Electroweak Phenomenology and Lattice Strong Dynamics

David Schaich (Syracuse)

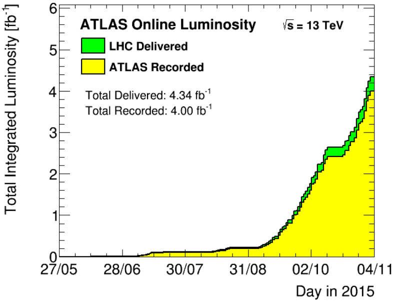
Humboldt / DESY Lattice Seminar
23 November 2015

arXiv:1405.4752, arXiv:1506.08791, arXiv:1510.06771
and work in progress with the Lattice Strong Dynamics Collaboration

Motivation: Electroweak symmetry breaking

LHC experiments have collected
$\sim 4\ \text{fb}^{-1}$ of data at $\sqrt{s} = 13$ TeV

Soon we will see new constraints on
physics beyond the standard model
. . . and possibly new discoveries!



One compelling possibility is new strong dynamics
that produces a composite Higgs boson

Protects the electroweak scale from sensitivity to quantum effects
(solving the hierarchy / fine-tuning problem)

Lattice gauge theory has a crucial role to play
in exploring and understanding new strong dynamics

Motivation: Composite Higgs vs. QCD

Electroweak symmetry breaking through new strong dynamics remains viable but must satisfy stringent experimental constraints

- The composite Higgs boson must have a mass of 125 GeV and standard-model-like properties
- Electroweak precision observables (e.g., *S* parameter) must be consistent with the standard model

If the new strong dynamics resembled QCD these conditions would not be satisfied

New strong dynamics different from QCD can be studied non-perturbatively by lattice calculations

Strategy for lattice studies of new strong dynamics

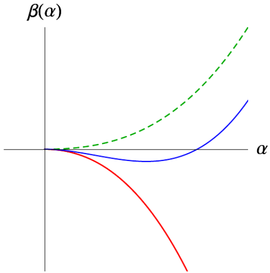
Systematically depart from familiar ground of lattice QCD

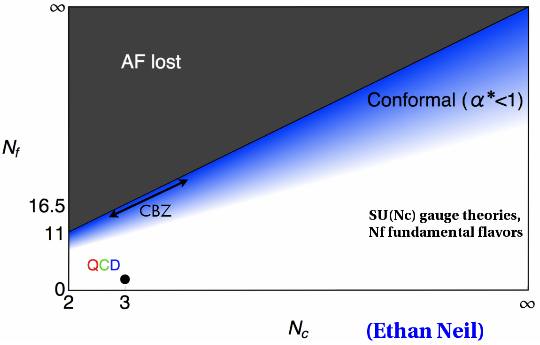
($N = 3$ with $N_F = 2$ light flavors in fundamental rep)

Identify generic features of non-QCD-like strong dynamics

Focus on near-conformal dynamics

Quick orientation:





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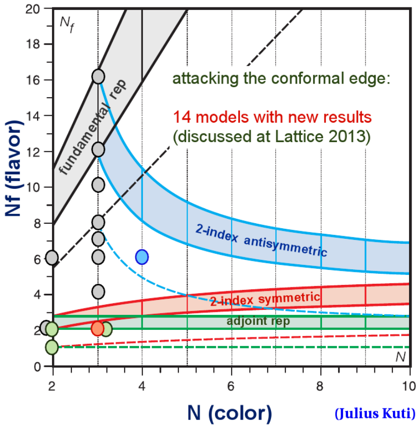


Focus on near-conformal dynamics

—Add more light flavors
$\longrightarrow N_F = 8$ fundamental

—Enlarge fermion rep
$\longrightarrow N_F = 2$ two-index symmetric

—Explore $N = 2$ and 4
$\longrightarrow$ (pseudo)real reps for cosets $SU(n)/Sp(n)$ and $SU(n)/SO(n)$

Lattice Strong Dynamics Collaboration

Argonne	Xiao-Yong Jin, James Osborn
Boston	Rich Brower, Claudio Rebbi, Evan Weinberg
Brookhaven	Meifeng Lin
Colorado	Anna Hasenfratz, Ethan Neil
Edinburgh	Oliver Witzel
Livermore	Evan Berkowitz, Enrico Rinaldi, Pavlos Vranas
RBRC	Ethan Neil, Sergey Syritsyn
Syracuse	DS
UC Davis	Joseph Kiskis
Yale	Thomas Appelquist, George Fleming, Andy Gasbarro

Exploring the range of possible phenomena
in strongly coupled gauge theories

IBM Blue Gene/Q @Livermore

USQCD cluster @Fermilab

Results to be shown are from
state-of-the-art lattice calculations

$\mathcal{O}$(100M core-hours) invested overall

Many thanks to DOE, NSF
and computing centers!

Cray *Blue Waters* @NCSA

Plan for this talk

1. Electroweak S parameter (arXiv:1405.4752)
   (Domain wall fermions on $32^3 \times 64$ lattices)

2. Higgs (singlet scalar) mass (arXiv:1510.06771 & ongoing)
   (nHYP-improved staggered fermions up to $64^3 \times 128$)

Common theme: Challenges of chiral extrapolation

3. Chiral condensate and WW scattering parameters (time permitting)

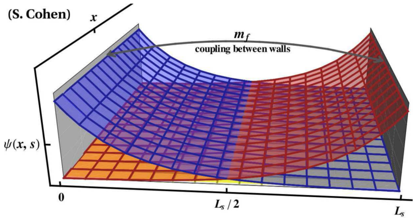
Additional studies can be reviewed by request
($N_F = 8$ phase diagram; discrete β function from gradient flow;
effective mass anomalous dimension $\gamma_{\text{eff}}(\lambda)$ from Dirac eigenmodes)

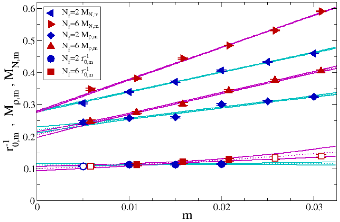
Electroweak precision observables — preliminaries

Good chiral & flavor symmetries important $\longrightarrow$ domain wall fermions

- Add fifth dimension of length L_s (expensive!)
- Exact chiral symmetry at finite lattice spacing in the limit $L_s \to \infty$
- At finite $L_s = 16$, "residual mass" $m_{res} \ll m_f$; $m = m_f + m_{res}$

$10^5 m_{res} = 2.6$ [2f]; 82 [6f]; 268 [8f]



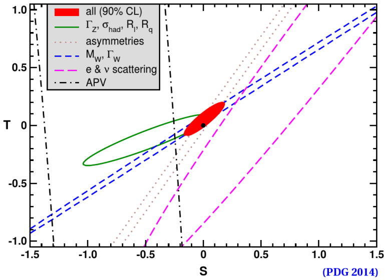


Compare more directly by approximately matching $m \to 0$ IR scales

$M_{V0} = 0.217(3)$ [2f]; $0.199(3)$ [6f]; $0.171(4)$ [8f]

Electroweak precision observable — the S parameter

Constrain the physics of electroweak symmetry breaking
from its effects on vacuum polarizations $\Pi(Q)$ of EW gauge bosons



S remains an important constraint
on new strong dynamics

Experiment: $S = 0.03 \pm 0.10$

Scaled-up QCD: $S \approx 0.43$

Can also analyze S as a low-energy constant (α_1 or L_{10})
of electroweak chiral lagrangian $\mathcal{L}_\chi$

The S parameter on the lattice

$$\mathcal{L}_\chi \supset \frac{\alpha_1}{2} g_1 g_2 B_{\mu\nu} \mathrm{Tr}\left[U \tau_3 U^\dagger W^{\mu\nu}\right] \longrightarrow$$

$$S = -16\pi^2 \alpha_1 = 4\pi N_D \lim_{Q^2 \to 0} \frac{d}{dQ^2} \Pi_{V-A}(Q^2) - \Delta S_{SM}(M_H)$$

- $N_D \geq 1$ is the number of doublets with chiral electroweak couplings
- $\Delta S_{SM}(M_H)$ subtracted so that $S = 0$ in the standard model
  Removes three eaten Goldstones, depends on Higgs mass
- $\Pi_{V-A}(Q^2)$ is transverse component of vacuum polarization tensor

$$\Pi^{\mu\nu}_{V-A}(Q) = Z \sum_x e^{iQ\cdot(x+\widehat{\mu}/2)} \mathrm{Tr}\left[\left\langle \mathcal{V}^{\mu a}(x) V^{\nu b}(0) \right\rangle - \left\langle \mathcal{A}^{\mu a}(x) A^{\nu b}(0) \right\rangle\right]$$

The S parameter on the lattice

$\mathcal{L}_\chi \supset \frac{\alpha_1}{2} g_1 g_2 B_{\mu\nu} \text{Tr}\left[U \tau_3 U^\dagger W^{\mu\nu}\right] \longrightarrow \gamma, Z$

γ, Z

$$S = -16\pi^2 \alpha_1 = 4\pi N_D \lim_{Q^2 \to 0} \frac{d}{dQ^2} \Pi_{V-A}(Q^2) - \Delta S_{SM}(M_H)$$

$\Pi_{V-A}(Q^2)$ is transverse component of vacuum polarization tensor

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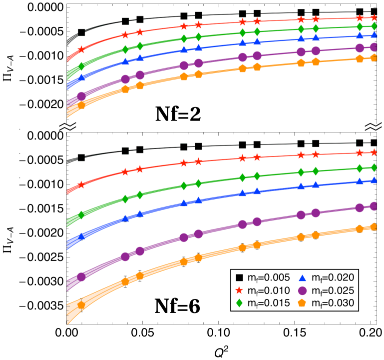
- Renormalization constant Z evaluated non-perturbatively
  Chiral symmetry of domain wall fermions $\Longrightarrow Z = Z_A = Z_V$
  $Z = 0.85$ [2f]; 0.73 [6f]; 0.70 [8f]

- Conserved currents $\mathcal{V}$ and $\mathcal{A}$ ensure that lattice artifacts cancel,
  combined with local currents V and A to reduce costs

Representative polarization function data, $\Pi_{V-A}(Q^2)$

$$S = 4\pi N_D \lim_{Q^2 \to 0} \frac{d}{dQ^2} \Pi_{V-A}(Q^2) - \Delta S_{SM}(M_H)$$



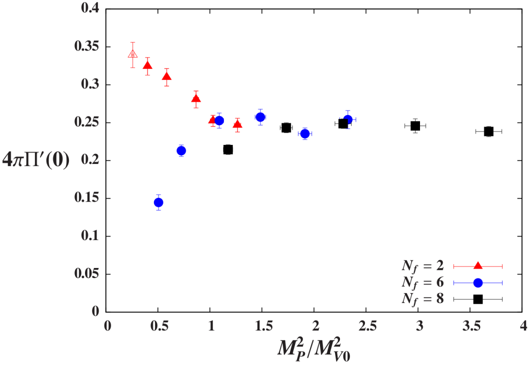
$Q^2 \to 0$ extrapolation via rational function

$$\Pi_{V-A}(Q^2) = \frac{a_0 + a_1 Q^2}{1 + b_1 Q^2 + b_2 Q^4}$$

Motivated by single-pole dominance and sum rules (cf. Aubin et al.)

Can already see contrast between $N_F = 2$ and $N_F = 6$ (may be non-negligible finite-volume effects for lightest $N_F = 6$ point)

Results for polarization function slopes $\Pi'_{V-A}(0)$



Vertical axis: $4\pi\Pi'_{V-A}(0)$

where

$$\Pi'(0) = \lim_{Q^2\to 0} \frac{d}{dQ^2}\Pi(Q^2)$$

$$S = 4\pi N_D \Pi'_{V-A}(0) - \Delta S_{SM}$$

Horizontal axis: M_P^2/M_{V0}^2 gives a more physical comparison than m

$M_{V0} \equiv \lim_{m\to 0} M_V$ approximately matched between $N_F = 2$, 6 and 8

$N_F = 6$ and 8 show significant reduction for $M_P \lesssim M_{V0}$,
and expected agreement in the quenched limit $M_P^2 \to \infty$

From slopes to S for $M_H = 125$ GeV

$$S = 4\pi N_D \lim_{Q^2 \to 0} \frac{d}{dQ^2} \Pi_{V-A}(Q^2) - \Delta S_{SM}(M_H)$$

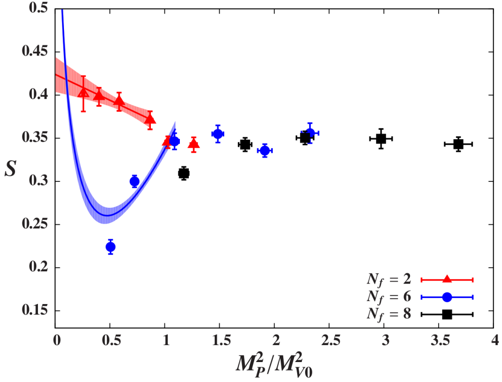
1. N_D doublets with chiral electroweak couplings contribute to S
   Scaled-up QCD often considers maximum $N_D = N_F/2$
   but only $N_D \geq 1$ is required for electroweak symmetry breaking

2. $\Delta S_{SM} = \frac{1}{12\pi} \int_{4M_P^2}^{\infty} \frac{ds}{s} \left[1 - \left(1 - \frac{M_{V0}^2}{s}\right)^3 \Theta(s - M_{V0}^2)\right] - \frac{1}{12\pi} \log\left(\frac{M_{V0}^2}{M_H^2}\right)$

   Integral diverges logarithmically as $M_P^2 \to 0$
   to cancel contribution of three eaten modes

   First term assumes $M_H \sim M_{V0} \sim$ TeV;
   second term corrects for $M_H = 125$ GeV $\ll$ TeV

Results for the S parameter



$N_F = 2$ result

$$\lim_{M_P^2 \to 0} S = 0.42(2)$$

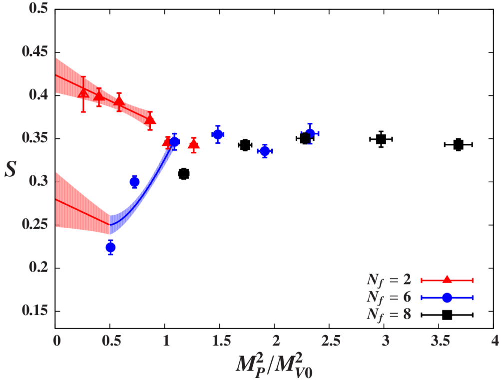
matches scaled-up QCD

Significant reductions as N_F increases

Linear + log fits to light points ($M_P \lesssim M_{V0}$) guide the eye, account for any chiral logs remaining after $\Delta S_{SM}(M_H)$

$$S = A + B\frac{M_P^2}{M_{V0}^2} + \frac{1}{12\pi}\left(\frac{N_F}{2} - 1\right)\log\left(\frac{M_{V0}^2}{M_P^2}\right) \qquad \text{for } N_D = 1$$

Challenges of chiral extrapolation



$N_F = 2$ result

$$\lim_{M_P^2 \to 0} S = 0.42(2)$$

matches scaled-up QCD

Significant reductions as N_F increases

- Lattice calculation involves $N_F^2 - 1$ degenerate pseudoscalars
- Only three massless Goldstones eaten by W and Z, $N_F^2 - 4$ PNGBs must acquire non-zero masses

For $N_F = 6$, imagine freezing 32 PNGBs at the blue curve's minimum, and taking only three to zero mass

Pushing $N_F = 8$ towards the chiral limit

Wish list after domain wall studies

- Want larger physical volumes to avoid finite-volume effects
- Want smaller masses to connect to chiral perturbation theory
- Want more statistics to analyze Higgs (singlet scalar)

Solution: Staggered fermions using nHYP-improved action

- $m = 0.00889$ on $24^3 \times 48$ with $\sim$24 700 thermalized MDTU
- $m = 0.00750$ on $32^3 \times 64$ with $\sim$24 600 thermalized MDTU
- $m = 0.00500$ on $32^3 \times 64$ with $\sim$46 600 thermalized MDTU
- $m = 0.00220$ on $48^3 \times 96$ with $\sim$19 600 thermalized MDTU
- $m = 0.00125$ on $64^3 \times 128$ with $\sim$2 000 thermalized MDTU

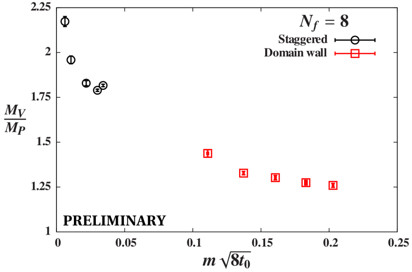
(no $64^3 \times 128$ disconnected analyses)

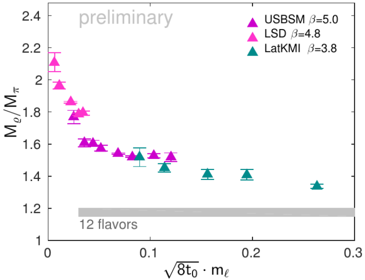
Pushing $N_F = 8$ towards the chiral limit

nHYP improvement reduces discretization artifacts,

allows larger lattice spacing $\longrightarrow$ larger physical volumes

Enables exploration of smaller masses (larger M_V / M_P)





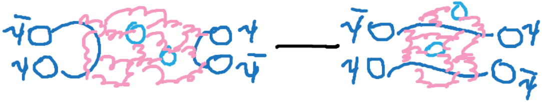
Horizontal axes use mass-dependent gradient flow scale $\sqrt{8t_0}$

Spontaneous chiral symmetry breaking

$\Longrightarrow$ M_V / M_P on vertical axes diverges in chiral limit

Higgs mass analysis — measurements and correlators

The Higgs is a scalar singlet (0^{++}) $\Longrightarrow$ disconnected diagrams



(Evan Weinberg)

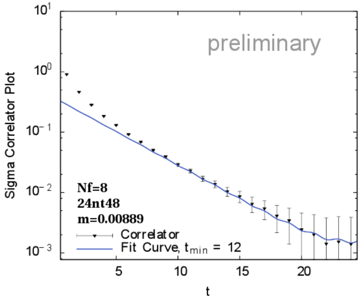
- For each disconnected measurement: 6 stochastic-U(1) sources diluted in time, color, and even/odd spatial sites
- Full scalar correlator is $S(t) = 2D(t) - C(t)$, combining connected and (vacuum-subtracted) disconnected correlators
- However, Higgs appears both in $S(t)$ and in $D(t)$ on its own
  $\Longrightarrow$ Fit each and include differences in systematic uncertainties [better plateaus in $D(t)$; more excited-state effects in $S(t)$]

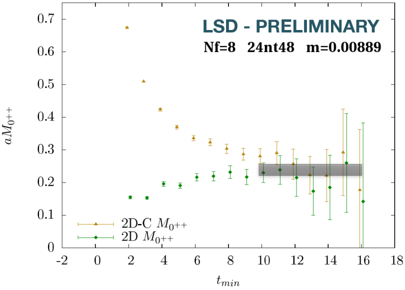
Higgs mass analysis — representative fits

For each of $D(t)$ and $S(t)$, carry out correlated fits to

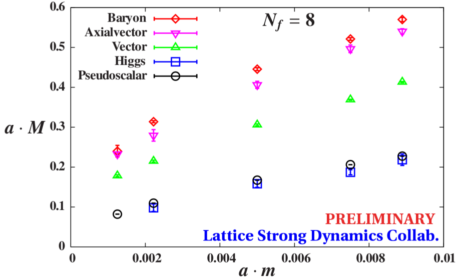
$$A_H \cosh\left[M_H\left(t - N_T/2\right)\right] + (-1)^t A_1 \cosh\left[M_1\left(t - N_T/2\right)\right] + v + \text{ excited states}$$

- Start with usual staggered state and parity partner
- Add free parameter v to control noise in vacuum subtraction [equivalent to fitting $D(t+1) - D(t)$ or $S(t+1) - S(t)$]
- Up to two excited states included in fits for $S(t)$





$N_F = 8$ spectrum results



Preliminary results
still in lattice units

Scale setting suggests
resonance masses $\sim$2–3 TeV

Large separation between
Higgs and resonances

Higgs degenerate with pseudo-Goldstones in accessible regime

Dramatically different from QCD-like dynamics,
where $M_H \approx 2M_P$ in this regime (dominated by two-pion scattering)

Typical chiral extrapolation integrates out everything except pions,
can't reliably be applied to these data

Challenges of chiral extrapolation

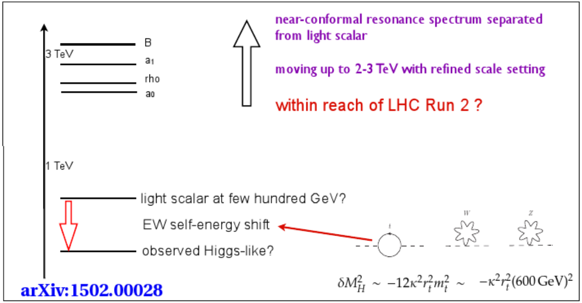
Without reliable chiral extrapolation we can only estimate

$M_H \sim$ few hundred GeV, with large error bars

Much lighter than scaled-up QCD, still somewhat far from 125 GeV

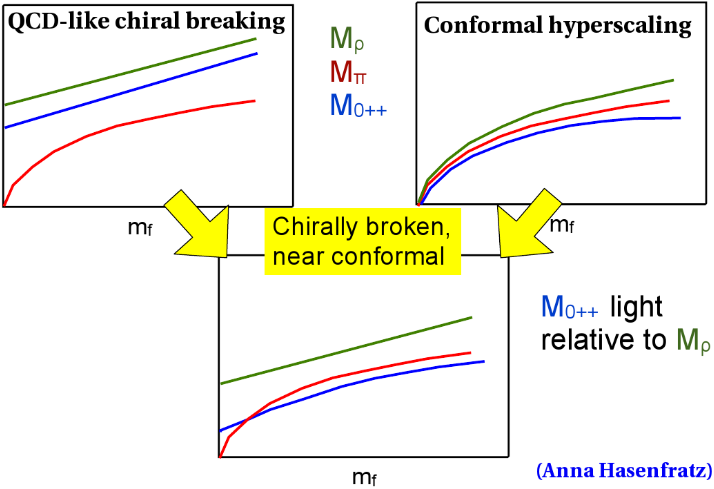
Of course, we **shouldn't** get exactly 125 GeV
since we haven't yet incorporated electroweak & top corrections

These reduce M_H,
but not yet consensus
on size of effect…



Emerging picture of near-conformal spectrum

Light scalar likely related to near-conformal dynamics
(unconfirmed interpretation as PNGB of approx. scale symmetry)



M_{0++} light relative to M_ρ

(Anna Hasenfratz)

Scale setting & electroweak effective theory

Let's review the standard approach impeded by the light Higgs

Integrating out resonances around $4\pi v$ scale gives chiral lagrangian

Dynamical d.o.f. are Goldstones π^a to be eaten by W and Z,
which appear through matrix field $U \equiv \exp\left[\, 2iT^a\pi^a/F \,\right]$

$$\mathcal{L}_{LO} = \frac{F^2}{4}\mathrm{Tr}\left[D_\mu U^\dagger D^\mu U\right] + \frac{F^2 B}{2}\mathrm{Tr}\left[m\left(U + U^\dagger\right)\right]$$

Decay constant F sets electroweak scale, W & Z masses
$F = v = 246$ GeV in simplest case (one electroweak doublet)

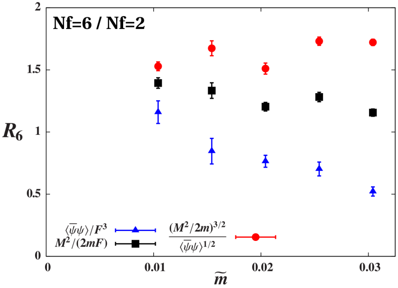
Chiral condensate $\left\langle\overline{\psi}\psi\right\rangle \propto F^2 B$ related to fermion mass generation
$\longrightarrow$ large $\left\langle\overline{\psi}\psi\right\rangle / F^3 \propto B/F$ helps to satisfy FCNC constraints

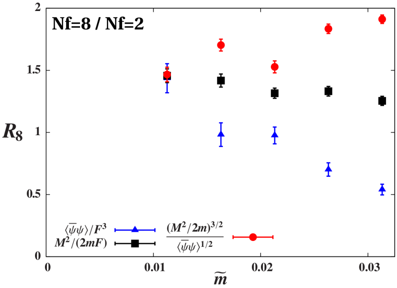
Chiral condensate enhancement

Three dimensionless ratios all approach $\langle\overline{\psi}\psi\rangle / F^3$ in the chiral limit

$$X^{(FM)} = \frac{M_P^2}{2mF_P} \qquad X^{(CM)} = \frac{\left(M_P^2/2m\right)^{3/2}}{\langle\overline{\psi}\psi\rangle^{1/2}} \qquad X^{(FM)} = \frac{\langle\overline{\psi}\psi\rangle}{F_P^3}$$

Condensate enhancement relative to $N_f = 2$ through "ratios of ratios"





Renormalized $R_{\overline{\text{MS}}} \approx 1.6$ in chiral limit for both $N_F = 6$ and $N_F = 8$

Electroweak chiral lagrangian NLO terms

With $X \equiv U\tau_3 U^\dagger$ and $V_\mu \equiv (D_\mu U)\,U^\dagger$, next-to-leading order includes
oblique corrections $\qquad S \propto \alpha_1, \qquad T \propto \beta_1, \qquad U \propto \alpha_8$
triple gauge vertices and dominant contributions to WW scattering

$$\mathcal{L}_1 = \frac{\alpha_1}{2} g_1 g_2 B_{\mu\nu} \mathrm{Tr}\left(X W^{\mu\nu}\right) \qquad \mathcal{L}_2 = \frac{i\alpha_2}{2} g_1 B_{\mu\nu} \mathrm{Tr}\left(X\left[V^\mu, V^\nu\right]\right)$$

$$\mathcal{L}_3 = i\alpha_3 g_2 \mathrm{Tr}\left(W_{\mu\nu}\left[V^\mu, V^\nu\right]\right) \qquad \mathcal{L}_4 = \alpha_4 \left\{\mathrm{Tr}\left(V_\mu V_\nu\right)\right\}^2$$

$$\mathcal{L}_5 = \alpha_5 \left\{\mathrm{Tr}\left(V_\mu V^\mu\right)\right\}^2 \qquad \mathcal{L}_6 = \alpha_6 \mathrm{Tr}\left(V_\mu V_\nu\right) \mathrm{Tr}\left(X V^\mu\right) \mathrm{Tr}\left(X V^\nu\right)$$

$$\mathcal{L}_7 = \alpha_7 \mathrm{Tr}\left(V_\mu V^\mu\right) \mathrm{Tr}\left(X V_\mu\right) \mathrm{Tr}\left(X V^\nu\right) \qquad \mathcal{L}_8 = \frac{\alpha_8}{4} g_2^2 \left\{\mathrm{Tr}\left(X W_{\mu\nu}\right)\right\}^2$$

$$\mathcal{L}_9 = \frac{i\alpha_9}{2} g_2 \mathrm{Tr}\left(X W_{\mu\nu}\right) \mathrm{Tr}\left(X\left[V^\mu, V^\nu\right]\right) \qquad \mathcal{L}_{10} = \frac{\alpha_{10}}{2} \left\{\mathrm{Tr}\left(X V_\mu\right) \mathrm{Tr}\left(X V_\nu\right)\right\}^2$$

$$\mathcal{L}_{11} = \alpha_{11} g_2 \epsilon^{\mu\nu\rho\lambda} \mathrm{Tr}\left(X V_\mu\right) \mathrm{Tr}\left(V_\nu W_{\rho\lambda}\right) \qquad \mathcal{L}_1' = \frac{\beta_1}{4} g_2^2 F^2 \left\{\mathrm{Tr}\left(X V_\mu\right)\right\}^2$$

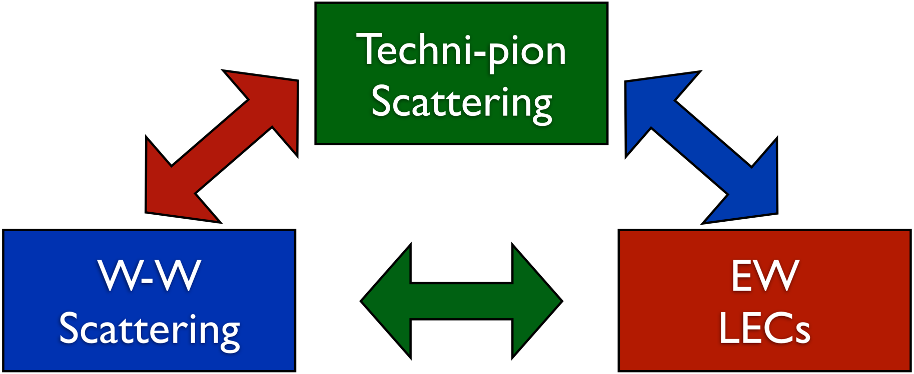
Simplest analysis is for WW scattering parameters α_4 and α_5

WW scattering from the lattice — The Big Picture

WW scattering is the most direct probe of EWSB dynamics,

though **not** the easiest to study at the LHC

Lattice calculations restricted to **low-energy** scattering



(M. Buchoff)

WW scattering from the lattice — EFT matching

—Hadronic chiral lagrangian has $m > 0$ and $g = 0$

—Electroweak chiral lagrangian has $m = 0$ and $g > 0$

Both reduce to same form in the limit $m \to 0$ and $g \to 0$

Hadronic EFT

$$m_d \to 0$$

$$p^2 \ll M_{ds}^2, M_{ss}^2$$

EW EFT

$$g, g' \to 0$$

$$p^2 \ll M_{ds}^2, M_{ss}^2$$

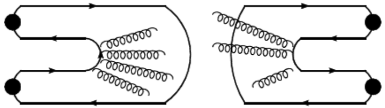
$$\frac{f^2}{4}\mathrm{tr}(\partial_\mu U^\dagger \partial^\mu U) + \alpha_5 \left[\mathrm{tr}(\partial_\mu U^\dagger \partial^\mu U)\right]^2 + \alpha_4 \left[\mathrm{tr}(\partial_\mu U^\dagger \partial_\nu U)\right]^2$$

Pseudoscalar scattering on the lattice — goal

"Maximal isospin" channel ($I = 2$ for $N_F = 2$)

Focus on S-wave scattering of identical charged pseudoscalars
$\longrightarrow$ simplest and cleanest scattering process

- Other isospin channels (e.g., $I = 0$) involve disconnected diagrams



- Other spin channels (e.g., D-wave) have smaller signals,
require higher precision

We want to extract the LECs ℓ_1 and ℓ_2 related to α_4 and α_5 in $\mathcal{L}_\chi$

These hide in the low-energy scattering length a_{PP}

Pseudoscalar scattering on the lattice — procedure

Recall Maiani & Testa (1990)

No asymptotically non-interacting states in euclidean spacetime
$\longrightarrow$ usual LSZ scattering formalism inapplicable

In a finite volume, measure energy of two-pseudoscalar state E_{PP},
projecting each correlator to zero momentum for S-wave scattering

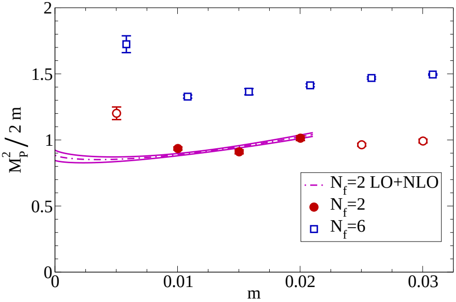
Access scattering phase shift δ from energy shift ΔE_{PP} (Lüscher, 1986)

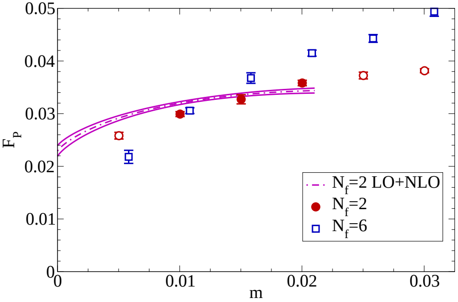
$$\Delta E_{PP} = E_{PP} - 2M_P = 2\sqrt{|\vec{k}|^2 + M_P^2} - 2M_P$$

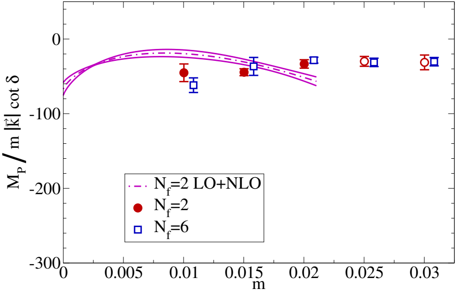
$$|\vec{k}| \cot\delta = \frac{1}{\pi L}\left[\sum_{\vec{\jmath}\neq 0}^{\Lambda_J} \frac{1}{|\vec{\jmath}|^2 - |\vec{k}|^2 L^2/(4\pi^2)} - 4\pi\Lambda_j\right] \qquad \text{(regularized } \zeta \text{ func.)}$$

Low-energy **scattering length** from $|\vec{k}| \cot\delta = \dfrac{1}{a_{PP}} + \mathcal{O}\left(\dfrac{|\vec{k}|^2}{M_P^2}\right)$

Joint chiral fit to M_P^2, F_P, $\langle\overline{\psi}\psi\rangle$ and $M_P a_{PP}$







- $a_{PP} \approx 1/|\vec{k}|\cot\delta$
- $\langle\overline{\psi}\psi\rangle$ plot in backup slide
- Only $N_F = 2$ fit feasible
- Fit restricted to solid points, $0.01 \leq m_f \leq 0.02$
- $\chi^2/\text{dof} = 83/6$

$N_F = 2$ WW scattering parameters from NLO chiral fit

Joint NLO chiral fit predicts sum of hadronic LECs $\ell_1 + \ell_2$

EFT matching discussed above relates this to the sum $\alpha_4 + \alpha_5$
(matching involves one-loop standard model calculation)

$$\alpha_4 + \alpha_5 = \left(3.34 \pm 0.17^{+0.08}_{-0.71}\right) \times 10^{-3} - \frac{1}{128\pi^2}\left[\log\left(\frac{M_H^2}{v^2}\right) + \mathcal{O}(1)_{SM}\right]$$

(dominant systematic error from chiral fit range)

Context for our $N_F = 2$ result

Unitarity bounds [hep-ph/0604255]:

$$\alpha_4 + \alpha_5 \geq 1.14 \times 10^{-3} \qquad \alpha_4 \geq 0.65 \times 10^{-3}$$

Expected LHC bounds [hep-ph/0606118]: (99% CL; 100/fb; 14 TeV)

$$-7.7 < \alpha_4 \times 10^3 < 15 \qquad -12 < \alpha_5 \times 10^3 < 10$$

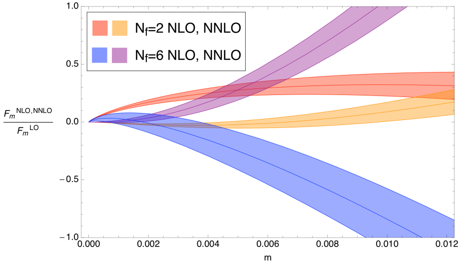
Complications for $N_F > 2$

- As for the S parameter, only charge one chiral doublet d
  Here we take the other $N_F - 2$ to be electroweak singlets s,
  leading to $N_F^2 - 4$ pseudoscalars with masses M_{ds} and M_{ss}

- Hadronic chiral perturbation theory (χPT) now involves **9** LECs
  with more complicated relations to α_4 and α_5

- Higher-order terms in χPT
  increase with N_F

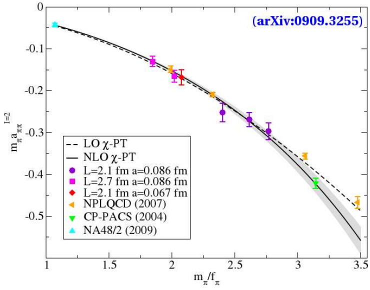
  Leads to smaller
  radius of convergence

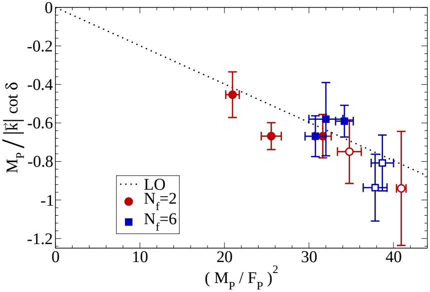


Strategy: Reorganize chiral expansion

Replace low energy constants $2mB$ and F by measured M_P and F_P

Expansion parameter $\propto M_P^2/F_P^2$, leading order is $M_P a_{PP} = -\frac{M_P^2}{16\pi F_P^2}$





—An old story in QCD (Weinberg, 1966)

—Allows direct comparison between $N_F = 2$ and $N_F = 6$ LECs

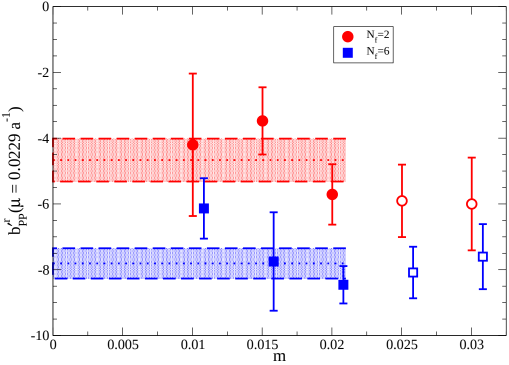
$N_F = 6$ scattering length only slightly smaller, but chiral logs differ...

Possible enhancement of WW scattering for $N_F = 6$

Combined LEC b'_{PP} must increase from $N_F = 2$ to $N_F = 6$,
to get similar a_{PP} despite different chiral logs

$$b'_{PP} = -256\pi^2 \left[L_0 + 2L_1 + 2L_2 + L_3 - 2L_4 - L_5 + 2L_6 + L_8\right]$$

contains α_4 and α_5, but we aren't able to isolate them



$b'_{PP} = -4.67 \pm 0.65^{+1.08}_{-0.05}$ [2f]; $b'_{PP} = -7.81 \pm 0.46^{+1.23}_{-0.56}$ [6f]

Recapitulation and outlook

- New strong dynamics related to electroweak symmetry breaking must behave unlike QCD
- Lattice calculations crucial to explore range of possibilities
- Focus on near-conformal gauge theories $\longrightarrow$ SU(3) with $N_F = 8$

Effects of increasing N_F compared to scaled-up QCD

- Evidence for dynamical reduction of electroweak S parameter
- Higgs boson is dramatically lighter, degenerate with PNGBs for currently accessible masses
- Chiral condensate ratio $\left\langle\overline{\psi}\psi\right\rangle / F^3$ is significantly enhanced
- WW scattering parameters possibly enhanced

Most pressing direction being pursued is to extend chiral effective theory to include a light scalar

Thank you!

Thank you!

Collaborators

Tom Appelquist, Evan Berkowitz, Rich Brower, George Fleming, Andy Gasbarro, Anna Hasenfratz, Xiao-Yong Jin, Joe Kiskis, Meifeng Lin, Ethan Neil, James Osborn, Claudio Rebbi, Enrico Rinaldi, Sergey Syritsyn, Pavlos Vranas, Evan Weinberg, Oliver Witzel

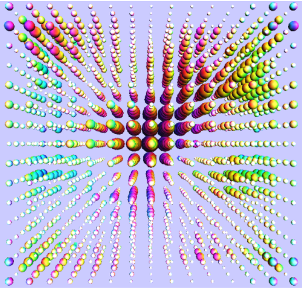
Funding and computing resources

Backup: Lattice gauge theory in a nutshell

Lattice gauge theory is a fully non-perturbative and gauge-invariant regularization of quantum field theory (QFT)

Any QFT observable is formally $\langle\mathcal{O}\rangle = \frac{1}{\mathcal{Z}}\int \mathcal{D}\Phi \;\; \mathcal{O}(\Phi) \;\; e^{-S[\Phi]}$

...but this is an infinite-dimensional integral



Regularize the theory by formulating it in a finite, discrete spacetime $\longrightarrow$ **the lattice**

Work in Euclidean spacetime (Wick rotation)

Spacing between lattice sites ("*a*") introduces UV cutoff scale $1/a$

Backup: Lattice gauge theory in a nutshell

Any QFT observable is formally $\langle \mathcal{O} \rangle = \dfrac{1}{\mathcal{Z}} \displaystyle\int \mathcal{D}\Phi \;\; \mathcal{O}(\Phi) \;\; e^{-S[\Phi]}$

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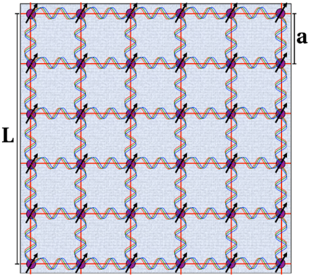


P. Vranas LLNL

Regularize the theory by formulating it in a finite, discrete spacetime $\longrightarrow$ **the lattice**

Work in Euclidean spacetime (Wick rotation)

Spacing between lattice sites ("*a*") introduces UV cutoff scale $1/a$

Lattice cutoff preserves hypercubic subgroup of full Lorentz symmetry

Remove cutoff by taking continuum limit $a \to 0$ (with $L/a \to \infty$)

Backup: Numerical lattice gauge theory calculations

$$\langle \mathcal{O} \rangle = \frac{1}{\mathcal{Z}} \int \mathcal{D}\Phi \;\; \mathcal{O}(\Phi) \;\; e^{-S[\Phi]}$$

Finite-dimensional integral $\Longrightarrow$ we can compute $\langle \mathcal{O} \rangle$ numerically

Importance sampling Monte Carlo

Approximate integral with a finite ensemble of field configurations $\{\Phi_i\}$

Algorithms choose each configuration Φ_i with probability $\frac{1}{\mathcal{Z}} e^{-S[\Phi_i]}$
to find those that make the most important contributions

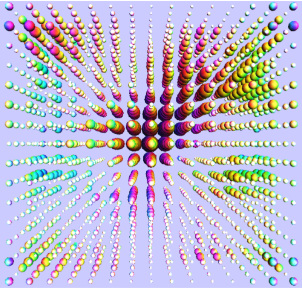
Then $\langle \mathcal{O} \rangle = \frac{1}{N} \sum_{i=1}^{N} \mathcal{O}(\Phi_i)$ with statistical uncertainty $\propto \sqrt{\frac{1}{N}}$

Generating ensembles $\{\Phi_i\}$ often dominates computational costs

These saved data can be reused to investigate many observables

Backup: Hybrid Monte Carlo (HMC) algorithm

Recall goal: Sample field configurations Φ_i with probability $\frac{1}{\mathcal{Z}} e^{-S[\Phi_i]}$



HMC is a Markov process, based on
Metropolis–Rosenbluth–Teller (MRT)

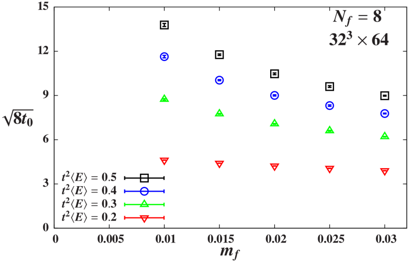
Fermions $\longrightarrow$ extensive action computation,
so best to update entire system at once

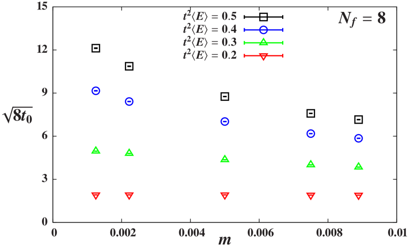
Use fictitious molecular dynamics evolution

1. Introduce a fictitious fifth dimension ("MD time" τ)
   and stochastic canonical momenta for all field variables
2. Run inexact MD evolution along a trajectory in τ
   to generate new four-dimensional field configuration
3. Apply MRT accept/reject test to MD discretization error

Backup: Gradient flow scale setting

Gradient flow scale $\sqrt{8t_0}$ defined by condition $t^2 \langle E(t) \rangle \Big|_{t=t_0} = c$





For both $N_F = 8$ domain wall (left) and staggered (right)

- $c \lesssim 0.3$ may be affected by discretization artifacts
- $c \gtrsim 0.3$ leads to significant mass dependence

Backup: A bit about the Wilson flow

Evolution of gauge links $U(x, \mu)$ in a "flow time" t:

$$\frac{d}{dt} V_t(x, \mu) = -g_0^2 \left[\frac{\delta}{\delta V_t(x, \mu)} S_W(V_t) \right] V_t(x, \mu),$$

where $V_{t=0}(x, \mu) = U(x, \mu)$ and S_W is the Wilson gauge action

$$S_W(U) = \frac{2N}{g_0^2} \sum_{\{P\}} \mathrm{ReTr}\left[1 - P(U)\right]$$

$$P_{x,\mu\nu}(U) = U_{x,\mu} U_{x+\widehat{\mu},\nu} U^\dagger_{x+\widehat{\nu},\mu} U^\dagger_{x,\nu}$$

Solution:

$$V_t(x, \mu) = \exp\left[-t g_0^2 \frac{\delta}{\delta U(x, \mu)} S_W(U) \right] U(x, \mu)$$

$\Longrightarrow$ numerical integration of **infinitesimal stout smearing steps**

Backup: Electroweak vacuum polarization functions

$$= ig_1 g_2 \cos\theta_w \sin\theta_w \Pi_{ee} \delta_{\mu\nu} + \dots$$

$$= ig_1 g_2 \left(\Pi_{3e} - \sin^2\theta_w \Pi_{ee}\right) \delta_{\mu\nu} + \dots$$

$$= \frac{ig_1 g_2}{\cos\theta_w \sin\theta_w} \left(\Pi_{33} - 2\sin^2\theta_w \Pi_{3e} + \sin^4\theta_w \Pi_{ee}\right) \delta_{\mu\nu} + \dots$$

$$= ig_2^2 \Pi_{11} \delta_{\mu\nu} + \dots$$

$$\Pi_{VV} = 2\Pi_{3e} \qquad \Pi_{AA} = 4\Pi_{33} - 2\Pi_{3e}$$

$$S = 4\pi N_D \lim_{Q^2 \to 0} \frac{d}{dQ^2} \left[\Pi_{VV}(Q^2) - \Pi_{AA}(Q^2)\right] - \Delta S_{SM}(M_H)$$

Backup: Scaling up QCD gives $S \gtrsim 0.4$

$N_F \geq 2$ fermions in fundamental rep of SU(N) for $N \geq 3$,
with $1 \leq N_D \leq N_F/2$ doublets given chiral electroweak charges

$$S \simeq 0.3 \frac{N_F}{2} \frac{N}{3} + \frac{N_D - 1}{12\pi} \log\left(\frac{M_V^2}{M_P^2}\right) + \frac{1}{12\pi} \log\left(\frac{\sim \text{TeV}^2}{M_H^2}\right)$$

1. Resonance contribution uses QCD phenomenology to model $R(s)$

$$4\pi \lim_{Q^2 \to 0} \frac{d}{dQ^2} \Pi_{V-A}(Q^2) = \frac{1}{3\pi} \int_0^\infty \frac{ds}{s} \left[R_V(s) - R_A(s)\right]$$

(essentially single-pole dominance with large-N scaling)

2. Chiral-log contribution based on leading-order chiral pert. theory
3. 125 GeV Higgs contributes $\sim$0.1 (leading-order estimate)

There is some subtlety regarding M_H (cf. arXiv:1211.1083)
for strong sector in isolation (no EW or radiative corrections)

Backup: Conserved and local domain wall currents

Conserved currents are point-split and summed over fifth dimension:

$$\mathcal{V}_\mu^a(x) = \sum_{s=0}^{L_s-1} j_\mu^a(x,s) \qquad\qquad \mathcal{A}_\mu^a(x) = \sum_{s=0}^{L_s-1} \mathrm{sign}\left(s - \frac{L_s-1}{2}\right) j_\mu^a(x,s)$$

$$j_\mu^a(x,s) = \overline{\Psi}(x+\widehat{\mu},s) P_{+\mu} \tau^a U_{x,\mu}^\dagger \Psi(x,s) - \overline{\Psi}(x,s) P_{-\mu} \tau^a U_{x,\mu} \Psi(x+\widehat{\mu},s)$$

where $P_{\pm\mu} \equiv \dfrac{1 \pm \gamma_\mu}{2}$

Local currents are constructed from boundaries of fifth dimension:

$$V_\mu^a(x) = \overline{q}(x)\gamma_\mu \tau^a q(x) \qquad\qquad A_\mu^a(x) = \overline{q}(x)\gamma_\mu \gamma_5 \tau^a q(x)$$

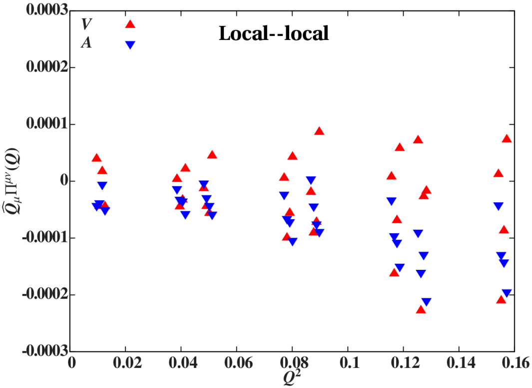
$$q(x) = \frac{1-\gamma_5}{2}\Psi(x,0) + \frac{1+\gamma_5}{2}\Psi(x, L_s - 1)$$

Backup: Non-conservation of local currents

$$\Pi^{\mu\nu}_{V-A}(Q) = Z \sum_x e^{iQ\cdot(x+\widehat{\mu}/2)} \mathrm{Tr}\left[\left\langle \mathcal{V}^{\mu a}(x) V^{\nu b}(0)\right\rangle - \left\langle \mathcal{A}^{\mu a}(x) A^{\nu b}(0)\right\rangle\right]$$

Local currents are simply $\overline{q}\gamma_\mu q$ defined on the domain walls

No Ward identity: $\widehat{Q}_\mu \left[\sum_x e^{iQ\cdot x} \left\langle V^a_\mu(x) V^a_\nu(0)\right\rangle\right] \neq 0$

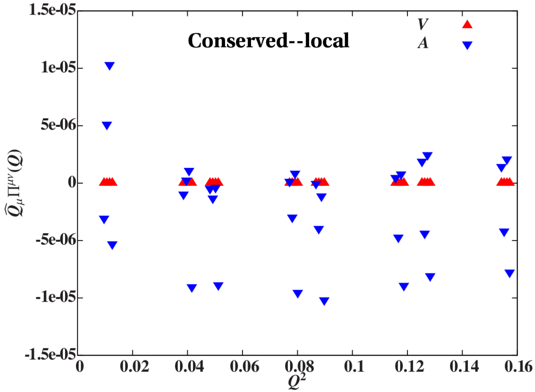


Backup: Ward identity for conserved currents

$$\Pi^{\mu\nu}_{V-A}(Q) = Z \sum_x e^{iQ\cdot(x+\widehat{\mu}/2)} \mathrm{Tr}\left[\left\langle \mathcal{V}^{\mu a}(x) V^{\nu b}(0)\right\rangle - \left\langle \mathcal{A}^{\mu a}(x) A^{\nu b}(0)\right\rangle\right]$$

Conserved currents are point-split, summed over fifth dimension

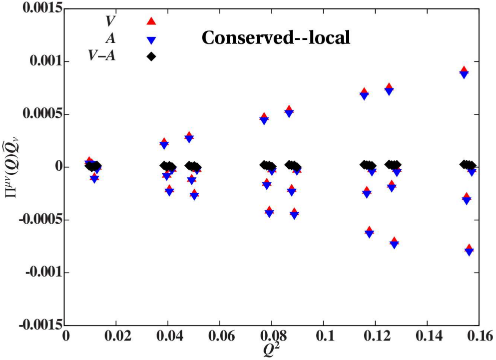
Obey Ward identity, PCAC: $\widehat{Q}_\mu \left[\sum_x e^{iQ\cdot(x+\widehat{\mu}/2)} \left\langle \mathcal{V}^a_\mu(x) V^a_\nu(0)\right\rangle\right] = 0$



Backup: Lattice artifacts cancel in mixed correlators

Plot shows divergence of local current in each correlator,

$$\left[\sum_x e^{iQ\cdot(x+\widehat{\mu}/2)} \left\langle \mathcal{V}_\mu^a(x) V_\nu^a(0) \right\rangle\right] \cdot \widehat{Q}_\nu \neq 0 \quad \text{for each } \nu$$

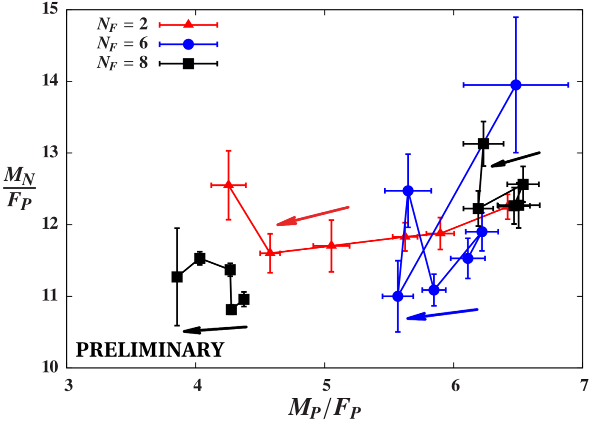


Cancellation seems due to conserved currents forming exact multiplet, also possible with overlap — even staggered (Y. Aoki @ Lattice 2013)

Backup: Finite-volume diagnostic plot

Arrows show direction of decreasing mass

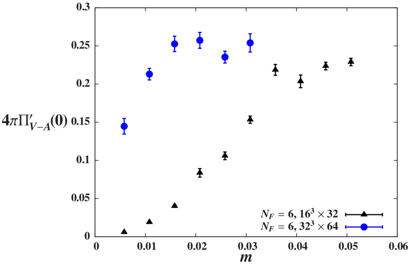
Expect finite-volume effects to push points up and to the right

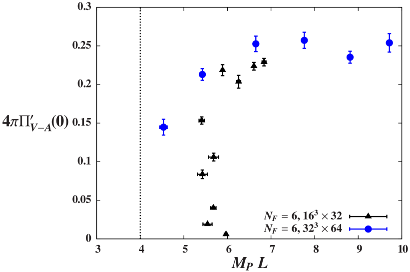


Finite-volume effects may be significant for lightest $N_F = 6$ point

Backup: Spurious $S \to 0$ from finite volume effects

Compare $N_F = 6$ results on $16^3 \times 32$ and $32^3 \times 64$ lattice volumes





$L = 16$ slopes $4\pi\Pi'_{V-A}(0)$ crash to zero as $m \longrightarrow 0$
attributable to spurious parity doubling from finite-volume effects

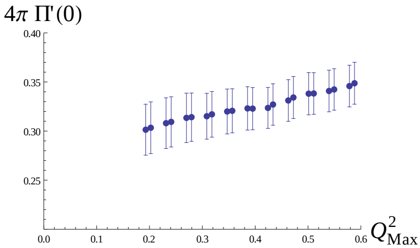
Simultaneously finite-volume effects freeze $M_P L \approx 5.5$

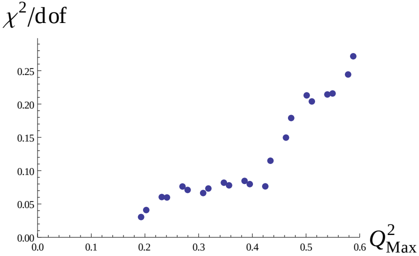
$L = 32$ results show no such effects in $M_P L$,
even for lightest $N_F = 6$ point where M_P/F_P increases

Backup: Padé fit Q^2-range dependence

Uncorrelated fits to "Padé-(1, 2)" rational function with $\chi^2/\text{dof} \ll 1$

$$\Pi_{V-A}(Q^2) = \frac{a_0 + a_1 Q^2}{1 + b_1 Q^2 + b_2 Q^4} = \frac{\sum_{m=0}^{1} a_m Q^{2m}}{1 + \sum_{n=1}^{2} b_n Q^{2n}}$$



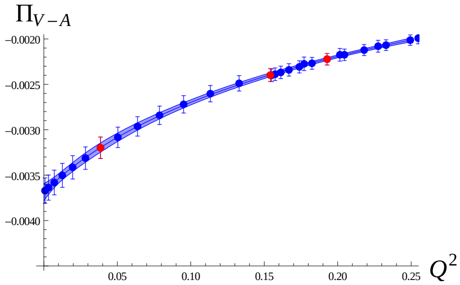


Results reported above use $Q^2_{Max} = 0.4$

Backup: Twisted boundary conditions for $\Pi_{V-A}(Q^2)$

Twisted boundary conditions (TwBCs)

- Introduce external abelian field (add phase at lattice boundaries)
- Allows access to arbitrary Q^2, not just lattice modes $2\pi n/L$



- Correlations $\Longrightarrow$ TwBCs do not improve Padé fit results for slope
- May help connect to chiral perturbation theory,
  where we need both small M_P and small Q^2

Backup: Chiral perturbation theory for $\Pi_{V-A}(Q^2)$

$\Pi_{V-A}(Q^2)$ in NLO hadronic χPT:

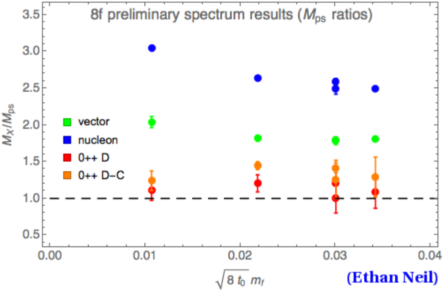
$$\Pi_{V-A}(M_{dd}^2, Q^2) = -F_P^2 - Q^2 \left[8L_{10}^r(\mu) + \frac{1}{24\pi^2}\left\{\log\left[\frac{M_{dd}^2}{\mu^2}\right] + \frac{1}{3} - H\left(\frac{4M_{dd}^2}{Q^2}\right)\right\}\right]$$

$$H(x) = (1+x)\left[\sqrt{1+x}\log\left(\frac{\sqrt{1+x}-1}{\sqrt{1+x}+1} + 2\right)\right]$$

Match with $S = -16\pi^2\alpha_1$ in electroweak chiral lagrangian:

$$S(\mu, M_{ds}) = \frac{1}{12\pi}\left[-192\pi^2\left(L_{10}^r(\mu) + \frac{1}{384\pi^2}\left\{\log\left[\frac{M_{ds}^2}{\mu^2}\right] + 1\right\}\right) + \log\left[\frac{\mu^2}{M_H}\right] - \frac{1}{6}\right].$$

Backup: Eight-flavor spectrum in dimensionless ratios



Preliminary results
still in lattice units

Scale setting suggests
resonance masses ∼2–3 TeV

Large separation between
Higgs and resonances

Higgs degenerate with pseudo-Goldstones in accessible regime

Dramatically different from QCD-like dynamics,
where $M_H \approx 2M_P$ in this regime (dominated by two-pion scattering)

Typical chiral extrapolation integrates out everything except pions,
can't reliably be applied to these data

Backup: NLO chiral expansions

For general N_F, $A = 2 - N_F + 2N_F^2 + N_F^3$

$$M_P a_{PP} = -\frac{2mB}{16\pi F^2}\left\{1 + \frac{2mB}{(4\pi F)^2}\left[b_{PP} - 2\frac{N_F - 1}{N_F^2} + \frac{A}{N_F^2}\log\left(\frac{2mB}{\mu^2}\right)\right]\right\}$$

$$M_P^2 = 2mB\left\{1 + \frac{2mB}{(4\pi F)^2}\left[b_M + \frac{1}{N_F}\log\left(\frac{2mB}{\mu^2}\right)\right]\right\}$$

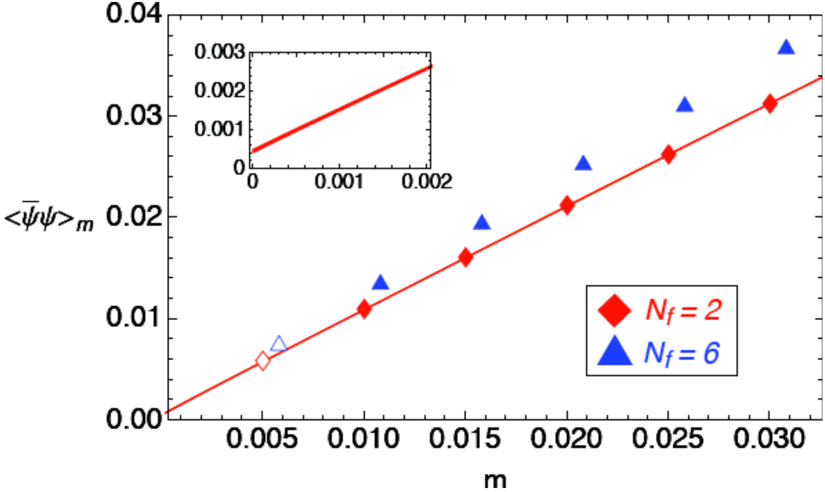
$$F_P = F\left\{1 + \frac{2mB}{(4\pi F)^2}\left[b_F - \frac{N_F}{2}\log\left(\frac{2mB}{\mu^2}\right)\right]\right\}$$

$$\langle\overline{\psi}\psi\rangle = \frac{F^2 2mB}{2m}\left\{1 + \frac{2mB}{(4\pi F)^2}\left[b_C - \frac{N_F^2 - 1}{N_F}\log\left(\frac{2mB}{\mu^2}\right)\right]\right\}$$

- LECs b are all linear combinations of low-energy constants L_i
- LECs' dependence on scale μ cancels the corresponding logs
- b_C includes "contact term" $\sim m/a^2$
- NNLO M_P^2 coefficients enhanced by N_F^2

(arXiv:0910.5424)

Backup: Chiral condensate with chiral fit

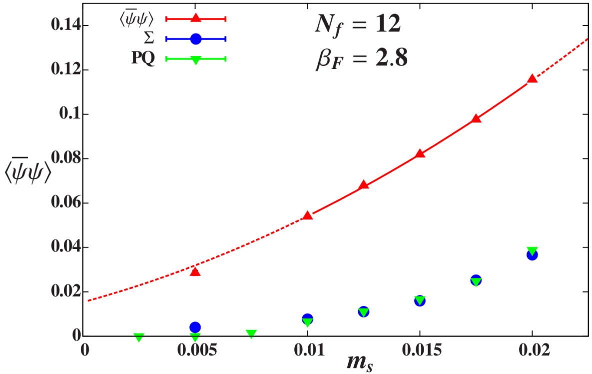


"Contact term" $\sim m/a^2$ clearly dominant ($\longrightarrow$ straight lines)

$N_F = 2$ joint NNLOχPT fit including F_P, M_P^2 and $\langle\overline{\psi}\psi\rangle$

Backup: $\langle\overline{\psi}\psi\rangle$ in three ways for $N_F = 12$

The chiral condensate directly probes chiral symmetry,
but this is explicitly broken by non-zero fermion mass on lattice



"Direct" $\langle\overline{\psi}\psi\rangle$ uses $m_{valence} = m_{sea}$

Σ measured from $m_v = 0$ eigenmodes

Partially quenched with $m_v \to 0$

"Contact term" $\sim m_v/a^2$ clearly dominates,
may lead to poorly controlled chiral extrapolation

Backup: Fermion mass dependence of $\left\langle\overline{\psi}\psi\right\rangle$

$\left\langle\overline{\psi}\psi\right\rangle$ depends on both valence mass m_v and sea mass m_s

Eigenspectrum $\rho(\lambda)$ of massless Dirac operator depends only on m_s

$$\left\langle\overline{\psi}\psi\right\rangle_{m_v;\ m_s} = m_v \int \frac{\rho(\lambda, m_s)}{\lambda^2 + m_v^2} d\lambda + m_v^5 \int \frac{\rho(\lambda, m_s)}{\left(\lambda^2 + m_v^2\right)\lambda^4} d\lambda + \gamma_1 m_v \Lambda^2 + \gamma_2 m_v + \mathcal{O}\left(1/\Lambda\right)$$

where $\Lambda = a^{-1}$ is the UV cutoff (Leutwyler & Smilga)

Quadratic UV divergence complicates chiral extrapolation

Can address with partially-quenched ($m_v \neq m_s$) measurements, to extrapolate $m_v \to 0$ with fixed m_s

Can also remove m_v dependence via $\Sigma_{m_s} = \pi\rho(0, m_s) = \left\langle\overline{\psi}\psi\right\rangle_{m_v=0;\ m_s}$

It is a good check that these two approaches agree!

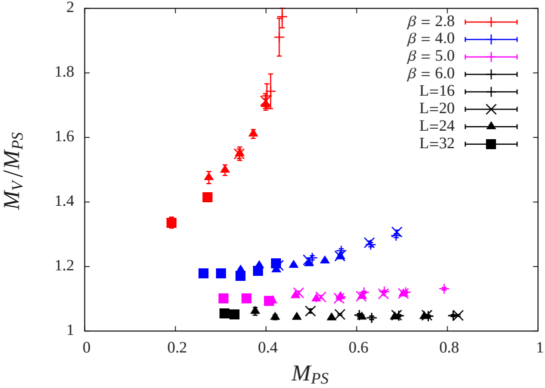
Backup: Dependence on gauge coupling for $N_F = 12$

Look at simple ratio M_V / M_P

plotted against relevant parameter (fermion mass $m \rightsquigarrow M_P$)

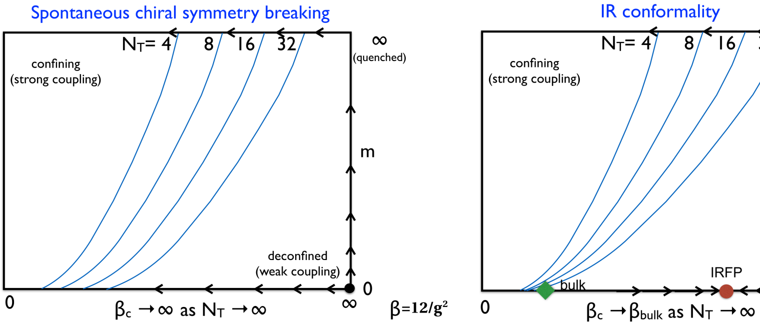
Even though β_F is formally irrelevant

it has significant effects for $M_P \gtrsim 0.2a^{-1}$



Backup: Thermal transitions to identify SχSB

May distinguish between chirally broken and IR-conformal cases from scaling $\Delta\beta_F$ of finite-temperature transitions as N_T increases



Plots show transitions and some RG flow lines in space of fermion mass m and gauge coupling β_F

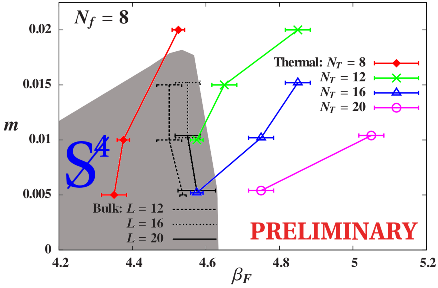
Contrast only clear near critical surface at $m = 0$

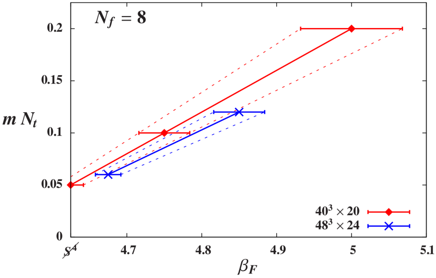
Backup: Search for $N_F = 8$ spontaneous χSB

QCD-like scaling at large $m \gtrsim 0.01$ does not persist as m decreases

Thermal transitions run into lattice phase before reaching chiral limit

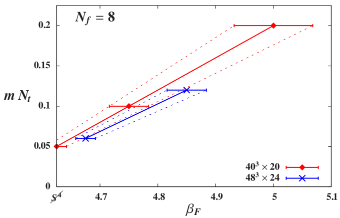
Even large lattice volumes up to $48^3 \times 24$ are insufficient
to establish spontaneous chiral symmetry breaking

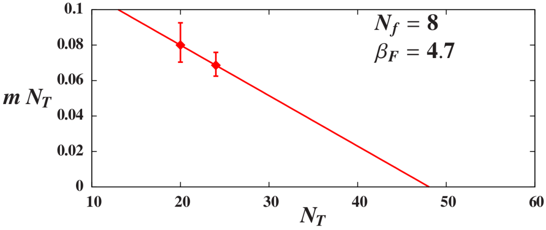




Backup: Search for $N_F = 8$ spontaneous χSB

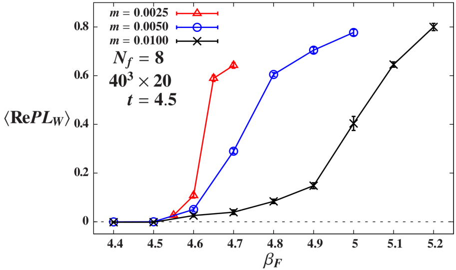
Extrapolating $m \to 0$ at fixed $\beta_F = 4.7$ suggests $N_T \gtrsim 48$ needed to establish spontaneous chiral symmetry breaking

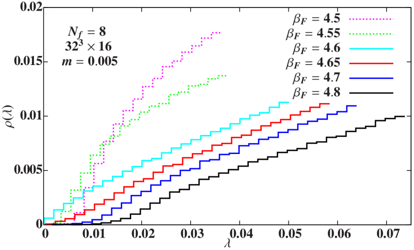


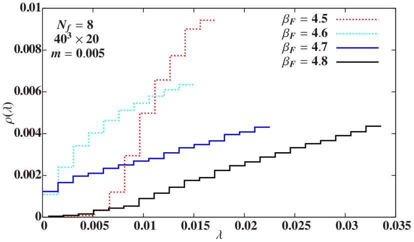


This behavior is extremely different from QCD but does not necessarily imply IR conformality

Backup: Sample $N_F = 8$ transition signals







Need $N_T = 20$ to observe chirally broken phase at $m = 0.005$

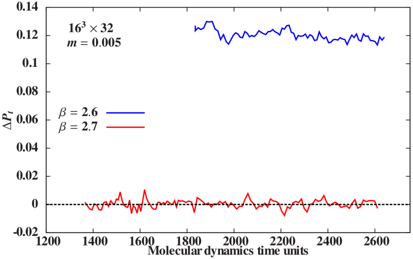
Backup: Order parameters for $\not{S}^4$ phase

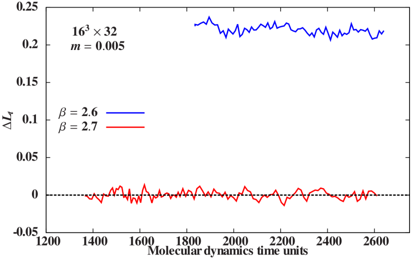
Staggered lattice actions possess exact single-site shift symmetry which is spontaneously broken in a novel lattice phase we encountered

Order parameters (any or all μ)

$$\Delta P_\mu = \langle \text{ReTr}\,\square_n - \text{ReTr}\,\square_{n+\mu} \rangle_{n_\mu\ \text{even}}$$

$$\Delta L_\mu = \langle \alpha_{\mu,n}\overline{\chi}_n U_{\mu,n}\chi_{n+\mu} - \alpha_{\mu,n+\mu}\overline{\chi}_{n+\mu}U_{\mu,n+\mu}\chi_{n+2\mu} \rangle_{n_\mu\ \text{even}}$$





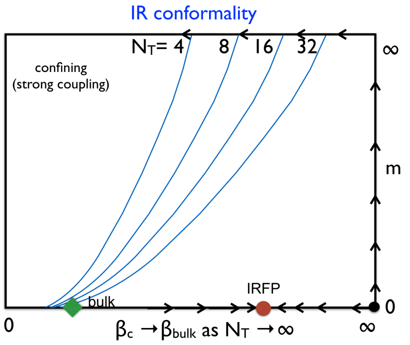
$\not{S}^4$ likely non-universal, though other groups see same phase structure

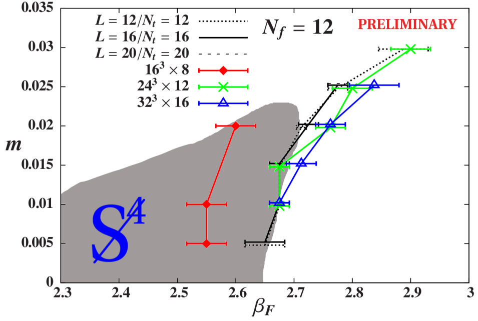
Backup: Thermal transitions for $N_F = 12$

Behave as expected for an IR-conformal system

Accumulate at zero-temperature bulk transition for small enough m

$N_T = 12$ and $N_T = 16$ transitions are indistinguishable



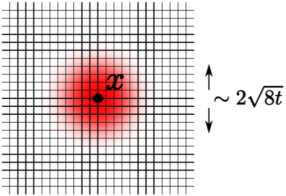


Backup: The gradient flow running coupling

In addition to a scale $\sqrt{8t_0}$,
the gradient flow defines a scale-dependent running coupling $g_c^2(L;a)$

Recall: The gradient flow
integrates an infinitesimal smoothing operation

Local observables measured after "flow time" t
depend on original fields within $r \simeq \sqrt{8t}$



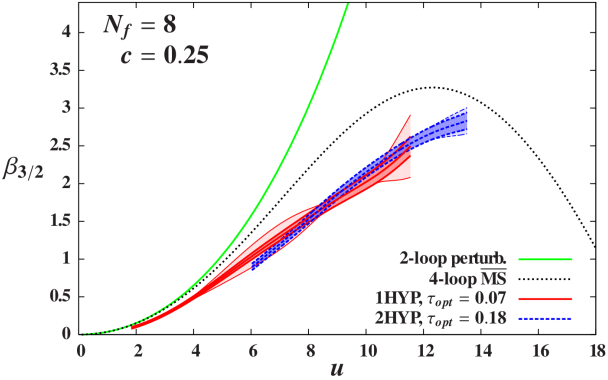
Perturbatively $g^2_{\overline{\text{MS}}}(\mu) \propto t^2 E(t)$ with $\mu = 1/\sqrt{8t}$
where $E = -\frac{1}{2}\text{Tr}\left[G_{\mu\nu}G^{\mu\nu}\right]$ is the energy density

Define running coupling $g_c^2(L;a)$ by fixing $c = L/\sqrt{8t}$

Backup: Discrete β function for $N_F = 8$

Continuum extrapolated $\beta_s(g_c^2)$ with scale change $s = 3/2$
increases monotonically for $g_c^2 \lesssim 14$

Although β_s is even smaller than IR-conformal four-loop $\overline{\text{MS}}$ prediction
any IR fixed point must be at stronger coupling



Backup: Scale-dependent $\gamma_{\text{eff}}(\lambda)$ from eigenmodes

λ defines an energy scale $\Longrightarrow$ fitting $\nu(\lambda)$ predicts effective anomalous dimension $\gamma_{\text{eff}}(\lambda)$ at that scale

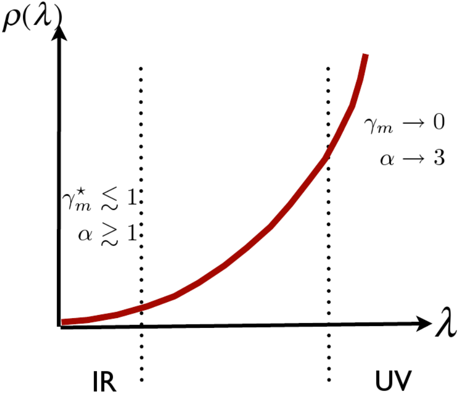
For IR-conformal systems

UV: Asymp. freedom $\Rightarrow \gamma_{\text{eff}}(\lambda) \to 0$ corresponding to $\alpha(\lambda) \to 3$

IR: Fixed point $\Longrightarrow \gamma_{\text{eff}}(\lambda) \to \gamma_m^\star$

$\gamma_m^\star$ scheme-independent

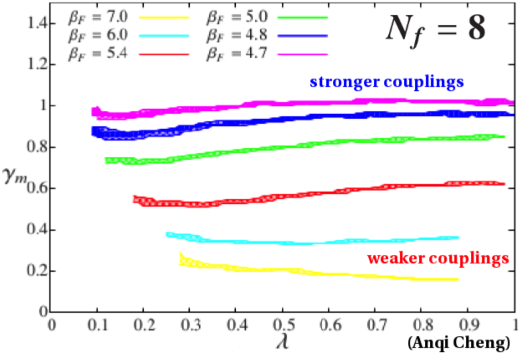
typically expect $\gamma_m^\star \lesssim 1$



Ideally monitor evolution from perturbative UV to strongly coupled IR

Backup: $\gamma_{\text{eff}}(\lambda)$ from eigenmodes for $N_F = 8$

Fit $\nu(\lambda) \propto \lambda^{1+\alpha}$ in a limited range of λ to find $1 + \gamma_{\text{eff}}(\lambda) = \dfrac{4}{1+\alpha(\lambda)}$



$\nu(\lambda)$ computed stochastically

Fit ranges included in error bands

Only retain regions where all volumes overlap

All systems have $m = 0$ and $\rho(0) = 0$

Behaves very differently compared to either $N_F = 12$ or QCD

γ_{eff} appears to evolve very slowly across a wide range of scales

Backup: $\gamma_{\text{eff}}(\lambda)$ for chirally broken systems

λ defines an energy scale

Fitting $\nu(\lambda) \propto \lambda^{1+\alpha(\lambda)}$ accesses $1 + \gamma_{\text{eff}}(\lambda) = \frac{4}{1+\alpha(\lambda)}$ at that scale

For chirally broken systems

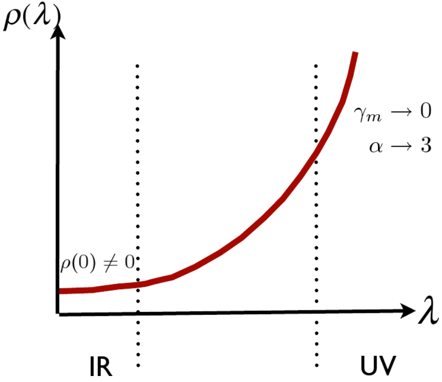
UV: Asymp. freedom $\Rightarrow \gamma_{\text{eff}}(\lambda) \to 0$

corresponding to $\alpha(\lambda) \to 3$

IR: $\langle\overline{\psi}\psi\rangle \propto \rho(0) > 0 \Longrightarrow \alpha(\lambda) \to 0$

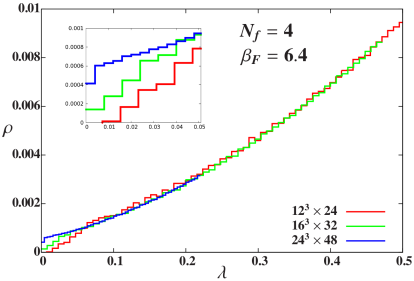
would produce $\gamma_{\text{eff}}(\lambda) \to 3$

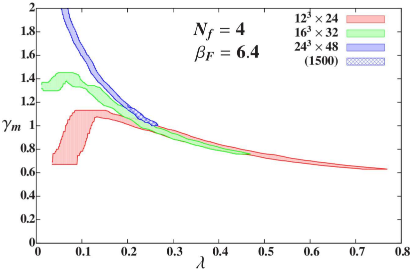
but $\rho(\lambda)$ no longer $\propto \lambda^{\alpha}$



Ideally monitor evolution from perturbative UV to chirally broken IR

Backup: Finite-volume effects in $\gamma_{\text{eff}}(\lambda)$ from $\nu(\lambda)$

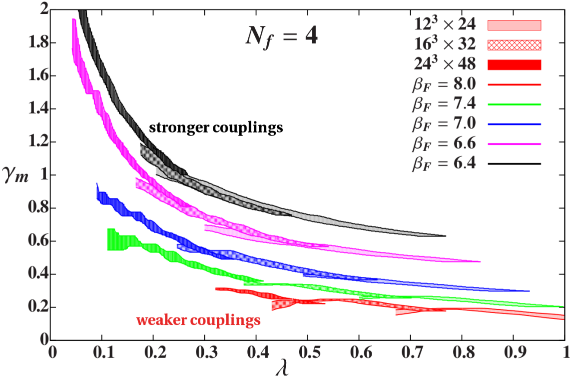




- As discussed above, $\langle\overline{\psi}\psi\rangle \propto \rho(\lambda \to 0) > 0 \Longrightarrow \gamma_{\text{eff}} \nearrow 3$,
  but scaling $\rho(\lambda) \propto \lambda^\alpha$ breaks down in this situation
- Finite-volume effects can produce a "gap" with $\rho(0) = 0$
  This is a different breakdown of the scaling, leading to $\gamma_{\text{eff}} \searrow 0$
- Both of these effects are unphysical
  and we remove the finite-volume transients from most γ_{eff} plots

Backup: $\gamma_{\text{eff}}(\lambda)$ for QCD-like $N_F = 4$

Fit $\nu(\lambda) \propto \lambda^{1+\alpha}$ in a limited range of λ to find $1 + \gamma_{\text{eff}}(\lambda) = \dfrac{4}{1+\alpha(\lambda)}$



1000 eigenvalues on each volume

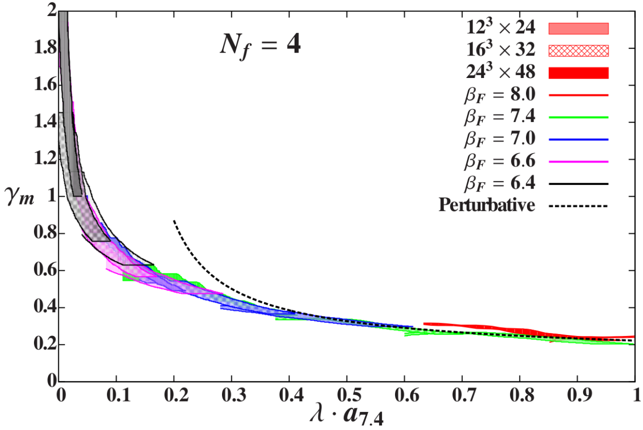
Fit ranges included in error bands

Only retain results free from finite-volume effects

$m = 0$ except for chirally broken systems at $\beta_F = 6.6$ and 6.4 where $\gamma_{\text{eff}} \nearrow 2$ becomes unphysically large

Backup: Rescaled $\gamma_{\text{eff}}(\lambda)$ for QCD-like $N_F = 4$

- Rescale $\lambda \to \left(\frac{a_{7.4}}{a}\right)^{1+\gamma_{\text{eff}}(\lambda)} \lambda$ to plot with constant lattice spacing
- Relative lattice spacings from gradient flow & MCRG matching
- Match to one-loop perturbation theory at $\lambda \cdot a_{7.4} = 0.8$



Universal curve from χSB to asymp. freedom

Strong test of method & control over systematics