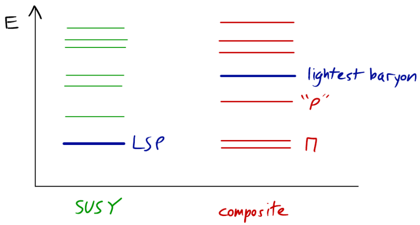
Shaded region is complementary constraint from collider searches

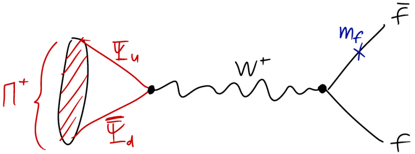
Stealth dark matter collider detection

Spectrum significantly different from MSSM-inspired models

Very little missing E_T at colliders

Main constraints from much lighter **charged** "Π" states





Rapid Π decays with $\Gamma \propto m_f^2$

Best current constraints recast stau searches at LEP

LHC can also search for $t\bar{b} + \bar{t}b$ from $\Pi^+\Pi^-$ Drell–Yan production

Outlook: An exciting time for lattice gauge theory

- Continuing computational progress is enhancing applications across a broad range of physics
- Lattice investigations of new strong dynamics find lighter Higgs and smaller *S* parameter than scaled-up QCD
- First large-scale lattice studies of $\mathcal{N} = 4$ SYM beginning to explore regimes inaccessible to other methods
- Non-perturbative composite dark matter form factors place lower bound on direct-detection cross section

Thank you!

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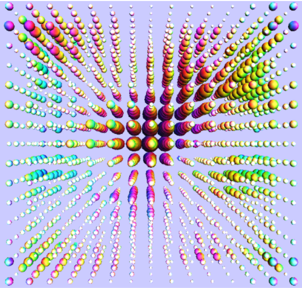
Backup: Hybrid Monte Carlo (HMC) algorithm

Recall goal: Sample field configuration Φ_i with probability $\frac{1}{\mathcal{Z}} e^{-S[\Phi_i]}$



HMC is a Markov process, based on Metropolis–Rosenbluth–Teller (MRT)

Since action computation is extensive, best to update entire system at once

Use fictitious molecular dynamics evolution

1. Introduce a fictitious fifth dimension (“MD time” τ) and stochastic canonical momenta for all field variables
2. Run inexact MD evolution along a trajectory in τ to generate new four-dimensional field configuration
3. Apply MRT accept/reject test to MD discretization error

Backup: Failure of Leibnitz rule in discrete space-time

Given that $\left\{Q_\alpha, \overline{Q}_{\dot\alpha}\right\} = 2\sigma^\mu_{\alpha\dot\alpha}P_\mu = 2i\sigma^\mu_{\alpha\dot\alpha}\partial_\mu$ is problematic,

why not try $\left\{Q_\alpha, \overline{Q}_{\dot\alpha}\right\} = 2i\sigma^\mu_{\alpha\dot\alpha}\nabla_\mu$ for a discrete translation?

Here $\nabla_\mu\phi(x) = \dfrac{1}{a}\left[\phi(x + a\widehat{\mu}) - \phi(x)\right] = \partial_\mu\phi(x) + \dfrac{a}{2}\partial_\mu^2\phi(x) + \mathcal{O}(a^2)$

Essential difference between ∂_μ and ∇_μ on the lattice, $a > 0$

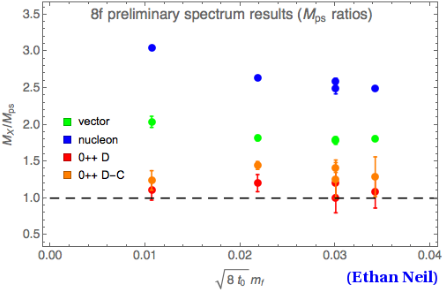
$$\begin{aligned}\nabla_\mu\left[\phi(x)\chi(x)\right] &= a^{-1}\left[\phi(x + a\widehat{\mu})\chi(x + a\widehat{\mu}) - \phi(x)\chi(x)\right] \\ &= \left[\nabla_\mu\phi(x)\right]\chi(x) + \phi(x)\nabla_\mu\chi(x) + a\left[\nabla_\mu\phi(x)\right]\nabla_\mu\chi(x)\end{aligned}$$

We only recover the Leibnitz rule $\partial_\mu(fg) = (\partial_\mu f)g + f\partial_\mu g$ when $a \to 0$

$\Longrightarrow$ "Discrete supersymmetry" breaks down on the lattice

(Dondi & Nicolai, "Lattice Supersymmetry", 1977)

Backup: Eight-flavor spectrum in dimensionless ratios



Work in progress using
lattice volumes up to $64^3 \times 128$

Scale setting suggests
resonance masses ∼2–3 TeV

Large separation between
Higgs and resonances

Higgs degenerate with pion in accessible regime

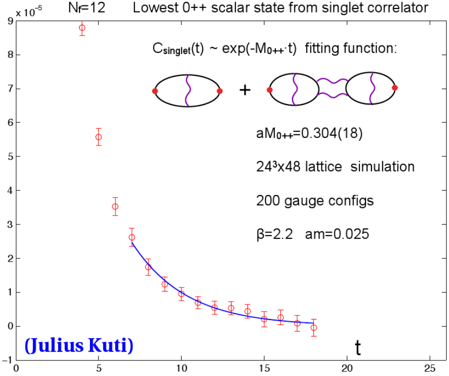
Dramatically different from QCD-like dynamics,
where $M_H \approx 2M_P$ in this regime (dominated by two-pion scattering)

Typical chiral extrapolation integrates out everything except pions,
can't reliably be applied to these data

Backup: Technical lattice challenge for Higgs state

Only the new strong sector is included in the lattice calculation
$\Longrightarrow$ The Higgs is a singlet that mixes with the vacuum

Leads to noisy data and relatively large uncertainties in Higgs mass



Fermion propagator computation is relatively expensive

"Disconnected diagrams" formally need propagators at all L^4 sites

In practice compute stochastically to reduce computational costs

Backup: Vacuum polarization is just current correlator

$$S = 4\pi N_D \lim_{Q^2 \to 0} \frac{d}{dQ^2} \Pi_{V-A}(Q^2) - \Delta S_{SM}(M_H)$$

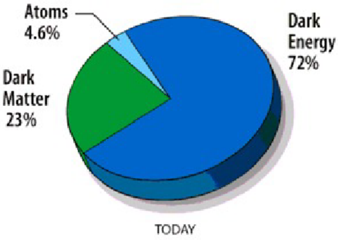
$$\Pi^{\mu\nu}_{V-A}(Q) = Z \sum_x e^{iQ\cdot(x+\widehat{\mu}/2)} \mathrm{Tr}\left[\left\langle \mathcal{V}^{\mu a}(x) V^{\nu b}(0)\right\rangle - \left\langle \mathcal{A}^{\mu a}(x) A^{\nu b}(0)\right\rangle\right]$$

$$\Pi^{\mu\nu}(Q) = \left(\delta^{\mu\nu} - \frac{\widehat{Q}^\mu \widehat{Q}^\nu}{\widehat{Q}^2}\right) \Pi(Q^2) - \frac{\widehat{Q}^\mu \widehat{Q}^\nu}{\widehat{Q}^2} \Pi^L(Q^2) \qquad \widehat{Q} = 2\sin(Q/2)$$

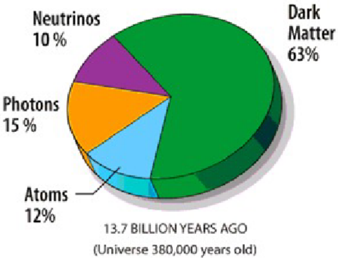
- Renormalization constant Z evaluated non-perturbatively
  Chiral symmetry of domain wall fermions $\Longrightarrow Z = Z_A = Z_V$
  $Z = 0.85$ [2f]; 0.73 [6f]; 0.70 [8f]; 0.71 [10f]
- Conserved currents $\mathcal{V}$ and $\mathcal{A}$ ensure that lattice artifacts cancel

Backup: Dark matter density in cosmological history

$$\frac{\Omega_{DM}}{\Omega_{baryons}} \approx \frac{\Omega_{DM}}{\Omega_{SM}} \approx 5 \text{ now}$$

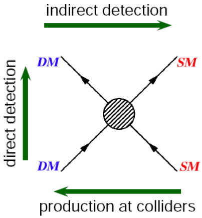


$$\frac{\Omega_{DM}}{\Omega_{baryons}} \approx 5 \text{ at recombination}$$



Simply because both are "matter" and evolve in the same way

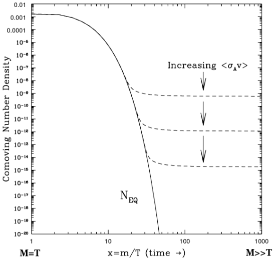
Backup: Thermal freeze-out for relic density



$T \gtrsim M_{DM}$: DM $\longleftrightarrow$ SM
Thermal equilibrium

$T \lesssim M_{DM}$: DM $\longrightarrow$ SM
Rapid depletion of Ω_{DM}

Hubble expansion $\longrightarrow$ dilution
leads to freeze-out



Requires coupling between standard model and dark matter

Mass and coupling of pure thermal relic are related: $\dfrac{M_{DM}}{100\ \text{GeV}} \sim 200\alpha$

(The "WIMP miracle" is $\alpha \sim \alpha_{EW} \sim 0.01 \Longrightarrow M_{DM} \sim 200$ GeV $\sim v$)

Thermal relic suppressed by **strong** coupling, easy for composite DM

Backup: Two roads to natural asymmetric dark matter

Basic idea: Dark matter relic density related to baryon asymmetry

$$\Omega_D \approx 5\Omega_B$$
$$\Longrightarrow M_D n_D \approx 5 M_B n_B$$

- $n_D \sim n_B \quad \Longrightarrow \quad M_D \sim 5M_B \approx 5$ GeV
  High-dimensional interactions relate baryon# and DM# violation

- $M_D \gg M_B \quad \Longrightarrow \quad n_B \gg n_D \sim \exp\left[-M_D/T_s\right]$
  Sphaleron transitions above $T_s \sim 200$ GeV distribute asymmetries

Both require coupling between standard model and dark matter