Lattice Strong Dynamics for the LHC

WW Scattering Parameters via Pseudoscalar Phase Shifts

David Schaich, 9 February 2012

arXiv:1201.3977 (LSD Collaboration)

Broad Outline

- LSD Philosophy and Program
- Exploring WW scattering from pion scattering on the lattice:
  Motivation Relations Results

Lattice Strong Dynamics Collaboration

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Performing non-perturbative studies of strongly interacting theories likely to produce observable signatures at the Large Hadron Collider

Setting the scene

At the perturbative level, familiar asymptotically free theories are

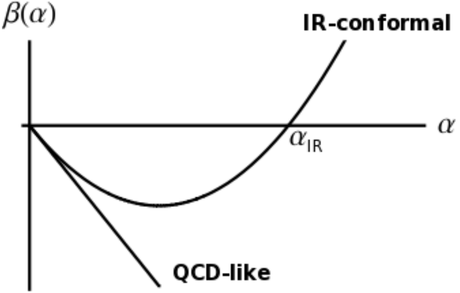
- QCD-like, with spontaneously broken chiral symmetry
- IR-conformal, with flow to an IR fixed point

$$\beta(\alpha) \equiv \frac{\partial\alpha}{\partial(\log\mu^2)} = -\beta_0 \frac{\alpha^2}{4\pi} - \beta_1 \frac{\alpha^3}{(4\pi)^2}$$

$$\beta_0 = \frac{11}{3} N_c - \frac{2}{3} N_f > 0$$

$$\beta_1 = \frac{34}{3} N_c^2 - \left[\frac{13}{3} N_c - \frac{1}{N_c}\right] N_f$$

(for fermions in fundamental rep.)



What is the range of possible behavior of strongly-coupled systems?

Goals of lattice gauge theory beyond QCD

What is the range of possible behavior of strongly-coupled systems?

Phenomenology

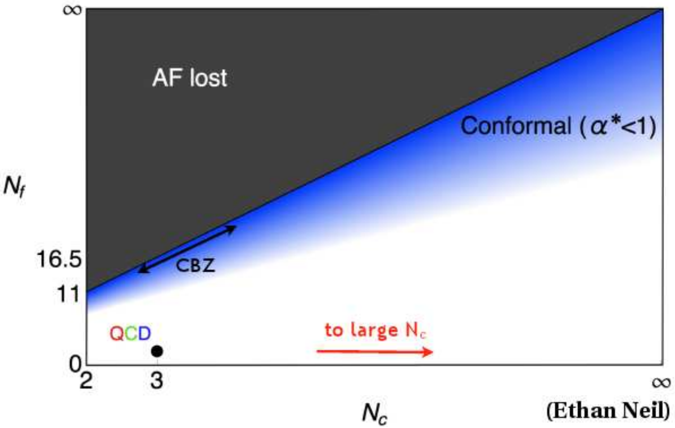
- How to tell whether or not the Higgs is composite?
- Observable signatures of walking/non-QCD dynamics:
  - Spectrum; S parameter; **WW scattering**; dark matter; ...
  - Anomalous dimensions if (at least approximately) IR-conformal

Theory

- What is the extent of the conformal window?
- Do theories "walk" near the edge of the conformal window?
- Lattice as tool to study generic strong interactions,
  complementing other approaches (e.g., gauge–gravity duality)

A Modest Observation

We can't explore all (or even many) conceivable models



(fermions in fundamental rep.; similar picture for other reps.)

Challenges facing lattice gauge theory beyond QCD

We can't explore all (or even many) conceivable models
Exploring *any* model beyond QCD is difficult

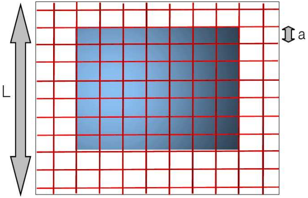
Practical difficulties

Coupling runs more slowly

$\Longrightarrow$ lattice artifacts more severe

- Strong coupling in IR (L)
  $\longrightarrow$ strong-coupling artifacts in UV (a)
- Weak coupling in UV (a)
  $\longrightarrow$ "finite-volume effects" in IR (L)

We don't know the answer



LSD Philosophy

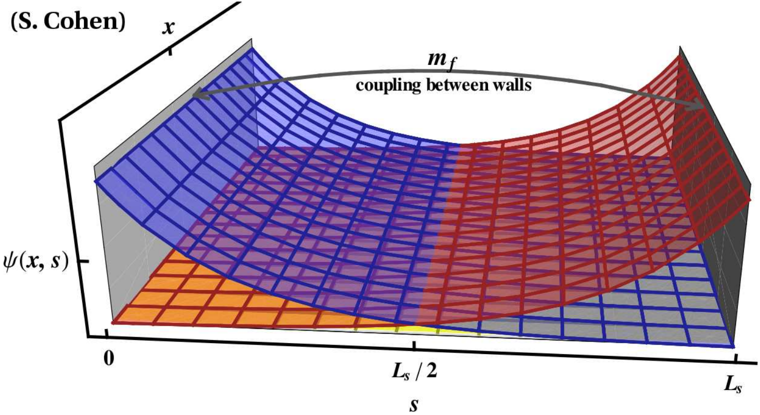
Challenges

- Large “theory space”
- Coupling runs more slowly $\Longrightarrow$ lattice artifacts more severe
- **We don’t know the answer**

Strategy

- Focus on QCD-like analyses, using lattice QCD as baseline
- Explore trends as N_f increases (this talk: $N_f = 2 \longrightarrow 6$)
- Match IR scale(s) for more direct comparison
- Use domain wall fermions for good chiral and flavor symmetries

Domain wall fermions

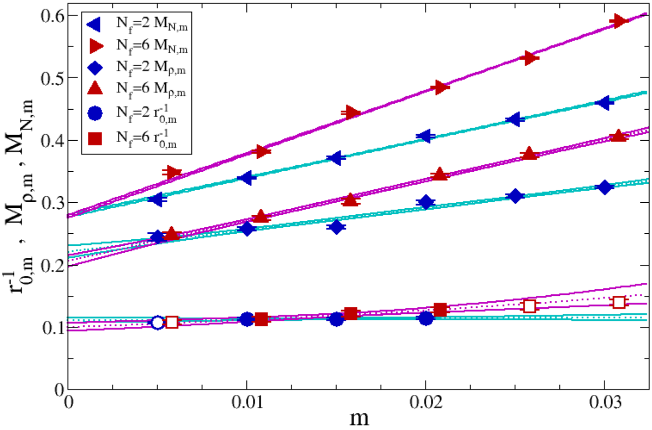


- Form a fifth dimension from L_s copies of the 4d gauge fields
- Exact chiral symmetry at finite lattice spacing in the limit $L_s \to \infty$
- At finite L_s, "residual mass" $m_{res} \ll m_f$; $\quad m = m_f + m_{res}$
- $L_s = 16$: **significant computational expense**

∼300M core-hours on clusters and supercomputers

Livermore Nat'l Lab, NSF Teragrid, USQCD (DoE), BU

Matching IR scales for more direct comparison



- Lattice spacing $a \approx 1/(5M_{V0})$ rather small $(M_{V0} = \lim_{m\to 0} M_V)$
- Even with large lattices ($32^3 \times 64 \times 16$), volumes are small
- Need relatively heavy pions to fit in box, $0.5 \lesssim M_P/M_{V0} \lesssim 1.5$

Summary of our situation

Compared to state-of-the-art lattice QCD...

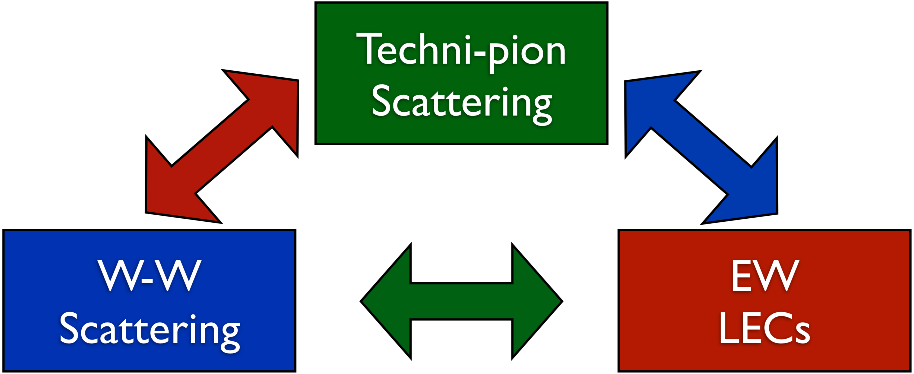
- Our physical volume is rather small
  $\longrightarrow$ significant systematic "finite-volume effects"
- Our pion masses are rather large
  $\longrightarrow$ difficult to extract information about chiral regime
- Our lattice action is computationally expensive
  $\longrightarrow$ we can only obtain limited statistics
- (For $N_f = 10$ we still have trouble with strong-coupling lattice artifacts)

Our calculations are exploratory, aiming for 10–20% uncertainties
Comparable to the state of lattice QCD 10–15 years ago

The point

Performing QCD-like analyses in lattice gauge theories
beyond QCD is **hard**, but not impossible

WW scattering from the lattice: The Big Picture



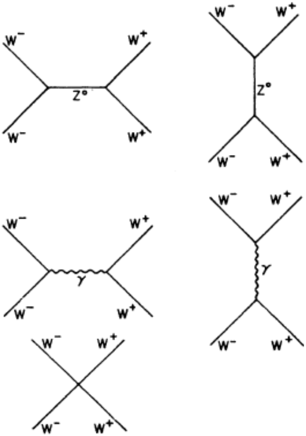
Our calculation involves five of the concepts in this picture

(and we'll see why we can't use the sixth)

I will gleefully gloss over details when they get too complicated

Why WW Scattering

Tree-level longitudinal
WW scattering amplitude grows $\sim E_{CM}^2/v^2$,
violating unitarity around TeV scale



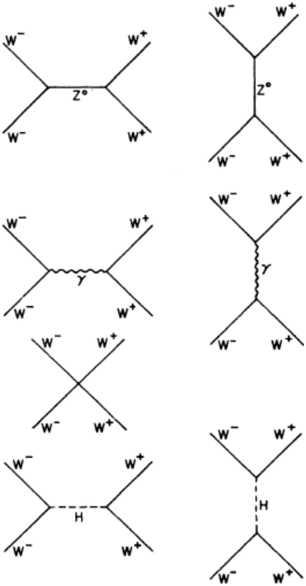
Why WW Scattering

Tree-level longitudinal
WW scattering amplitude grows $\sim E_{CM}^2/v^2$,
violating unitarity around TeV scale

Cured by Higgs boson or new physics

WW scattering guaranteed
to contain information about EWSB
Most direct probe (though **not** easiest)

Target: higher-order BSM contributions



How WW Scattering: a tale of two EFTs

Chiral Effective Field Theories

Describe dynamics of Nambu–Goldstone bosons (NGBs) (pions) resulting from spontaneous symmetry breaking

Low-energy description valid up to energies $\sim 4\pi f$, where f is the symmetry-breaking scale

Example: chiral symmetry breaking in QCD

$$SU(2)_L \times SU(2)_R \longrightarrow SU(2)_V$$

$(f = f_\pi)$ results in **hadronic chiral lagrangian**

For WW scattering, we need **electroweak chiral lagrangian** from

$$SU(2)_L \times U(1)_Y \longrightarrow U(1)_{em}$$

$(f = v)$

Electroweak chiral lagrangian

Higgs sector (at least approximately) respects a "custodial" symmetry

$$SU(2)_L \times SU(2)_C \longrightarrow SU(2)_c$$

(violations would produce $M_W^2 \neq M_Z^2 \cos^2\theta_w$)

A bit of formalism

NGBs wrapped up in $U = \exp\left[i\pi^a(x)\tau^a/v\right]$

Transformation: $U \longrightarrow LUR^\dagger$ with $L \in SU(2)_L$ and $R \in SU(2)_C$

Covariant derivative: $D_\mu U = \partial_\mu U + ig_2 \frac{\tau_a}{2} W_\mu^a U - ig_1 U \frac{\tau_3}{2} B_\mu$

Leading order $\mathcal{L}_0 = \frac{v^2}{4} \mathrm{tr}\left[(D_\mu U)^\dagger (D^\mu U)\right] - \frac{1}{2}\mathrm{tr}\left[W_{\mu\nu}W^{\mu\nu}\right] - \frac{1}{4}B_{\mu\nu}B^{\mu\nu}$

WW scattering in the electroweak chiral lagrangian

Custodial symmetry cleans up seven terms allowed by $SU(2)_L \times U(1)_Y$

Remaining terms contributing to WW scattering:

$$\begin{aligned}
\mathcal{L}_{WW} = {} & \frac{v^2}{4} \mathrm{tr}\left[(D_\mu U)^\dagger (D^\mu U)\right] - g_2^2 \mathrm{tr}\left[W_\mu, W_\nu\right]^2 \\
& + 2ig_2 \mathrm{tr}\left[(\partial_\mu W_\nu - \partial_\nu W_\mu)\left[W_\mu, W_\nu\right]\right] + \frac{1}{2}\alpha_1 g_1 g_2 B_{\mu\nu} \mathrm{tr}\left[U\tau_3 U^\dagger W^{\mu\nu}\right] \\
& + \frac{1}{2} i\alpha_2 g_1 B_{\mu\nu} \mathrm{tr}\left[U\tau_3 U^\dagger \left[(D^\mu U) U^\dagger, (D^\nu U) U^\dagger\right]\right] \\
& + i\alpha_3 g_2 \mathrm{tr}\left[W_{\mu\nu}\left[(D^\mu U) U^\dagger, (D^\nu U) U^\dagger\right]\right] \\
& + \alpha_4 \left(\mathrm{tr}\left[(D_\mu U) U^\dagger (D_\nu U) U^\dagger\right]\right)^2 + \alpha_5 \left(\mathrm{tr}\left[(D_\mu U) U^\dagger (D^\mu U) U^\dagger\right]\right)^2
\end{aligned}$$

LEP: low-energy constants α_1, α_2 and α_3 are negligible

Constrained by M_W, M_Z, and anomalous three-gauge-boson vertices

Relation to hadronic chiral lagrangian

In the limit $g_1, g_2 \longrightarrow 0$,

$$\mathcal{L}_{WW} \to \frac{v^2}{4}\text{tr}\left[\partial_\mu U^\dagger \partial^\mu U\right] + \alpha_4 \left(\text{tr}\left[\partial_\mu U^\dagger \partial_\nu U\right]\right)^2 + \alpha_5 \left(\text{tr}\left[\partial_\mu U^\dagger \partial^\mu U\right]\right)^2$$

This **is** the massless two-flavor hadronic chiral lagrangian

$f_\pi \to v$; $\ell_1 \to 4\alpha_5 + \mathcal{O}(g_2)$ and $\ell_2 \to 4\alpha_4 + \mathcal{O}(g)$.

Goal: calculate ℓ_1 and ℓ_2 on the lattice to find α_4 and α_5

Two-flavor result: $\alpha_4 + \alpha_5 = \left(3.34 \pm 0.17^{+0.08}_{-0.71}\right) \times 10^{-3} +$ caveats

Unitarity bounds [arXiv:hep-ph/0604255]:

$$\alpha_4 + \alpha_5 \geq 1.14 \times 10^{-3} \qquad \alpha_4 \geq 0.65 \times 10^{-3}$$

Expected LHC bounds [arXiv:hep-ph/0606118]: (100/fb at 14 TeV)

$$-7.7 < \alpha_4 \times 10^3 < 15 \qquad -12 < \alpha_5 \times 10^3 < 10$$

Now things start to get complicated

Complications from going beyond QCD

Our goal is to go **beyond** $N_f = 2$ QCD (scaled up $f_\pi \to v$)

For general (massless) N_f, we have $N_f^2 - 1$ NGBs from

$$SU(N_f)_L \times SU(N_f)_R \longrightarrow SU(N_f)_V$$

Resulting chiral lagrangian has many more low-energy constants L, but we can relate $\ell_1 = -2L_0 + 4L_1 + 2L_3$ $\ell_2 = 4L_0 + 4L_2$

Only three massless NGBs eaten in Higgs mechanism,
$N_f^2 - 4$ must be massive pseudo-NGBs

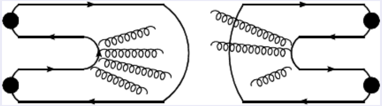
To recover electroweak chiral lagrangian,
integrate out pseudo-NGBs along with other TeV-scale physics
$\Longrightarrow \alpha_4$ and α_5 pick up M_P dependence

Restrictions from working on the lattice

"Simplest, cleanest, and best" scattering process

Restrict to S-wave scattering of identical charged pseudoscalars ("$I = 2$" or "maximal isospin" scattering)

- Other isospin channels (e.g., $I = 0$) involve quark-line-disconnected diagrams



  Extremely expensive to evaluate on lattice
- Other spin channels (e.g., D-wave) have smaller signals, require higher precision

Complications from working in euclidean spacetime

Usual (Lehmann, Symanzik and Zimmermann) scattering formalism does not hold in euclidean spacetime

No asymptotically non-interacting "in" and "out" states

(Maiani and Testa, 1990)

In a finite volume, measure M_P and E_{PP}
(projecting correlators onto zero momentum for S-wave scattering)

Access scattering phase shift δ from energy shift ΔE_{PP} (Lüscher, 1986)

$$\Delta E_{PP} = E_{PP} - 2M_P = 2\sqrt{|\vec{k}|^2 + {M_P}^2} - 2M_P$$

$$|\vec{k}| \cot \delta = \frac{1}{\pi L} \left[\sum_{\vec{\jmath} \neq 0}^{\Lambda_J} \frac{1}{|\vec{\jmath}|^2 - |\vec{k}|^2 L^2/(4\pi^2)} - 4\pi \Lambda_j \right]$$

(Λ_j regularizes zeta function in the UV)

Scattering length from scattering phase shift

Having measured M_P and E_{PP} to extract $|\vec{k}|\cot\delta$,
the S-wave effective range expansion gives the scattering length a_{PP}

$$|\vec{k}|\cot\delta = \frac{1}{a_{PP}} + \frac{1}{2}M_P^2 r_{PP}\left(\frac{|\vec{k}|^2}{M_P^2}\right) + \mathcal{O}\left(\frac{|\vec{k}|^2}{M_P^2}\right)^2$$

$$a_{PP} \approx \frac{1}{|\vec{k}|\cot\delta} \qquad \text{for } |\vec{k}|^2 \ll M_P^2$$

Approximation invalid above inelastic threshold $\Delta E_{PP} > 2M_P$

Lattice scattering inherently low-energy

Obstructs direct connection between NGB and $W_L W_L$ scattering:
Equivalence Theorem valid at high energies $s/M_W^2 \to \infty$

Chiral expansion for scattering length

Chiral perturbation theory predicts *m*-dependence of scattering length

$$M_P a_{PP} = -\frac{M^2}{16\pi^2 F^2}\left\{1 + \frac{M^2}{16\pi^2 F^2}\left[b_{PP} - 2\frac{N_f - 1}{N_f^2} + \frac{2 - N_f + 2N_f^2 + N_f^3}{N_f^2}\log\left(\frac{M^2}{\mu^2}\right)\right]\right.$$

(renormalization scale $\mu \longrightarrow F$)

Expression involves low-energy constants

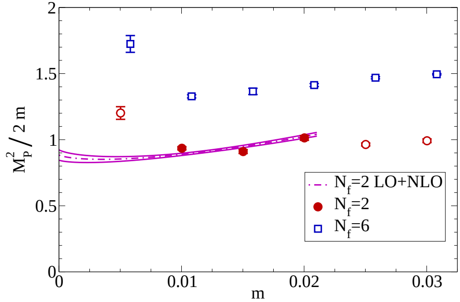
$$F \equiv \lim_{m\to 0} F_P \qquad M^2 \equiv 2m \lim_{m\to 0} \left\langle \overline{\psi}\psi \right\rangle / F^2 = 2mB$$

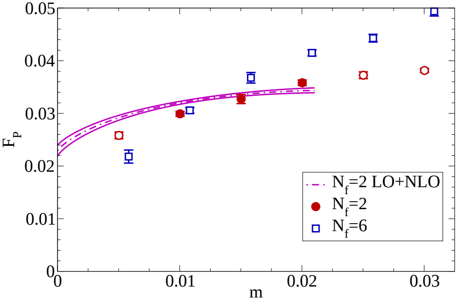
$$b_{PP} = -256\pi^2 \left[(N_f - 2)\left\{L_4 - L_6\right\} + L_0 + 2L_1 + 2L_2 + L_3\right]$$

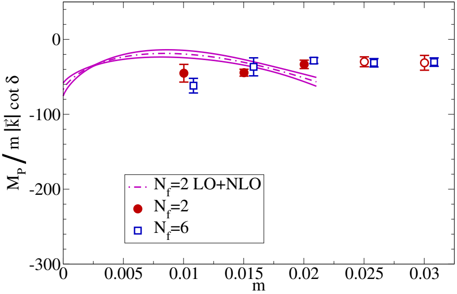
For $N_f = 2$, $b_{PP} \longrightarrow -128\pi^2 \left[\ell_1 + \ell_2\right]$

Similar exponsions for M_P^2, F_P and $\left\langle \overline{\psi}\psi \right\rangle$ (with LECs b_M, b_F, b_C)

Joint chiral fit to $M_P^2/2m$; F_P; $\langle\overline{\psi}\psi\rangle$; and $M_P/m|\vec{k}|\cot\delta$







$\langle\overline{\psi}\psi\rangle$ very boring, not shown

Only $N_f = 2$ fit feasible

Fit range restricted to

$0.01 \leq m_f \leq 0.02$

(solid points)

$\chi^2/\mathbf{dof} = \mathbf{83/6}$

Translating $N_f = 2$ $\pi\pi$ results to WW scattering

- Relate general-N_f chiral expansions above to electroweak LECs
$$b_{PP}, L_i \longrightarrow \ell_1, \ell_2 \longrightarrow \alpha_4, \alpha_5$$
- Remove eaten modes from spectrum
One-loop standard model subtraction introduces Higgs mass M_H

$$\alpha_4 + \alpha_5 = \left(3.34 \pm 0.17^{+0.08}_{-0.71}\right) \times 10^{-3} - \frac{1}{128\pi^2}\left[\log\left(\frac{M_H^2}{v^2} + \mathcal{O}(1)_{SM}\right)\right]$$

(dominant systematic error from chiral fit range)

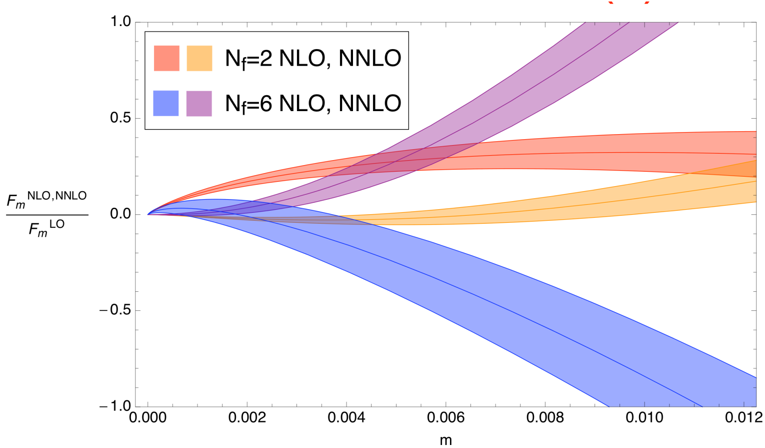
Unitarity bounds [arXiv:hep-ph/0604255]:

$$\alpha_4 + \alpha_5 \geq 1.14 \times 10^{-3} \qquad\qquad \alpha_4 \geq 0.65 \times 10^{-3}$$

Expected LHC bounds [arXiv:hep-ph/0606118]: (100/fb at 14 TeV)

$$-7.7 < \alpha_4 \times 10^3 < 15 \qquad\qquad -12 < \alpha_5 \times 10^3 < 10$$

Why we can only fit $N_f = 2$



Next-to-leading and higher order terms in chiral perturbation theory increase with N_f at fixed m

Compare $N_f = 6$ by reorganizing chiral expansion

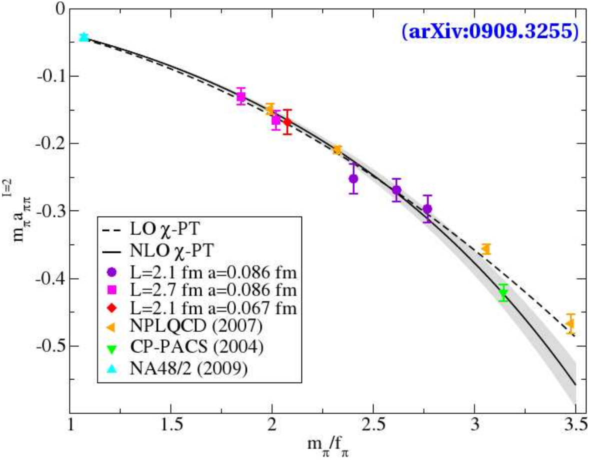
Solve chiral expansions for measured M_P and F_P
replace low-energy constants M^2 and F by $x \equiv M_P^2/F_P^2$:

$$M_P a_{PP} = -\frac{x}{16\pi^2}\left\{1 + \frac{x}{16\pi^2}\left[b'_{PP} - 2\frac{N_f - 1}{N_f^2} + 2\frac{1 - N_f + N_f^2}{N_f^2}\log\left(\frac{M_P^2}{\mu^2}\right)\right]\right\}$$

Now $b'_{PP} = -256\pi^2\left[L_0 + 2L_1 + 2L_2 + L_3 - 2L_4 - L_5 + 2L_6 + L_8\right]$
No explicit factors of N_f in b'_{PP},
all N_f dependence due to dynamics affecting LECs L_i
Unable to untangle L_i to recover $\ell_1, \ell_2 \longrightarrow \alpha_4, \alpha_5$

Reorganized expansion controversial in QCD

Leading order is $M_P a_{PP} = -\frac{M_P^2}{16\pi F_P^2}$ (Weinberg, 1966)

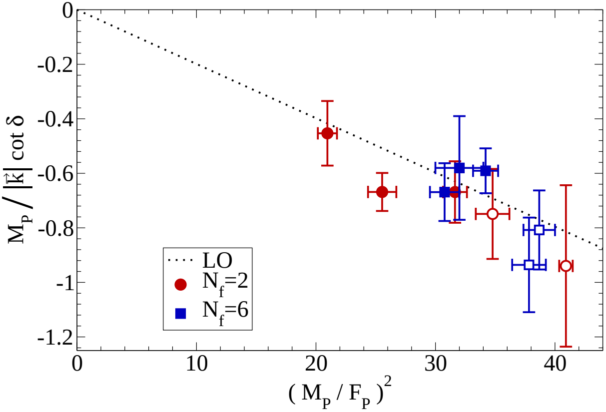


Puzzling persistence of leading-order relation

well beyond expected radius of convergence

Our results in reorganized expansion

Leading-order relation is straight line for $M_P/(|\vec{k}|\cot\delta)$ vs. M_P^2/F_P^2



Leading order continues describing data far better than expected

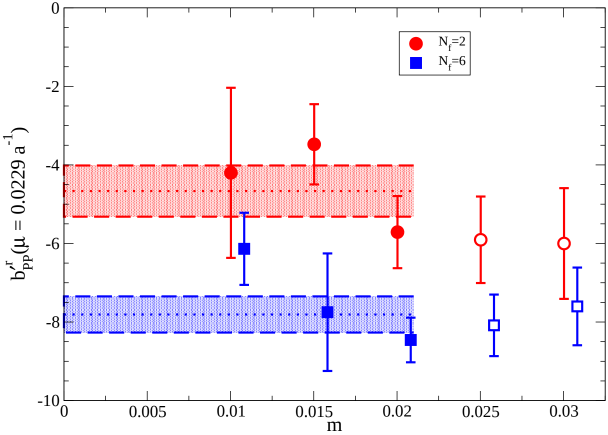
Small upward shift (somewhat less-repulsive scattering)

visible for $N_f = 6$ compared to $N_f = 2$

Comparing $N_f = 6$ to $N_f = 2$

Small shift in $M_P/(|\vec{k}|\cot\delta)$ signals large difference in LEC b'_{PP}

$N_f = 6$ LEC must cancel larger chiral log term



$b'_{PP} = -4.67 \pm 0.65^{+1.08}_{-0.05}$ (2f); $b'_{PP} = -7.81 \pm 0.46^{+1.23}_{-0.56}$ (6f)

The end – and the beginning

Results

- For two-flavor scaled-up QCD

$$\alpha_4 + \alpha_5 = \left(3.34 \pm 0.17^{+0.08}_{-0.71}\right) \times 10^{-3} - \frac{1}{128\pi^2}\left[\log\left(\frac{M_H^2}{v^2} + \mathcal{O}(1)_{SM}\right)\right]$$

- $N_f = 6$ shows somewhat less repulsive NLO interaction

Definitely exploratory, and many improvements can be done or dreamt

- Separate $N_f = 2$ results for α_4 and α_5
- Untangle $N_f = 6$ LECs to access α_4 and α_5

Strategies

Besides the obvious (larger volumes, lighter masses, more statistics):
D-wave scattering; pion form factors; higher-order expansions

Backup: LSD ensembles and measurements of S

	$N_f = 2$			$N_f = 6$		
m_f	$M_P L$	N_{cfg}	N_{meas}	$M_P L$	N_{cfg}	N_{meas}
0.010	4.4	564	564	5.4	221	882
0.015	5.3	148	444	6.6	112	414
0.020	6.4	131	131	7.8	81	324
0.025	7.0	67	268	8.8	89	267
0.030	7.8	39	154	9.7	72	259

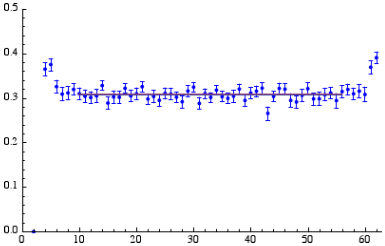
$32^3 \times 64 \times 16 \longrightarrow m_{res} \approx 3 \times 10^{-5}$ (2f); 8×10^{-4} (6f)

Backup: correlation functions and fitting

$$C_P(t) = B\cosh\left(E_{PP}t\right)$$

$$\cosh(M_P) = \frac{C_P(t+1) - C_P(t-1)}{2C_P(t)}$$

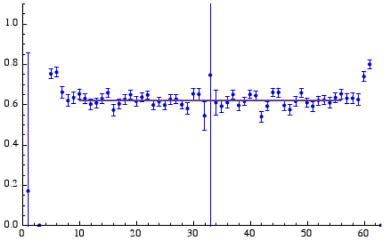
$$M_P = 0.3075(5)$$



$$C_{PP}(t) = A + B\cosh\left(E_{PP}t\right)$$

$$2\cosh(E_{PP}) = \frac{C_{PP}(t+2) - C_{PP}(t-2)}{C_{PP}(t+1) - C_{PP}(t-1)}$$

$$E_{PP} = 0.6210(10)$$



Backup: NLO chiral expansions for M_P, F_P and $\langle\overline{\psi}\psi\rangle$

For general N_f,

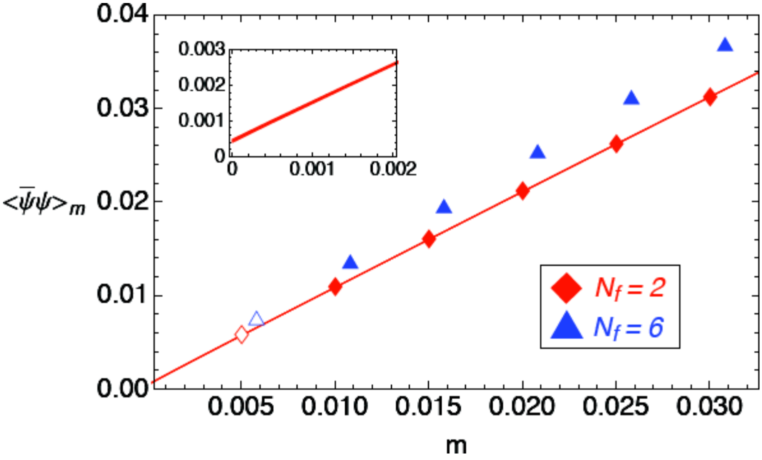
$$M_P^2 = M^2\left\{1 + \frac{M^2}{(4\pi F)^2}\left[b_M + \frac{1}{N_f}\log\left(\frac{M^2}{\mu^2}\right)\right]\right\}$$

$$F_P = F\left\{1 + \frac{M^2}{(4\pi F)^2}\left[b_F - \frac{N_f}{2}\log\left(\frac{M^2}{\mu^2}\right)\right]\right\}$$

$$\langle\overline{\psi}\psi\rangle = \frac{F^2M^2}{2m}\left\{1 + \frac{M^2}{(4\pi F)^2}\left[b_C - \frac{N_f^2-1}{N_f}\log\left(\frac{M^2}{\mu^2}\right)\right]\right\}$$

- Like b_{PP} above, b_M, b_F and b_C are all linear combinations of low-energy constants L_i
- b_C includes "contact term" $m\Lambda^2 \sim ma^{-2}$
- Like $M_P a_{PP}$ above, LECs are scale μ-dependent, but full expressions are not
- NNLO M_P^2 coefficients enhanced by N_f^2 (arXiv:0910.5424)

Backup: Chiral condensate with chiral fit



Joint NNLOχPT fit to $N_f = 2$ F_P, M_P^2, $\langle\overline{\psi}\psi\rangle$

Linear term clearly dominant